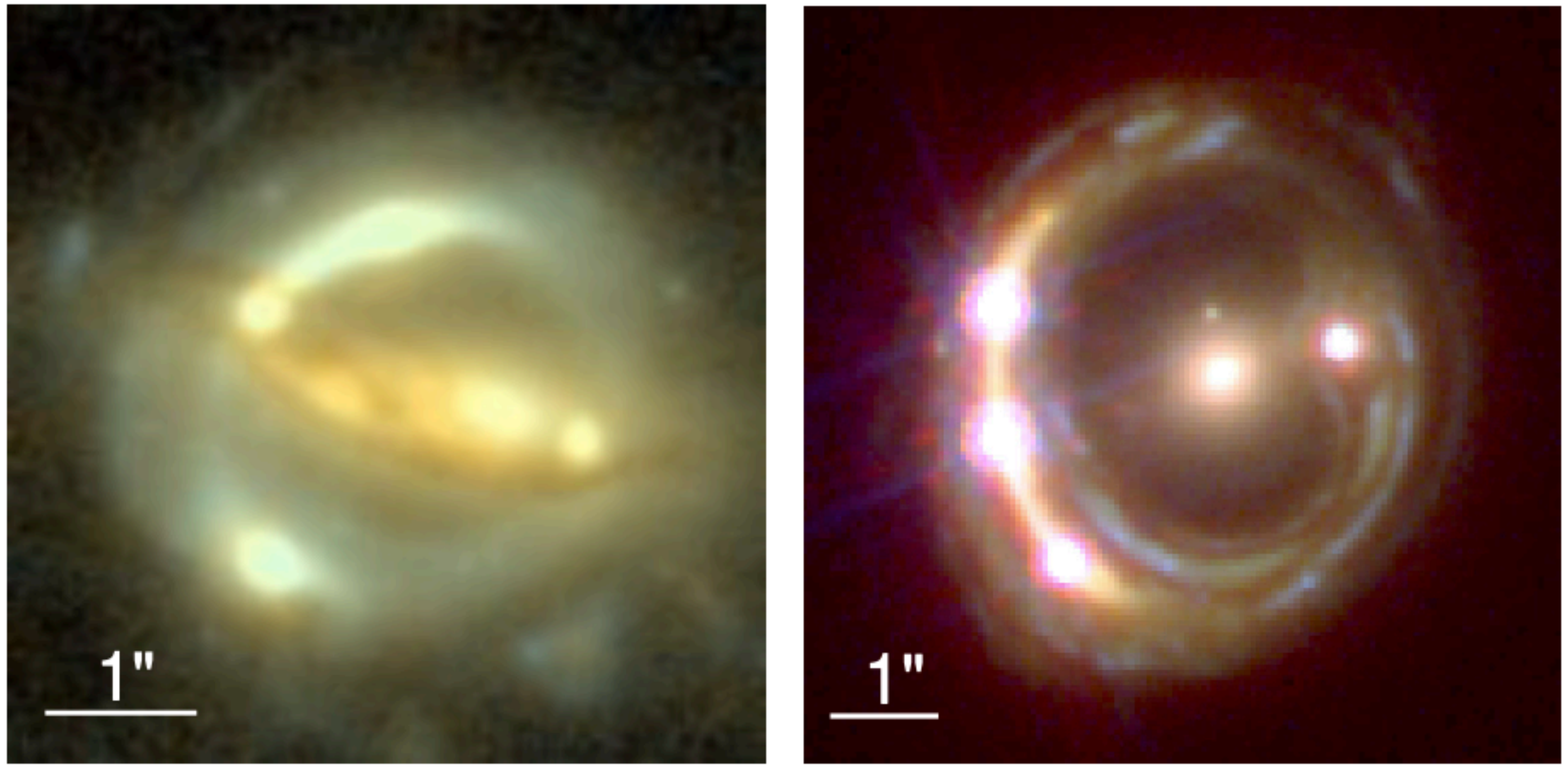
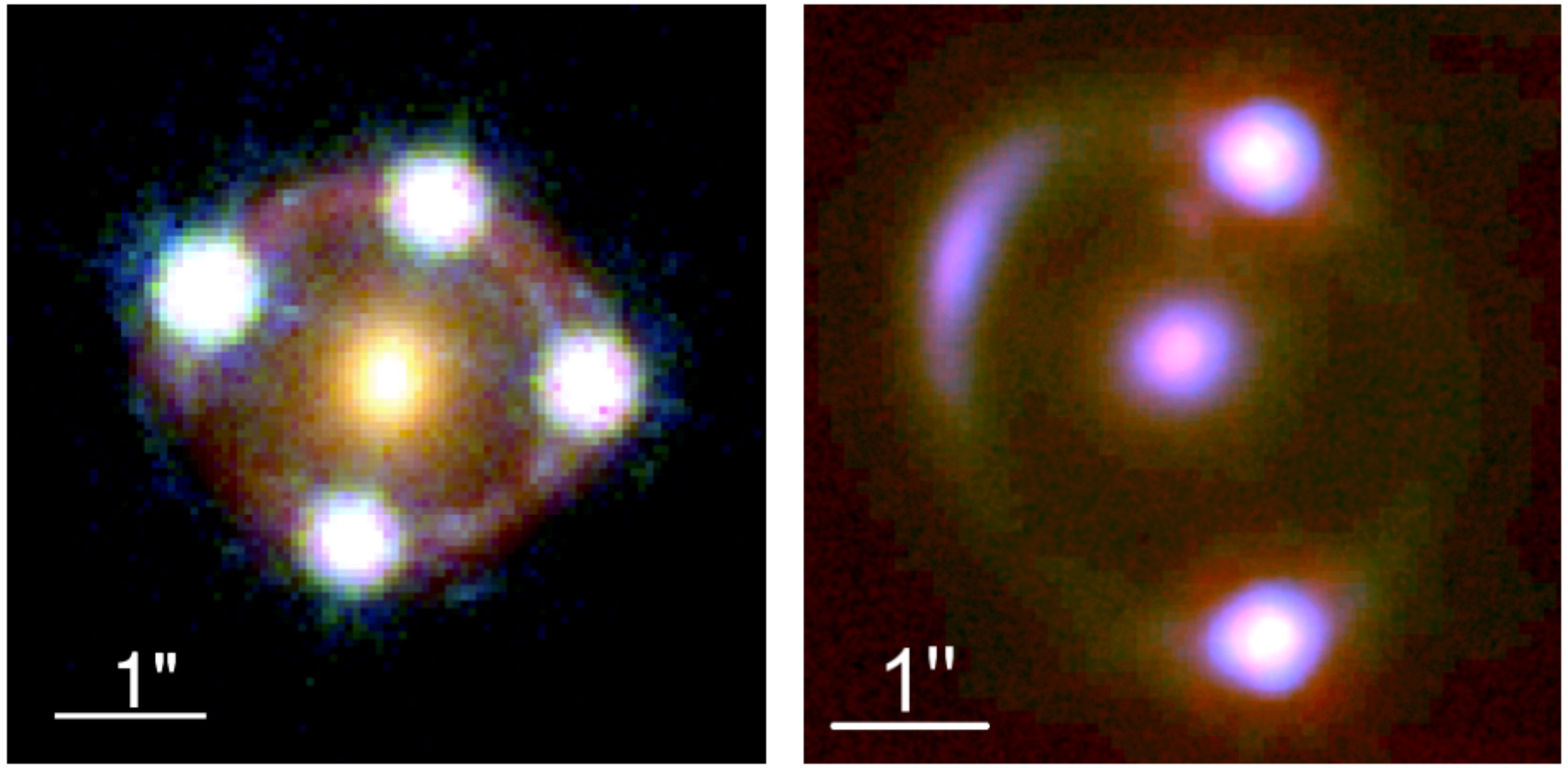
H_0 from gravitational lensing time delay

Yanhan Guo, Siyi Zhao, Jiaqi Zou, Zhuo Cheng
4.29



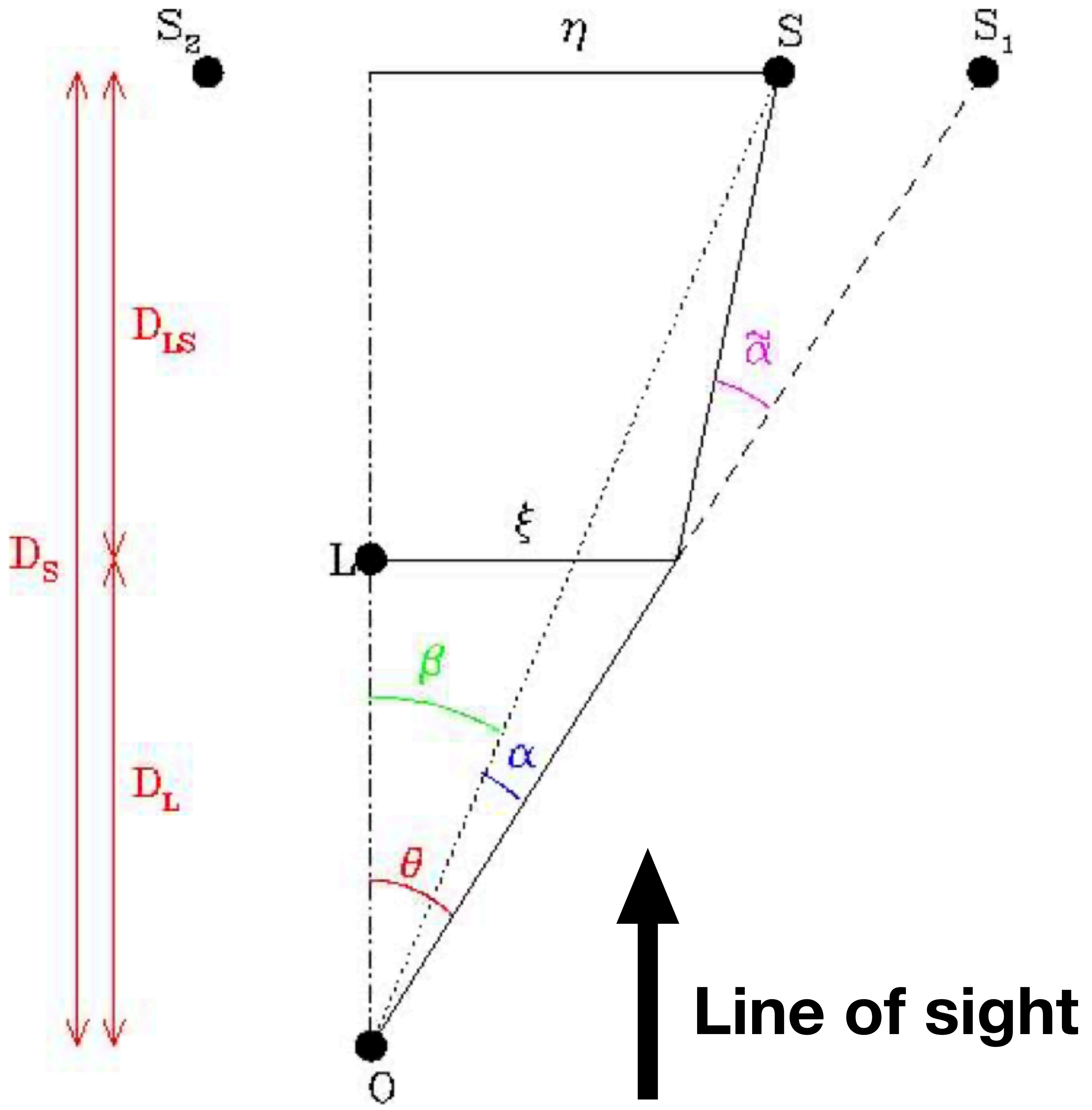
(a) B1608+656 (b) RXJ1131−1231



(c) HE 0435−1223 (d) SDSS 1206+4332

Gravitational lensing time delay

- Lensing equation: $\beta = \theta - \alpha(\theta)$
- Time delay: $\tau(\overrightarrow{\theta}, \overrightarrow{\beta}) = \frac{D_{\Delta t}}{c}[\frac{1}{2}(\overrightarrow{\theta} - \overrightarrow{\beta})^2 - \psi(\overrightarrow{\theta})]$
- Time-delay distance: $D_{\Delta t} \equiv (1 + z_L)\frac{D_L D_s}{D_{Ls}}$



Line of sight

H_0 from gravitational lensing time delay

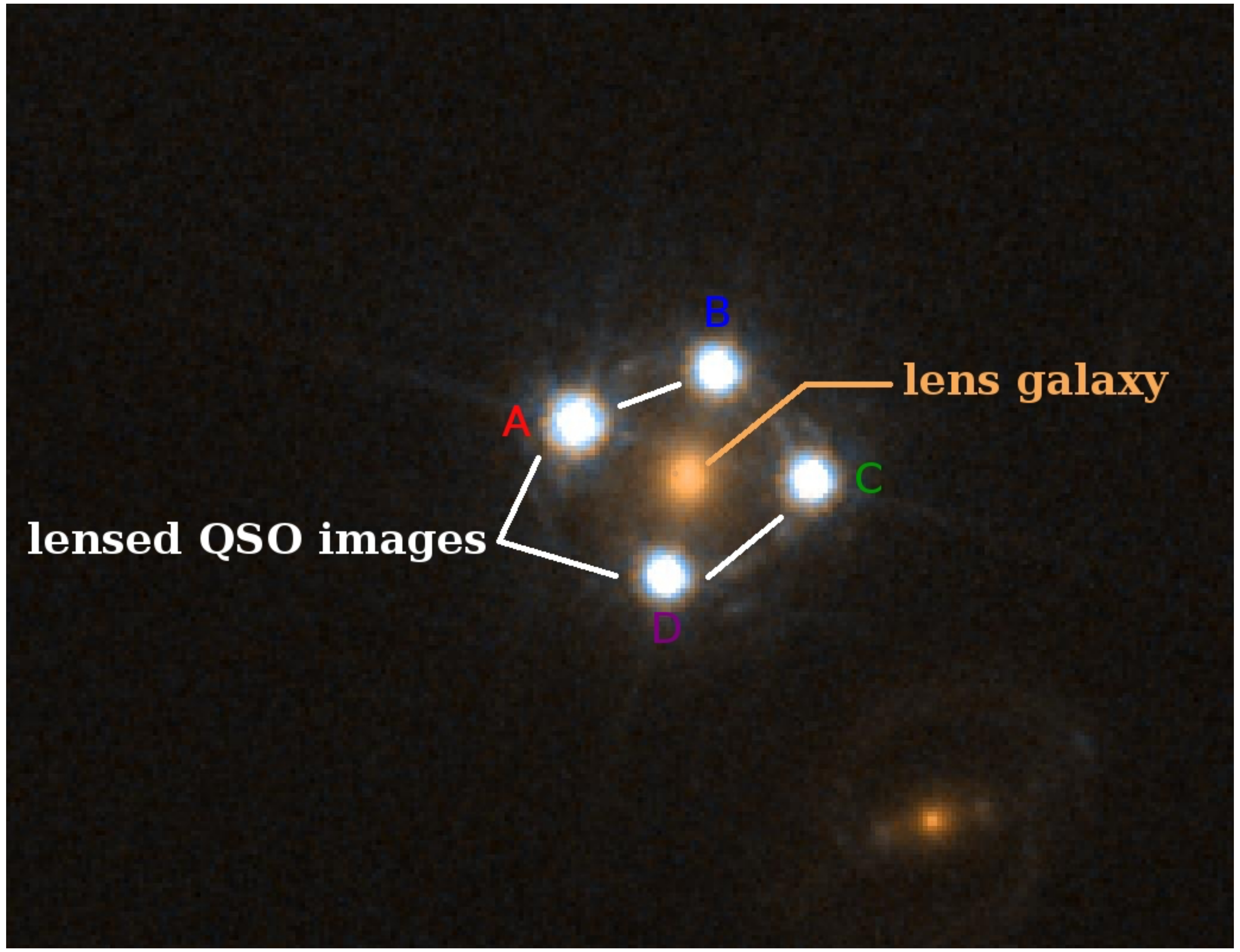
$$\tau(\overrightarrow{\theta}, \overrightarrow{\beta}) = \frac{D_{\Delta t}}{c}[\frac{1}{2}(\overrightarrow{\theta} - \overrightarrow{\beta})^2 - \psi(\overrightarrow{\theta})]$$

- Accurate time delay measurement: $\tau(\overrightarrow{\theta}, \overrightarrow{\beta})$
- Precise mass model of lense: $\psi(\overrightarrow{\theta})$ and $\overrightarrow{\beta}$

Time delay measurement

COSmological MOnitoring of GRAvItational Lenses(COSMOGRAIL)

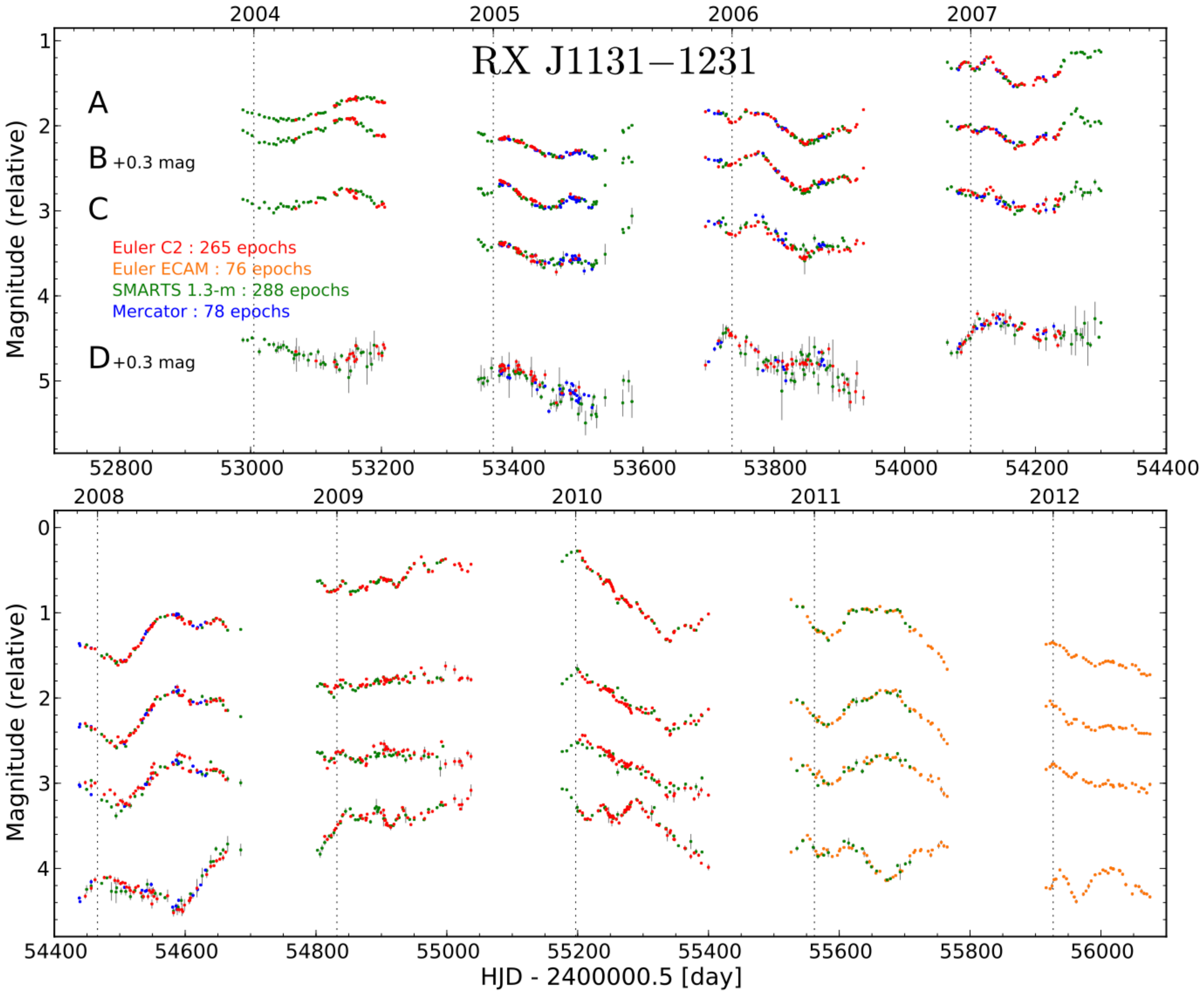
- Monitor dozens of lensed quasars, to measure time delays with an accuracy below 3%
- The most recent, and perhaps most impacting result of this project is a 2.4% determination of H_0

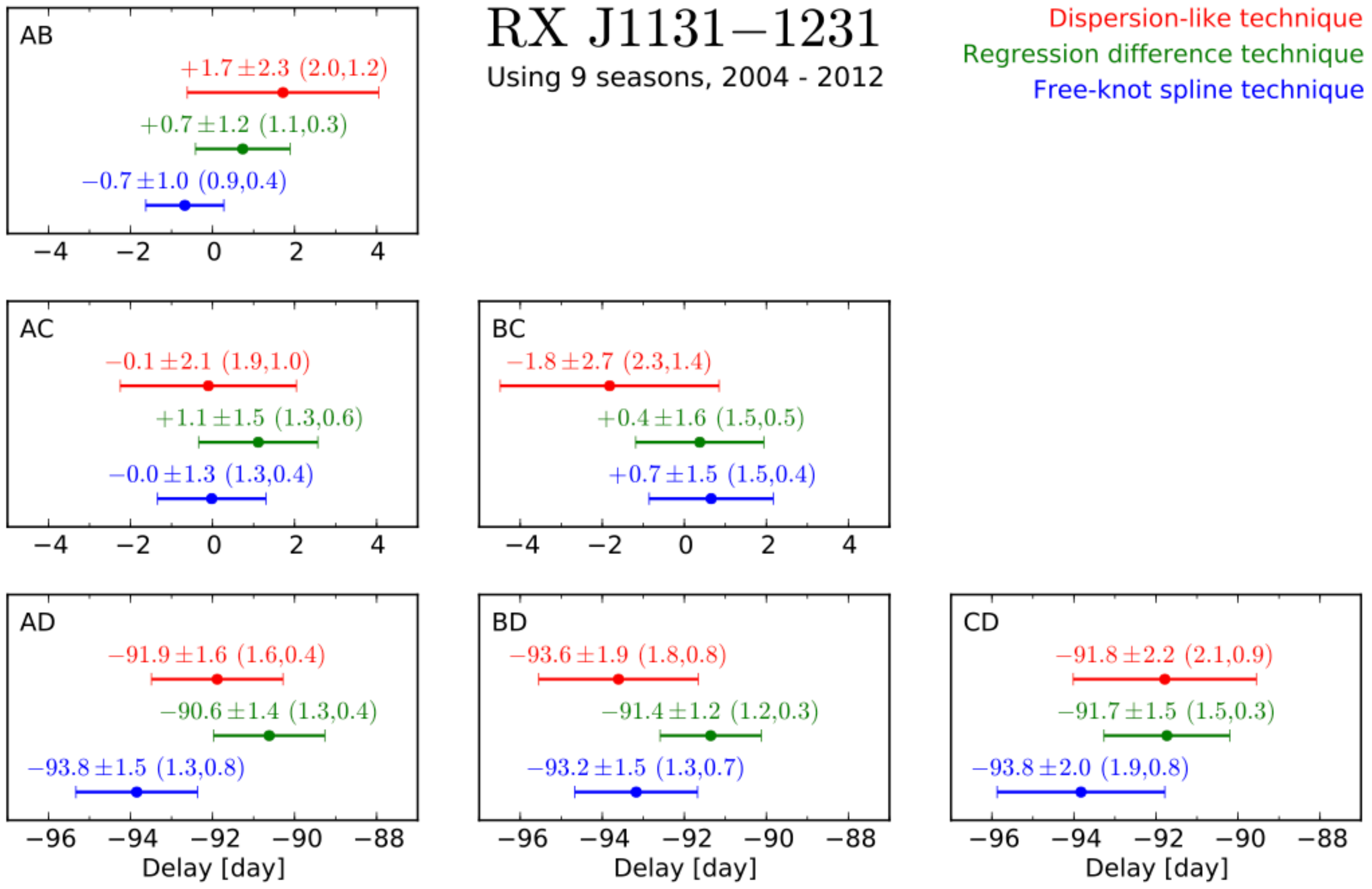


https://www.epfl.ch/labs/lastro/scientific-activities/cosmograil/

Time delay measurement

An example from COSMOGRAIL





Time delay measurement

- Long-term dedicated photometric monitoring of the systems
- Several years of monitoring are generally required to overcome microlensing variability

The effect of microlensing variability: For RX J1131−1231, Mosquera & Kochanek(2011) estimated a time scale of ≈ 11 years for the crossing of a stellar Einstein radius

Lens Modeling

$$\tau(\vec{\theta}, \vec{\beta}) = \frac{D_{\Delta t}}{c}[\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta})]$$

- **Mass profiles of lenses:**

Power law: $\rho(r) \propto r^{-n}$

Exponential Disk: $\Sigma(\theta) = \Sigma_0 \exp(-\theta/\theta_0)$

NFW Profile: $\rho(r) = \dfrac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$

- **For more accurate lens models: stellar dynamical measurements**

$$\tau(\overrightarrow{\theta}, \overrightarrow{\beta}) = \frac{D_{\Delta t}}{c}[\frac{1}{2}(\overrightarrow{\theta} - \overrightarrow{\beta})^2 - \psi(\overrightarrow{\theta})]$$

LOS Structure and External Convergence

Two types of perturbations :

- Structures that affect the lens potential significantly, which should be included in the gravitational potential $\psi(\overrightarrow{\theta})$
- Other LOS structures, which can be approximated by a κ term in time-delay distance: $D_{\Delta t} = D_{\Delta t}^{model}/(1 - \kappa_E)$

Comparison with other methods of measuring H_0

Advantages:

- No need for any primary or secondary calibrator
- No effects from the intergalactic or interstellar medium

Disadvantages:

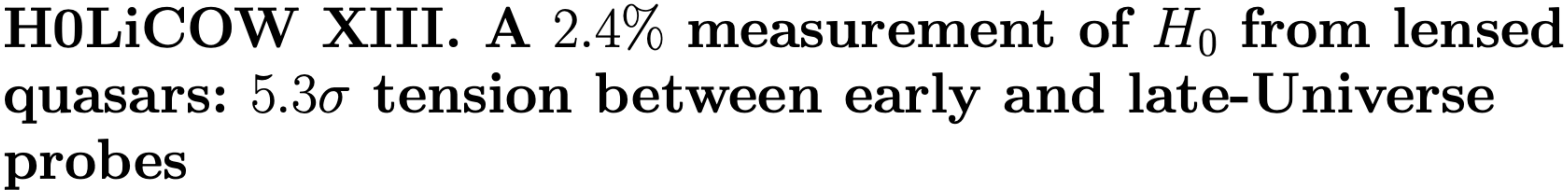
$$\tau(\overrightarrow{\theta}, \overrightarrow{\beta}) = \frac{D_{\Delta t}}{c}[\frac{1}{2}(\overrightarrow{\theta} - \overrightarrow{\beta})^2 - \psi(\overrightarrow{\theta})]$$

- Hard to accurately measure the time delay
- Inaccurate len models
- LOS Structure and External Convergence : $D_{\Delta t} = D_{\Delta t}^{model}/(1 - \kappa_E)$

error

Present state

- The H0LiCOW team measured $H_0 = 73.3^{+1.7}_{-1.8}\mathrm{km/s/Mpc}$ and reported a 2.4% precision (Wong et al. 2019)



H0LiCOW XIII. A 2.4% measurement of H_0 from lensed quasars: 5.3σ tension between early and late-Universe probes

- others have claimed that their 2.4% precision measurement may have substantially underestimated the uncertainty (Kochanek 2020, Birrer et al. 2020)

Take home message

- Gravitational lensing time delay can be used to measure H_0
- Wong et al. 2019 claimed that they find $H_0 = 73.3^{+1.7}_{-1.8}\mathrm{km/s/Mpc}$, and others have claimed that their 2.4% precision measurement may have substantially underestimated the uncertainty

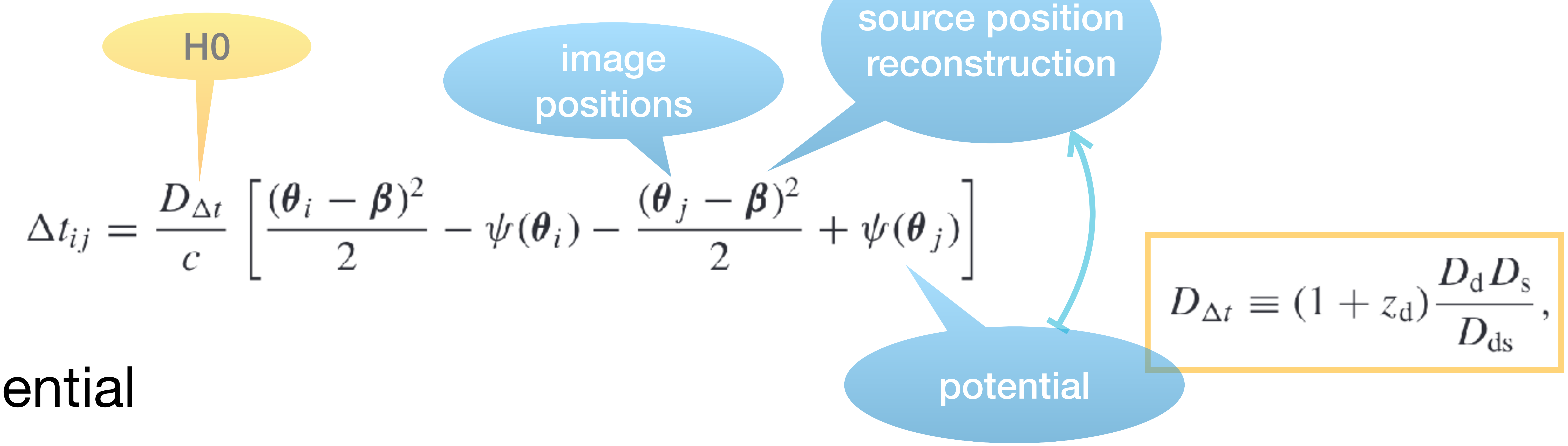
H0LiCOW 2.4% Measurement of H0 and Questions

Siyi ZHAO 2022.4

Method review

From time delay measurement to time delay distance.

- Time delay
  - Geometry
  - Gravitational potential
    - Lens model: an effective single lens
    - External convergence



$$\Delta t_{ij} = \frac{D_{\Delta t}}{c}\left[\frac{(\boldsymbol{\theta}_i - \boldsymbol{\beta})^2}{2} - \psi(\boldsymbol{\theta}_i) - \frac{(\boldsymbol{\theta}_j - \boldsymbol{\beta})^2}{2} + \psi(\boldsymbol{\theta}_j)\right]$$

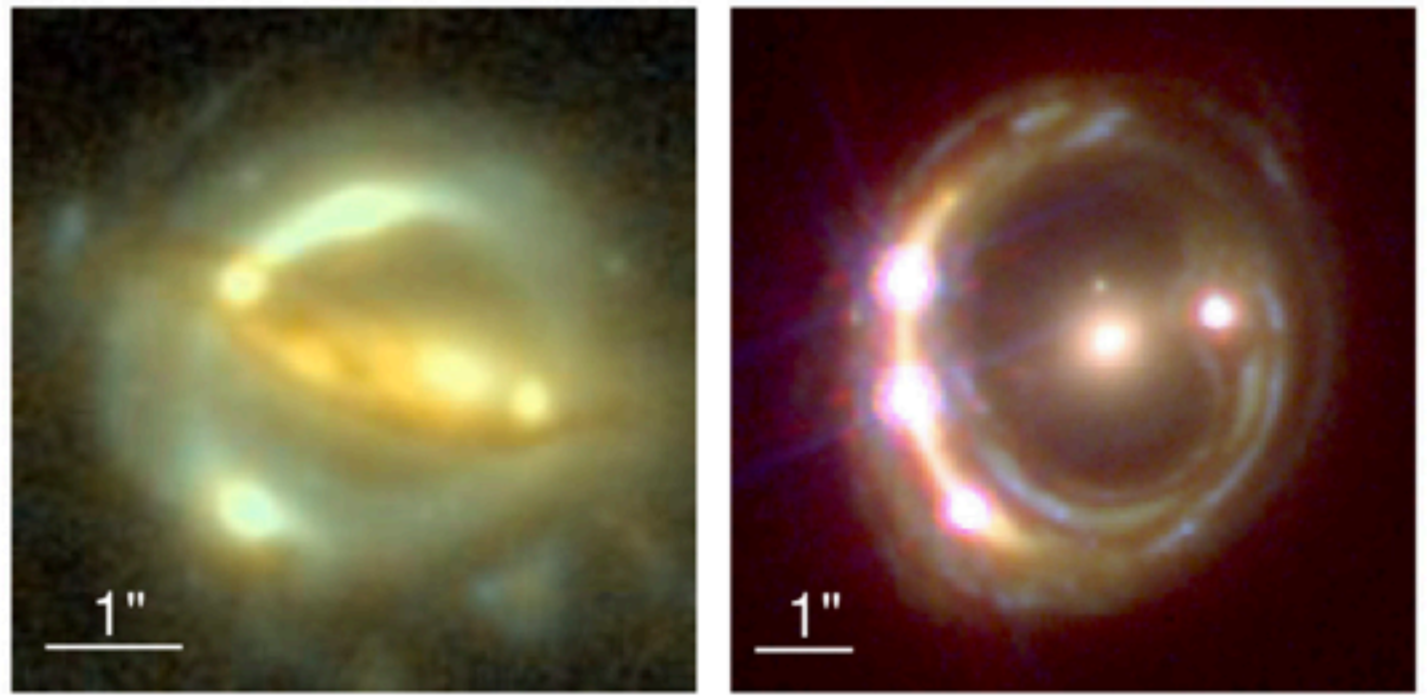
$$\boxed{D_{\Delta t} \equiv (1 + z_{\mathrm{d}})\frac{D_{\mathrm{d}} D_{\mathrm{s}}}{D_{\mathrm{ds}}},}$$

$$D_{\Delta t} = \frac{D_{\Delta t}^{\mathrm{model}}}{1 - \kappa_{\mathrm{ext}}}.$$

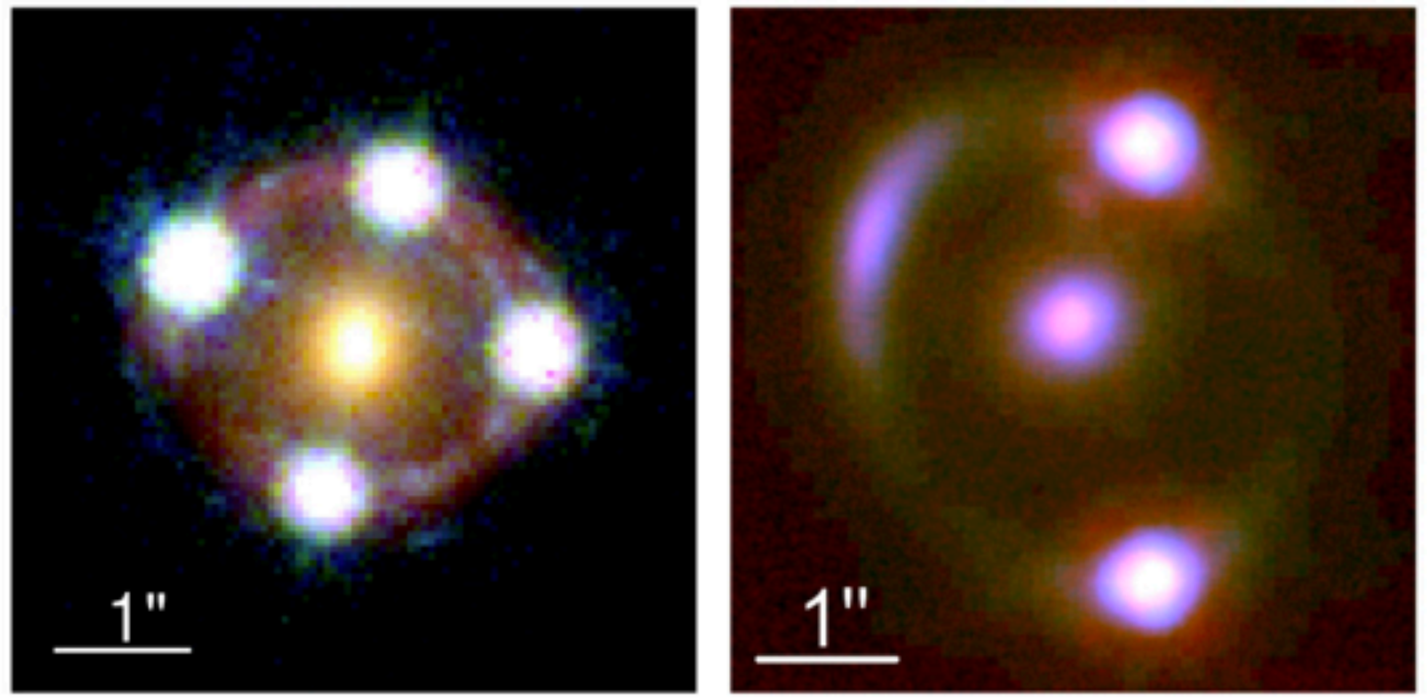
Lens model

Often a single lens plane dominates

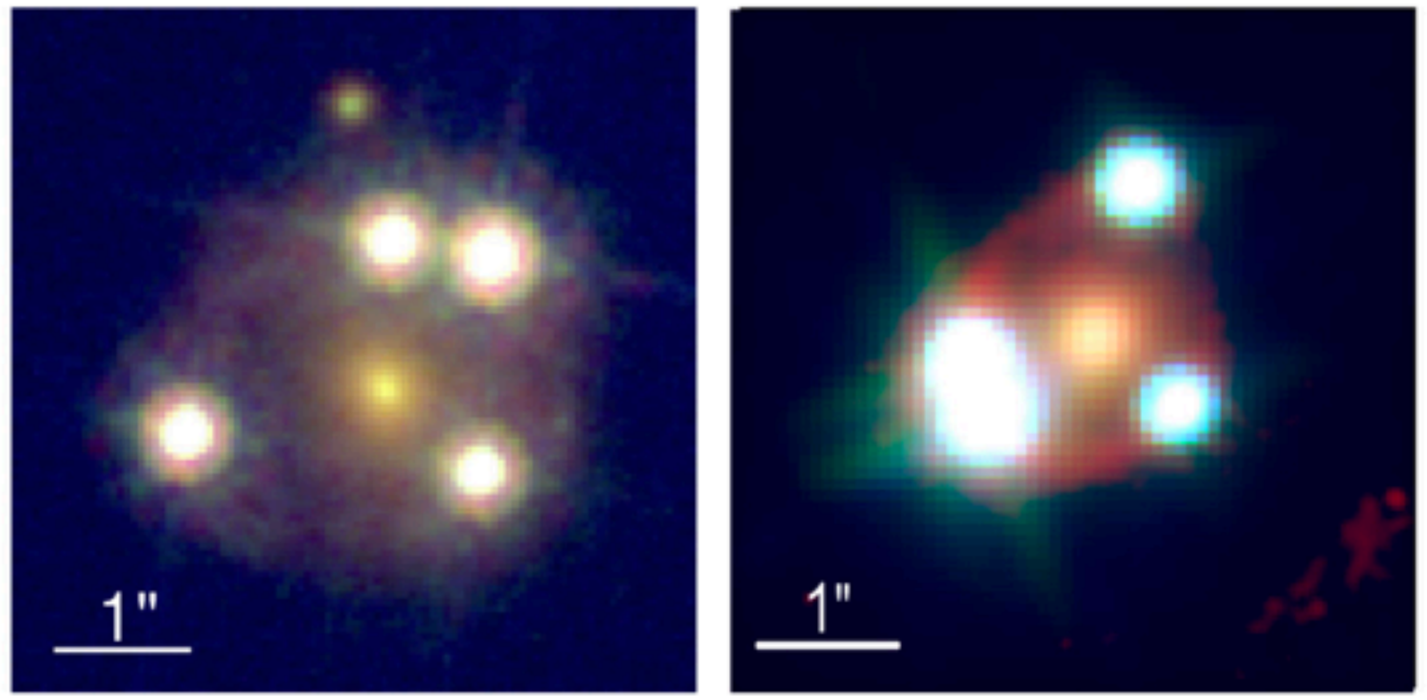
- a singular elliptical power-law model
- a **baryonic** component linked to the stellar light distribution plus an elliptical NFW halo representing the **dark matter** component
- for a complex system, B1608+656, started from the power-law model and performed **a pixelated lens potential reconstruction**



(a) B1608+656 (b) RXJ1131−1231



(c) HE 0435−1223 (d) SDSS 1206+4332



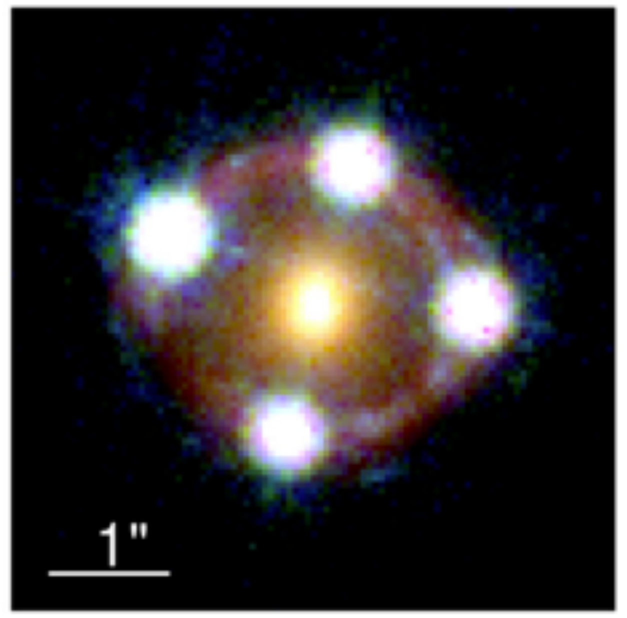
(e) WFI2033−4723 (f) PG 1115+080

H0LiCOW paper XIII (Wong et al., 2020)

Lens model

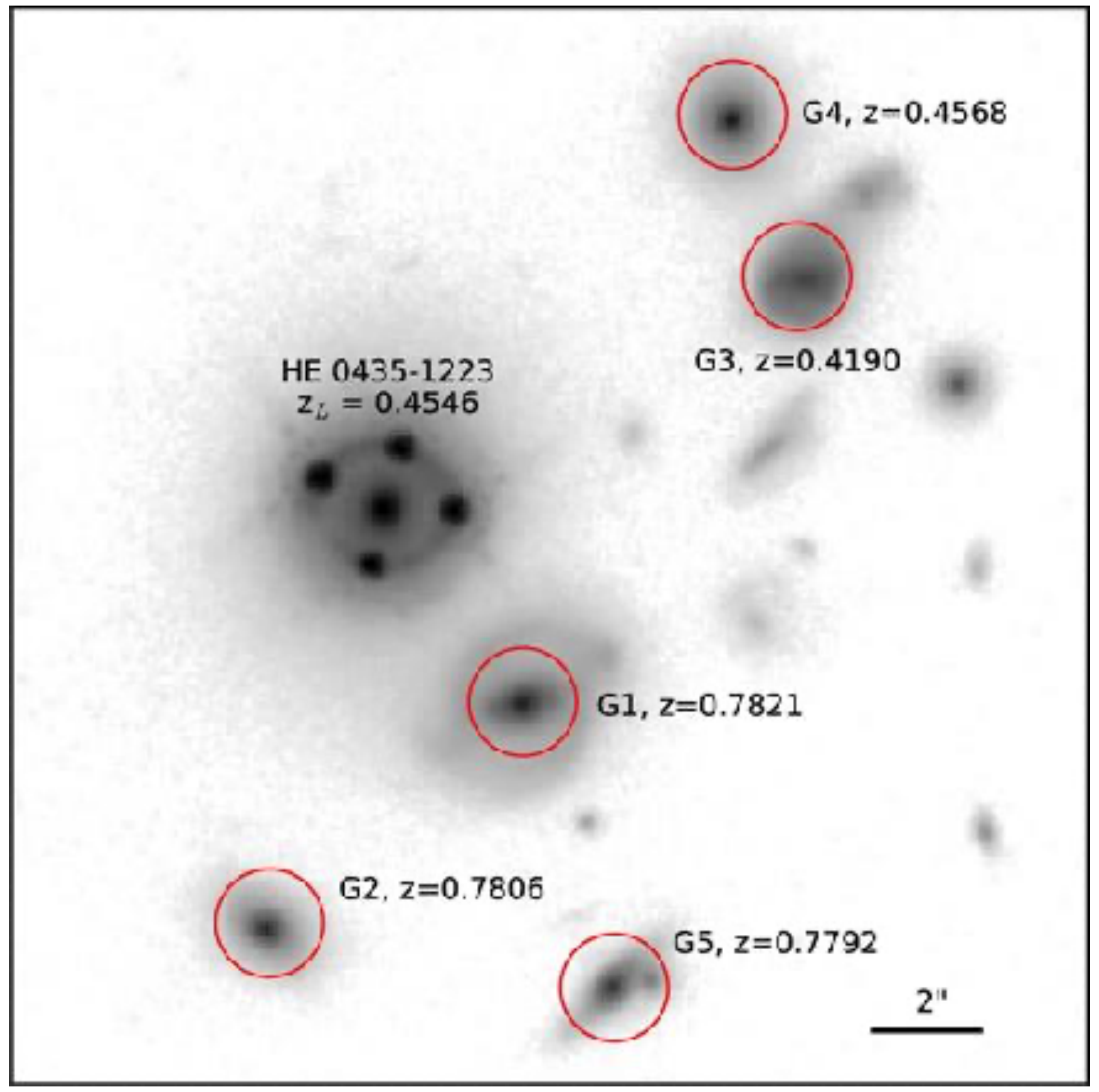
Effectively single lens

(Wong et al., 2020)

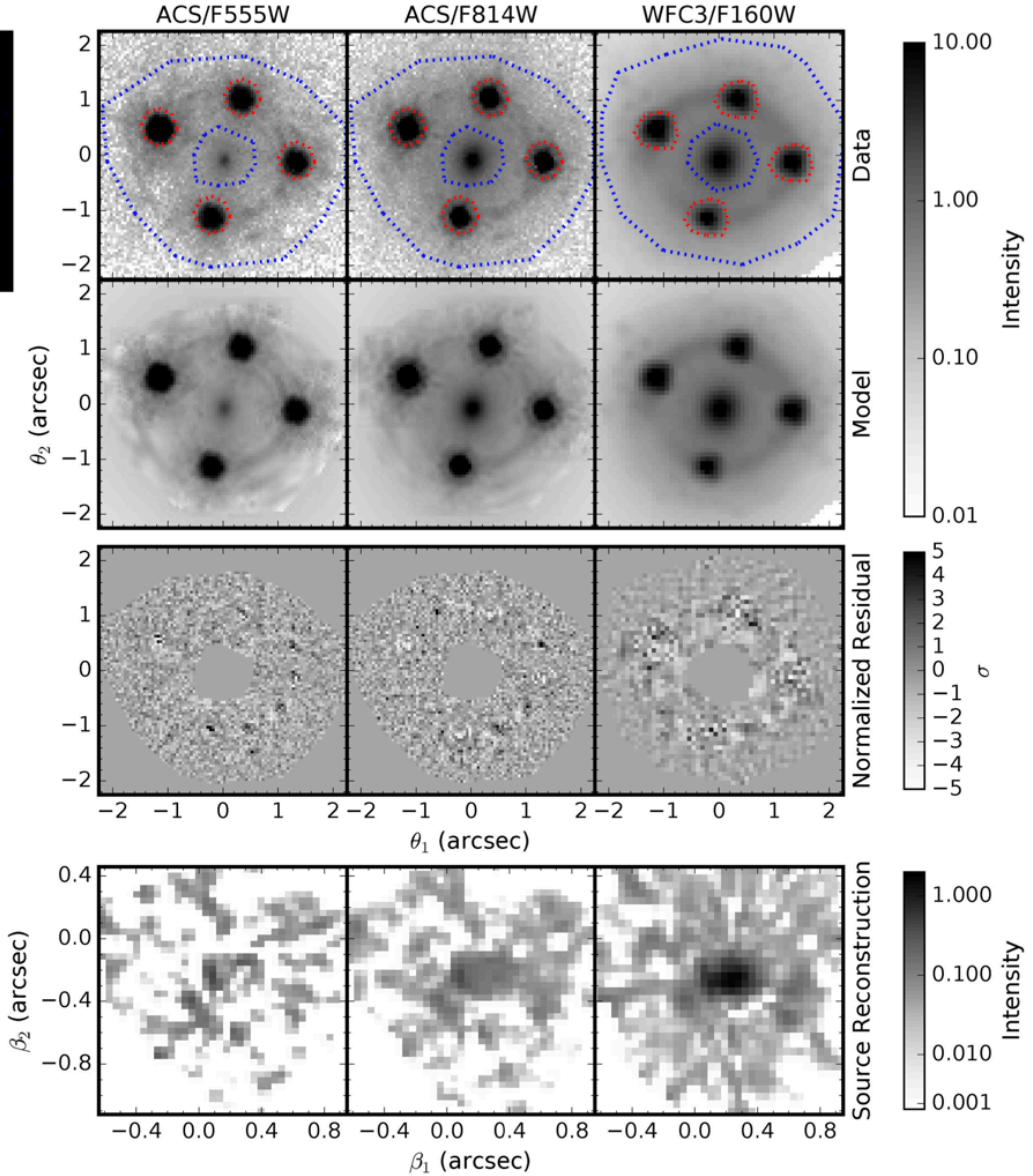


(c) HE 0435−1223

- a singular elliptical power-law model
- include the influence of the nearby massive perturbing galaxies in projection. (G1, for HE 0435−1223)



H0LiCOW paper IV (Wong et al., 2017)



Assumptions

Tested to control the systematics

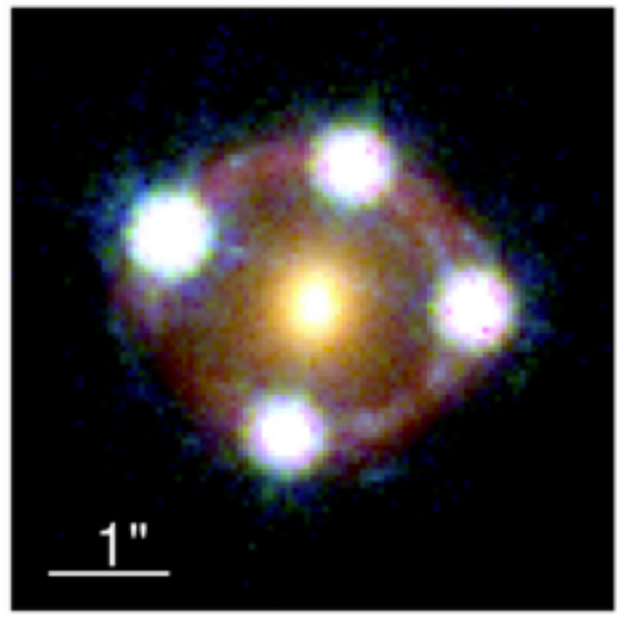
- lens galaxy light profile
- combinations of nearby perturbers
- mass profile parameterization
- source reconstruction
- weighting of the pixels in the image plane

(Wong et al., 2020)

External convergence

External mass distribution -> a convergence parameter

$$D_{\Delta t} = \frac{D^{\mathrm{model}}_{\Delta t}}{1-\kappa_{\mathrm{ext}}}.$$



(c) HE 0435−1223

- count weighted galaxy numbers
- relative over-density of the lens field by comparing with the random LOS
- select LOS from simulation catalogs to get the PDF of κ_ext.

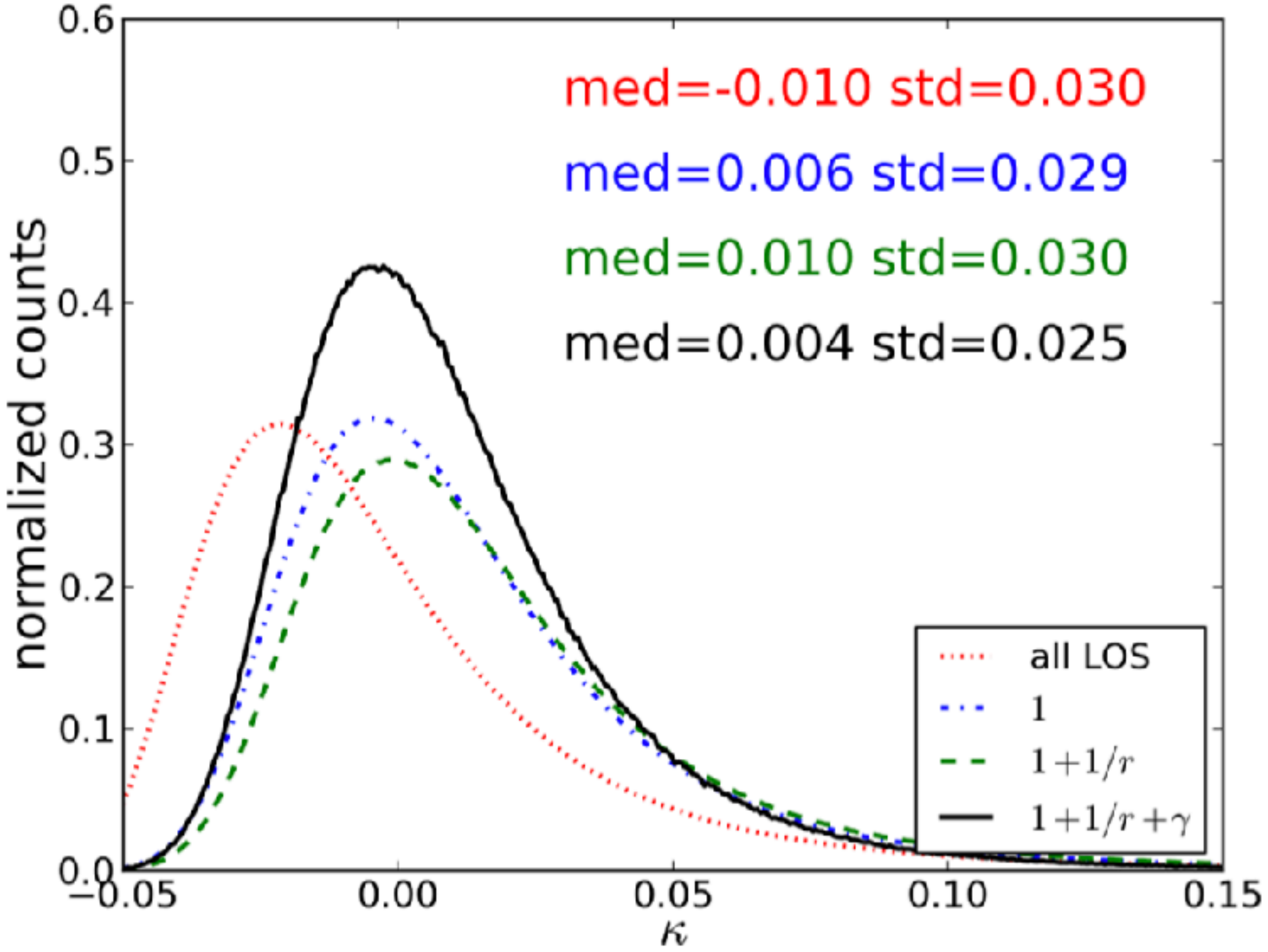


Figure 13. Example of the variation of $P(\kappa_{\mathrm{ext}})$ with the addition of constraints, for aperture radius 45 arcsec, $i < 24$ mag.

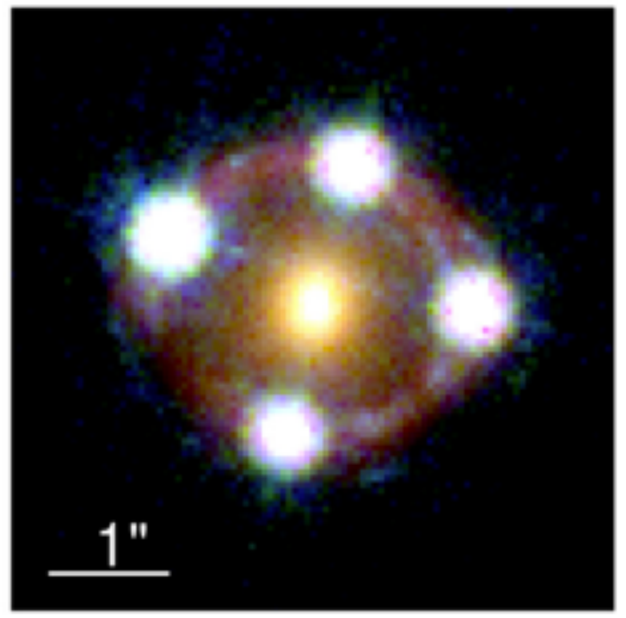
H0LiCOW paper III (Rusu et al., 2017)

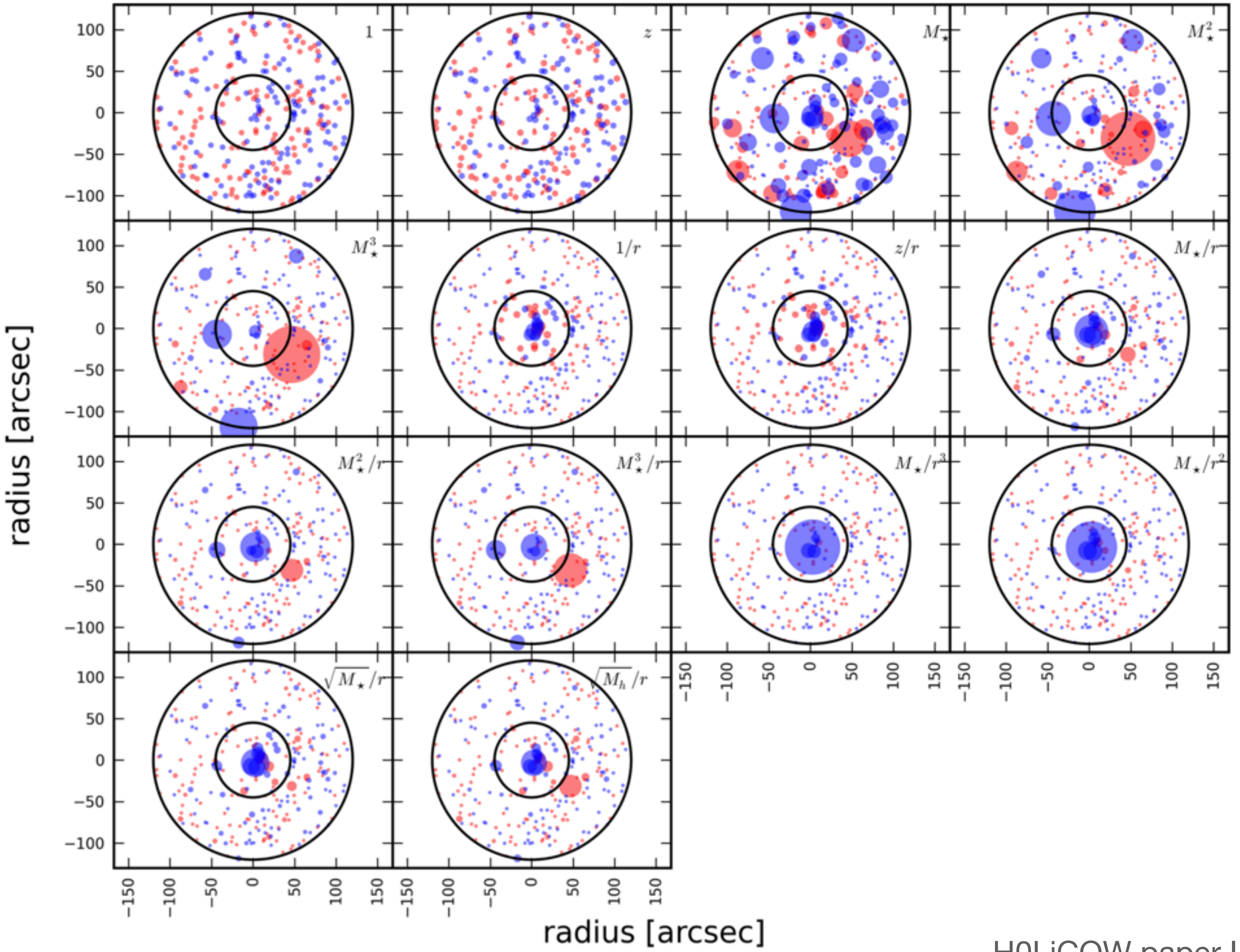
External convergence

Assumptions: quantities to weighted

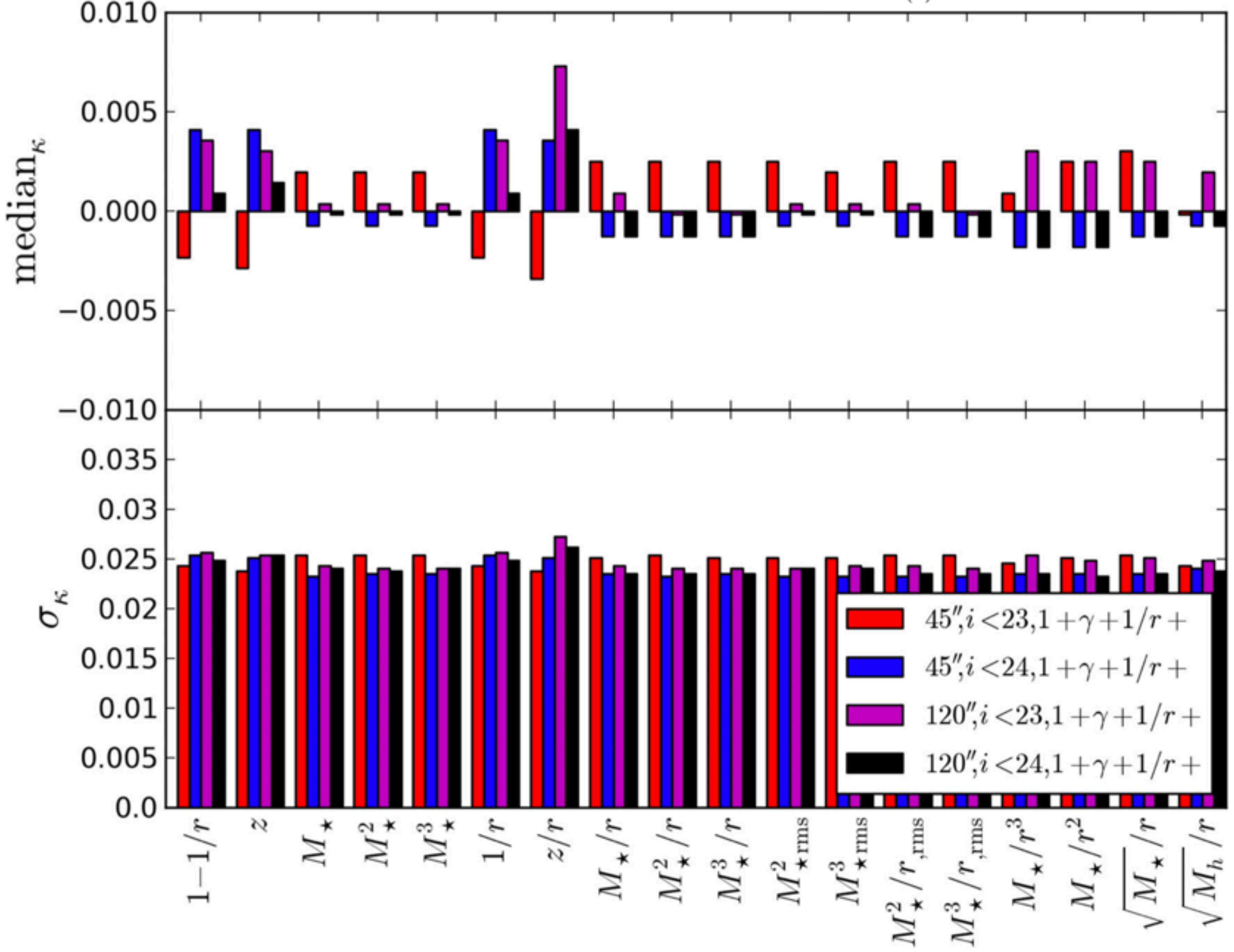
$$D_{\Delta t} = \frac{D_{\Delta t}^{\text{model}}}{1 - \kappa_{\text{ext}}}.$$

(Wong et al., 2020)





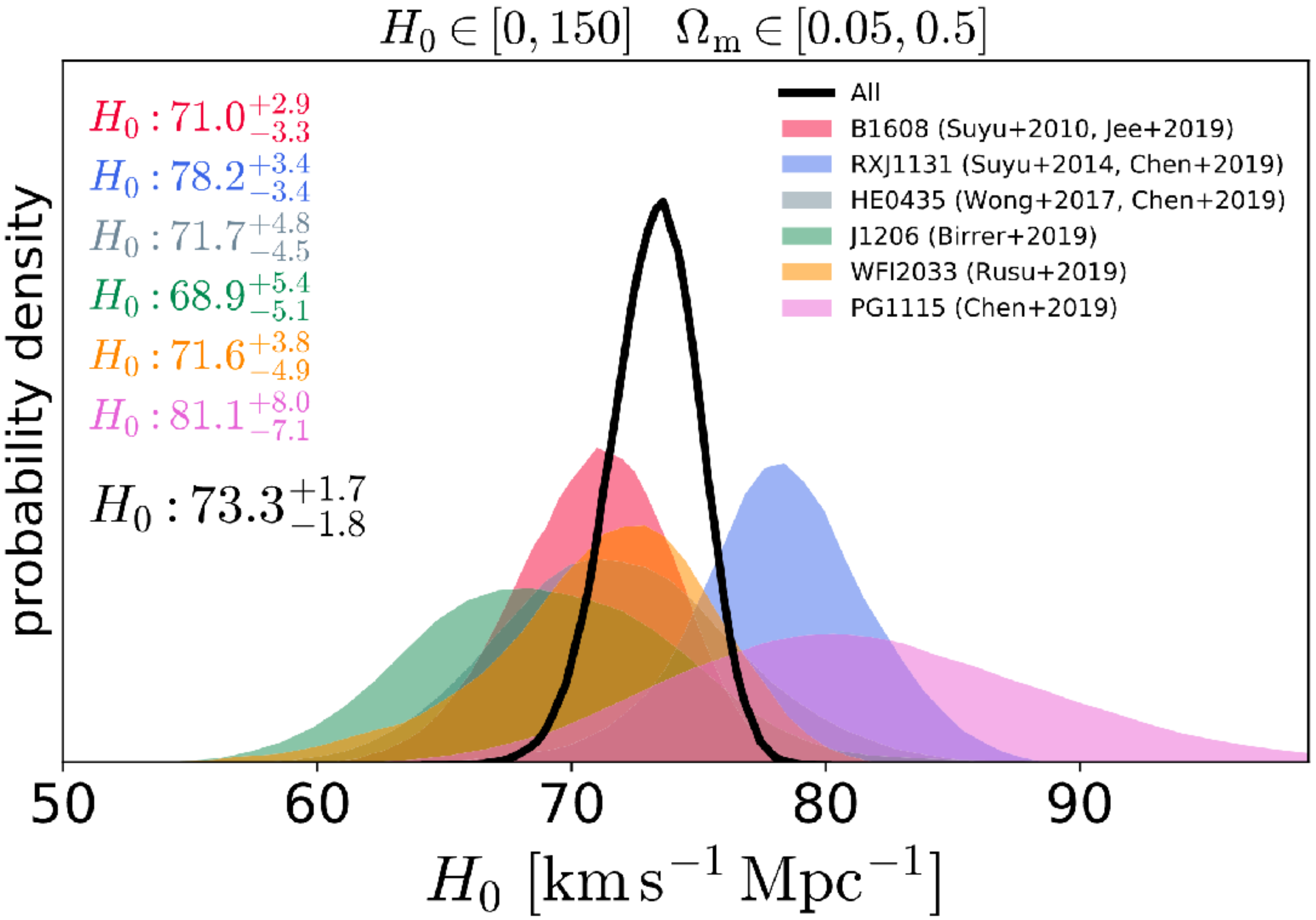
(c) HE 0435−1223



H0LiCOW paper III (Rusu et al., 2017)

H0LiCOW 2.4% results

For ΛCDM



H0LiCOW paper XIII
(Wong et al., 2020)

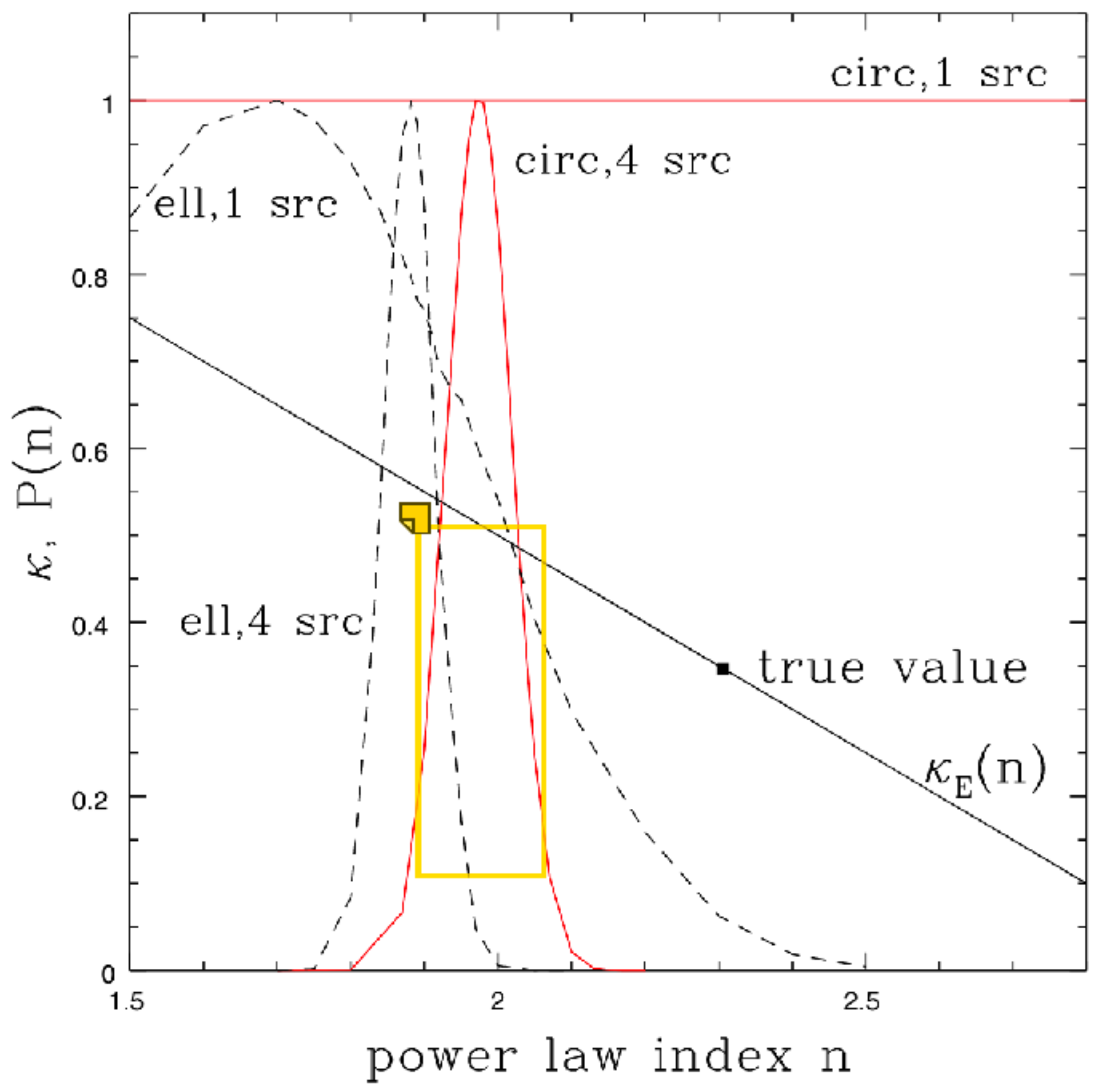
Problem: mass-sheet degeneracy

A constant mass sheet as a thin lens

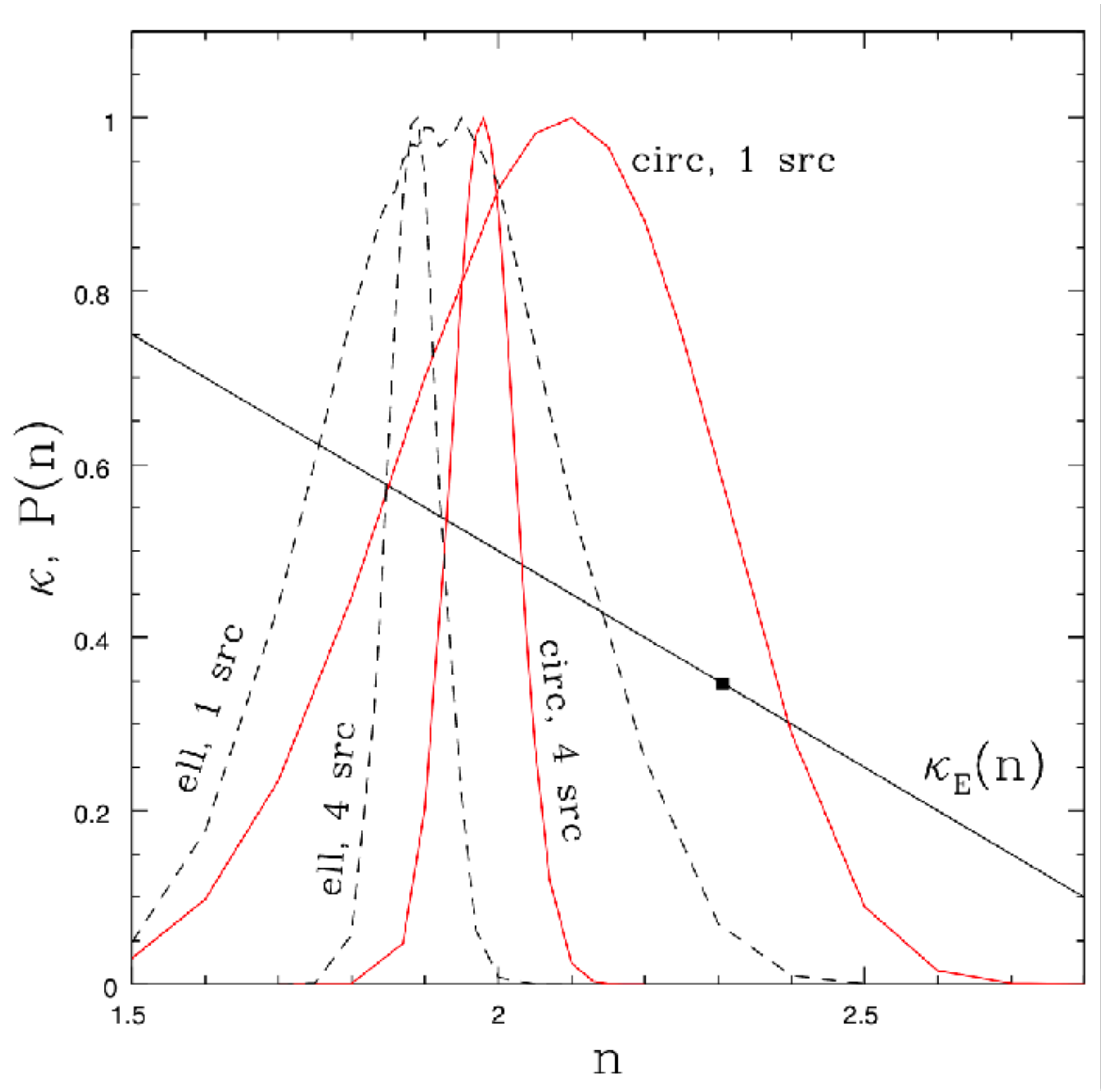
- a set of transformation
- a scaling factor λ
- angle of deflection $\vec{\alpha} \to \vec{\alpha}_\lambda = \lambda\vec{\alpha} + (1-\lambda)\vec{\theta}\,,$
- source position $\vec{\theta}_S \to \vec{\theta}_{S,\lambda} = \lambda\vec{\theta}_S\,.$
- lens mass $\kappa \to \kappa_\lambda = \lambda\kappa + (1-\lambda)\,.$
- internal MSD + external MSD

Question: Incorrect constraints

Dynamical constraints fail when lens constraints are strong.



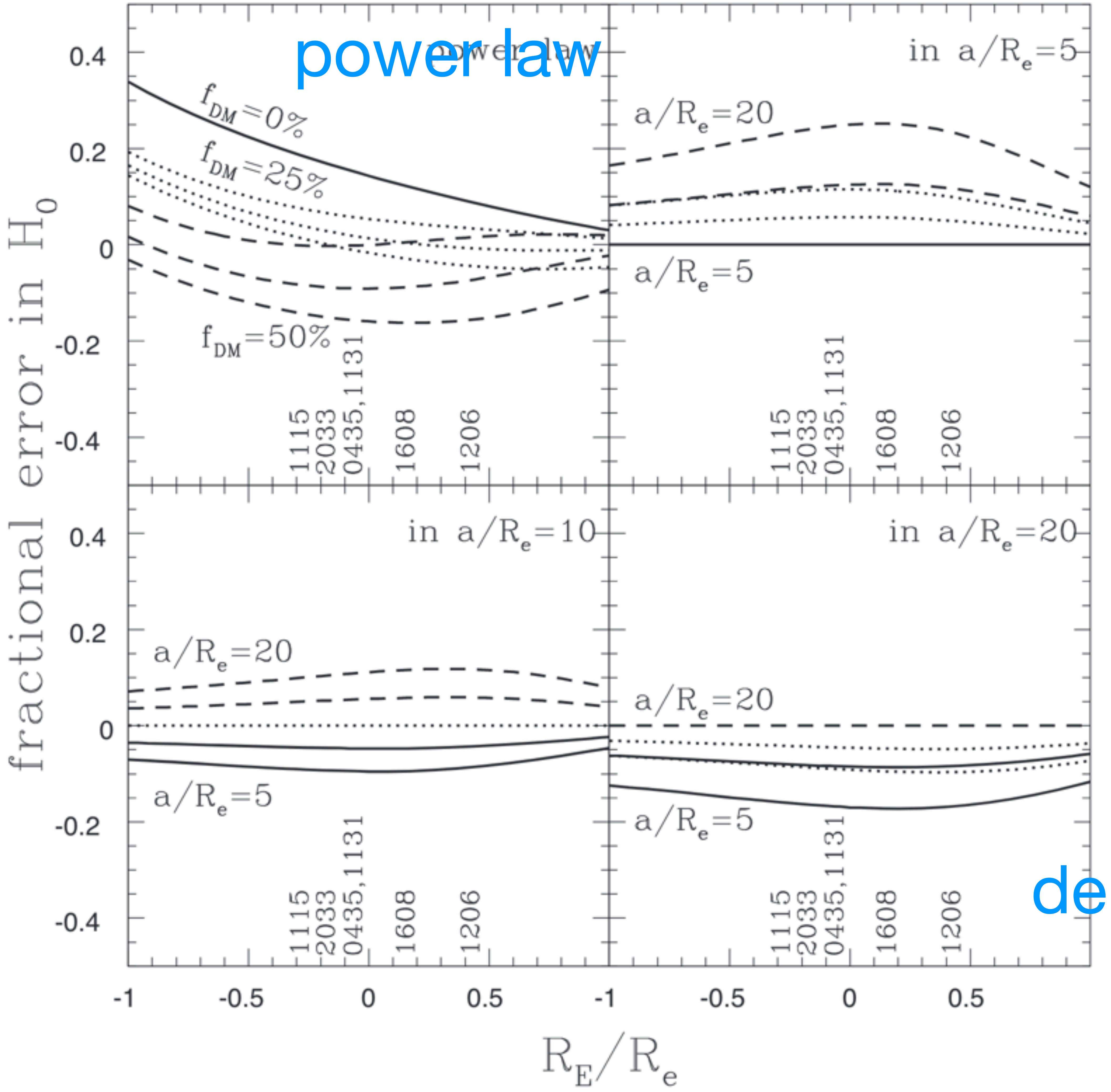
30% fractional error in H0



(Kochanek, 2020)

Question: bigger fractional error in H0

Typical scale of the systematic error in H0 is ~ 10%



More accurate models are wanted!

de Vaucouleurs + NFW

(Kochanek, 2020)

Take home message

- We model the lens with an effective single lens + an external convergence parameter κ.
- The model can include the influence of the nearby massive galaxies.
- The external convergence can be worked out by counting galaxy numbers.
- H0LiCOW collaboration constrained H0 to 2.4% by 6 lens systems.
- Kochanek (2020) shows their results are biased and should have larger uncertainty.

Time-Delay Cosmography

Cosmology with strong gravitational lenses

Jiaqi Zou
2022.04.29

Earlier time-delay vs Later time-delay?

Wong et al. 2019:

- $H_0 = 73.7^{+1.7}_{-1.8} \mathrm{km\ s^{-1}\ Mpc^{-1}}$
- a 2.4% measurement

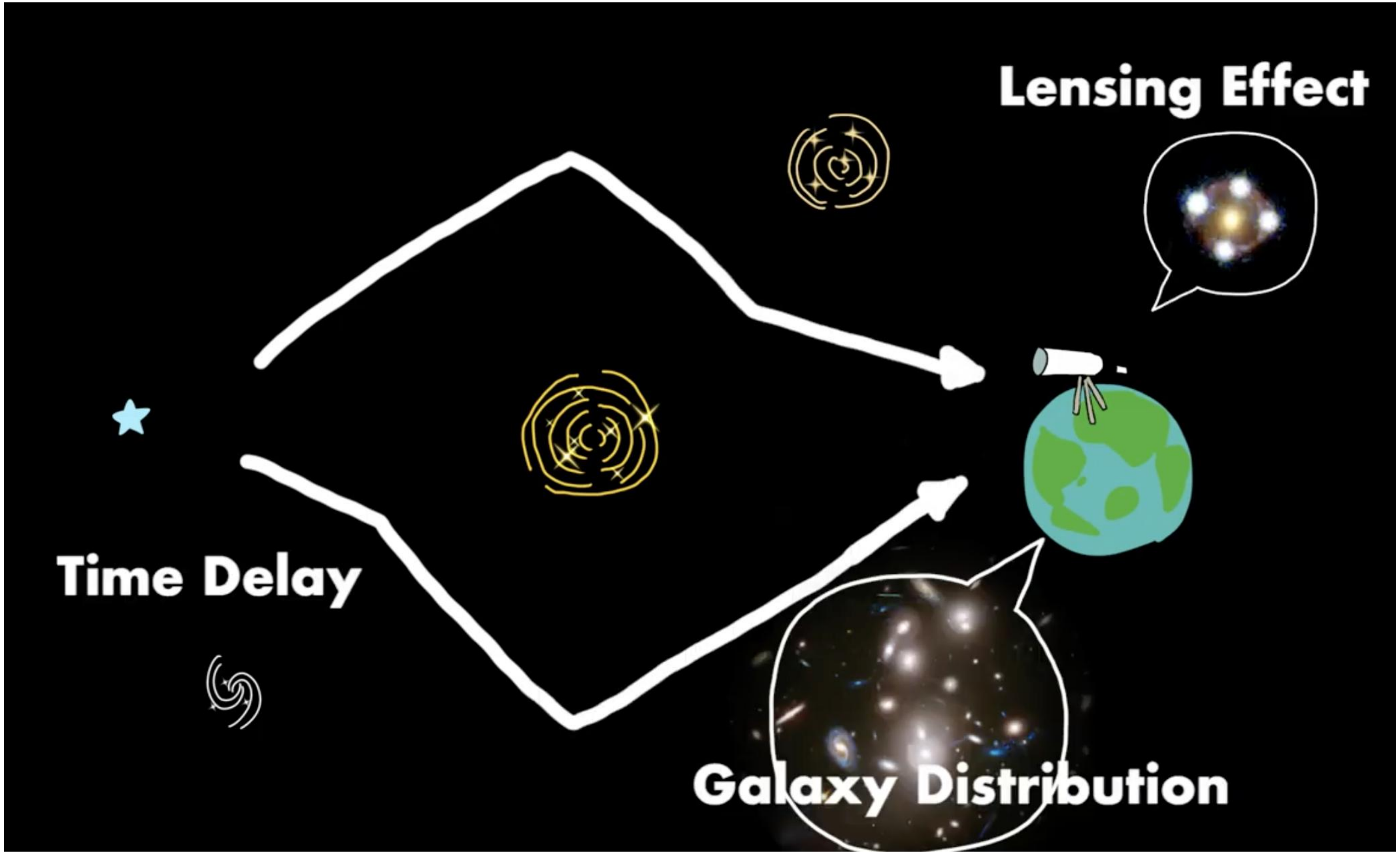
Birrer et al. 2020:

- $H_0 = 67.4^{+4.1}_{-3.2} \mathrm{km\ s^{-1}\ Mpc^{-1}}$
- a ~5% measurement

Why?

more detailed consideration of the modeling of the lens mass distribution

Modifications of Later time-delay results



The Residual uncertainty: mass sheet degeneracy

Mass sheet degeneracy(MSD)

Defination

a uniform, projected mass distribution on the radial mass distribution

a linear source displacement $\boldsymbol{\beta} \rightarrow \lambda \boldsymbol{\beta}$

$$\kappa_\lambda(\theta) = \lambda \times \kappa(\theta) + (1 - \lambda)\,.$$

Origin

- Line-of-sight structure (κs) not related to the main deflector (External)
- The mass profile of the main deflector itself (Internal)

$$\lambda = (1 - \kappa_{\mathrm{s}}) \times \lambda_{\mathrm{int}}.$$

How to model it?

- External MSD:

galaxy number counts

weak lensing

- Internal MSD:

kinematics--velocity dispersion (derived from Jeans equation assuming spherical symmetry and no rotation)

$$\frac{\partial(\rho_* \sigma_{\mathrm{r}}^2(r))}{\partial r} + \frac{2\beta_{\mathrm{ani}}(r)\rho_*(r)\sigma_{\mathrm{r}}^2(r)}{r} = -\rho_*(r)\frac{\partial \Phi(r)}{\partial r},$$

$$\frac{\delta\lambda}{\lambda} = 2\frac{\delta\sigma^{\mathrm{P}}}{\sigma^{\mathrm{P}}}.$$

Hierarchical Bayesian cosmography

Data D = {D_img, D_td, D_spec, D_los}

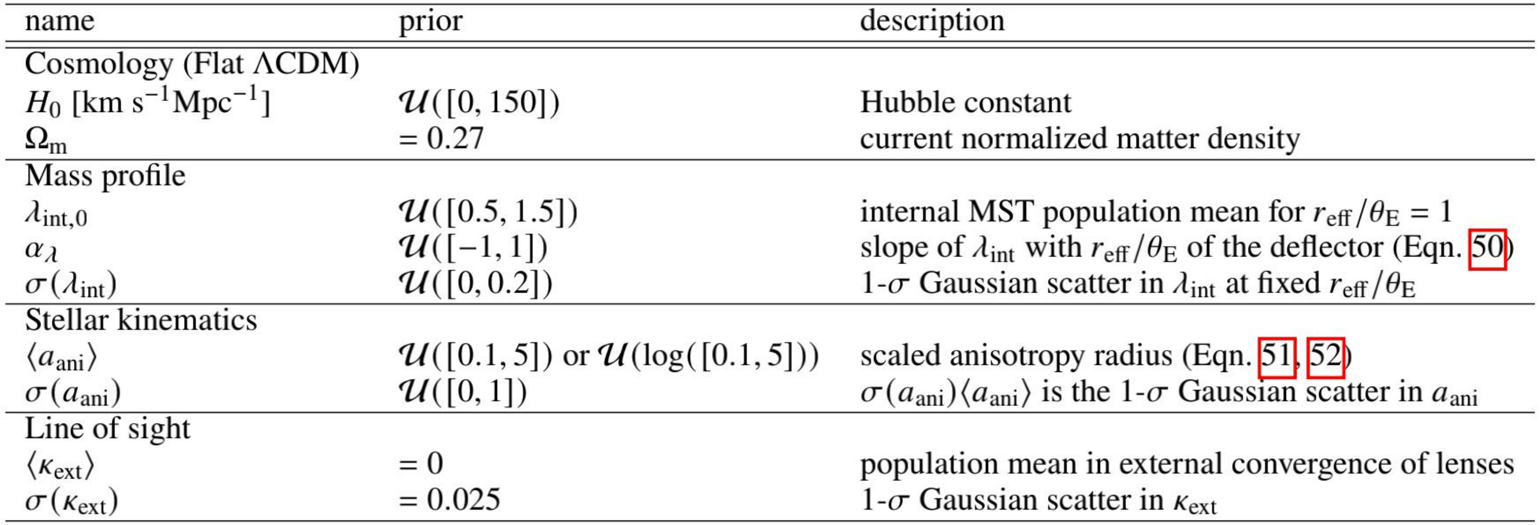
Hierarchical sampling procedures

(1) Population level: An overall internal MSD relative to a chosen mass profile, λint

(2) Population level: Stellar anisotropy distribution in the sample of lenses

(3) Individual level: The line-of-sight structure selection and distribution of the lens sample.

hyper-parameters



name	prior	description
Cosmology (Flat ΛCDM)		
H_0 [km s^{-1}Mpc^{-1}]	$\mathcal{U}([0, 150])$	Hubble constant
Ω_m	$= 0.27$	current normalized matter density
Mass profile		
$\lambda_{\mathrm{int},0}$	$\mathcal{U}([0.5, 1.5])$	internal MST population mean for $r_\mathrm{eff}/\theta_\mathrm{E} = 1$
α_λ	$\mathcal{U}([-1, 1])$	slope of λ_int with $r_\mathrm{eff}/\theta_\mathrm{E}$ of the deflector (Eqn. 50)
$\sigma(\lambda_\mathrm{int})$	$\mathcal{U}([0, 0.2])$	1-σ Gaussian scatter in λ_int at fixed $r_\mathrm{eff}/\theta_\mathrm{E}$
Stellar kinematics		
$\langle a_\mathrm{ani} \rangle$	$\mathcal{U}([0.1, 5])$ or $\mathcal{U}(\log([0.1, 5]))$	scaled anisotropy radius (Eqn. 51, 52)
$\sigma(a_\mathrm{ani})$	$\mathcal{U}([0, 1])$	$\sigma(a_\mathrm{ani})\langle a_\mathrm{ani} \rangle$ is the 1-σ Gaussian scatter in a_ani
Line of sight		
$\langle \kappa_\mathrm{ext} \rangle$	$= 0$	population mean in external convergence of lenses
$\sigma(\kappa_\mathrm{ext})$	$= 0.025$	1-σ Gaussian scatter in κ_ext

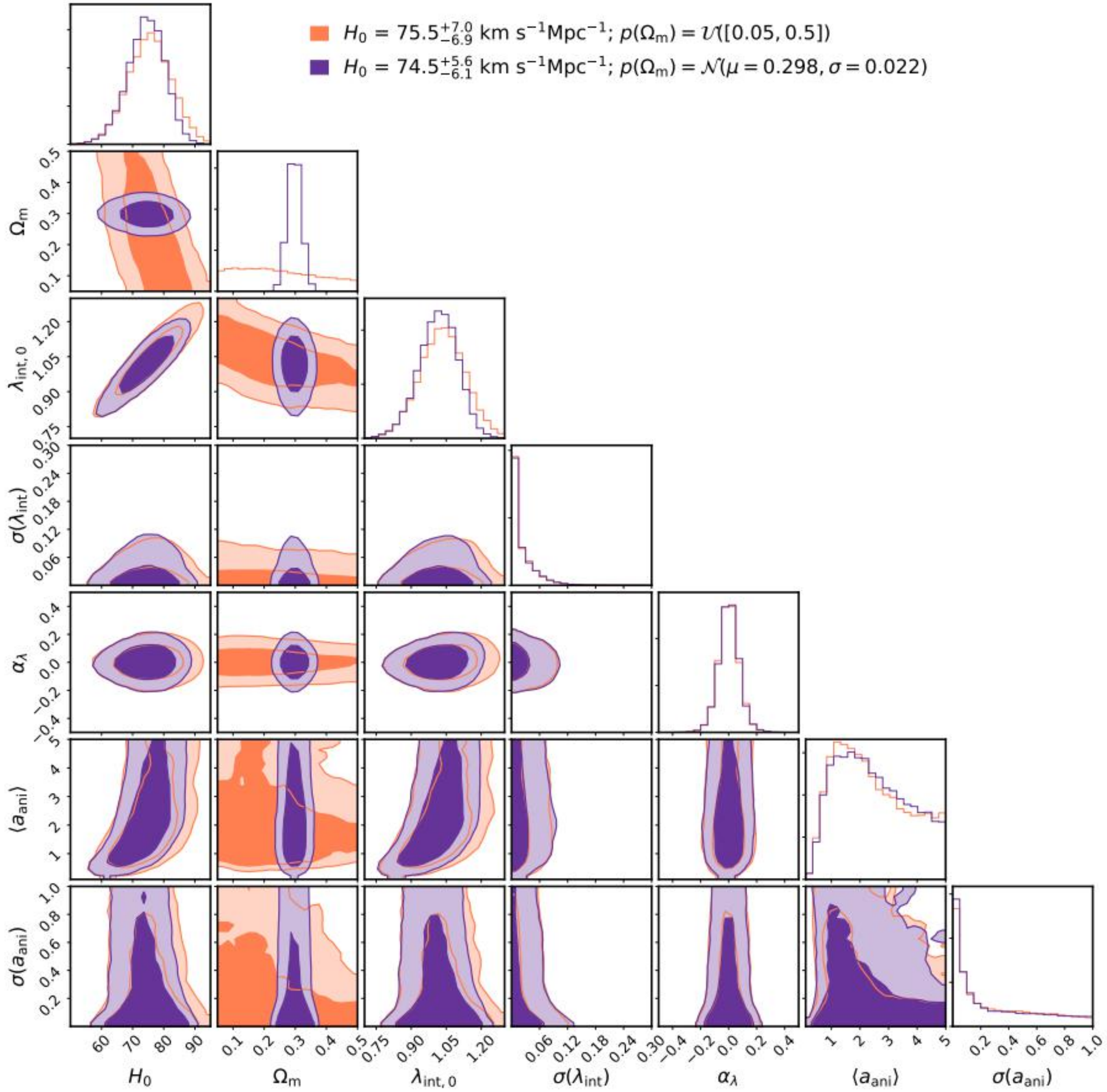
Hierarchical analysis of TDCOSMO

Lens sample: 6

Precision: 9%

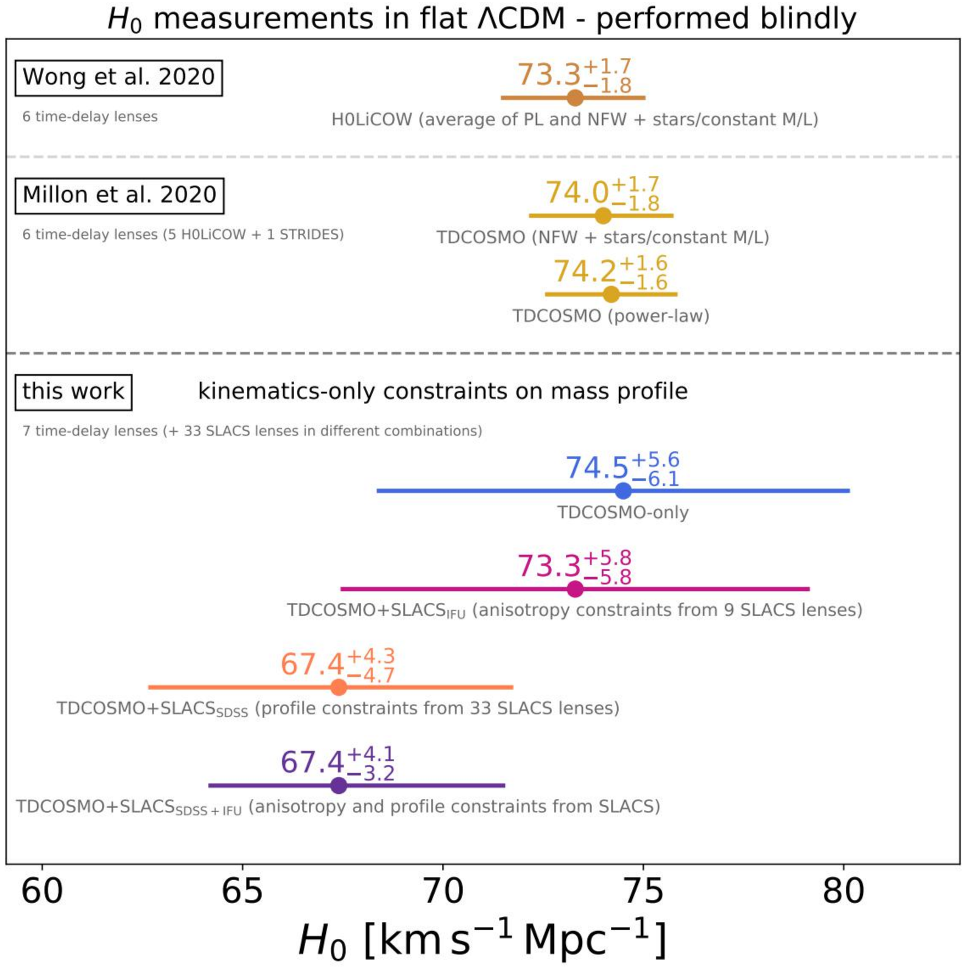
Main factors:
1) relaxed the assumption of NFW+stars or power-law mass density profiles
2) considered the impact of covariance between lenses when accounting for uncertainties potentially arising from assumptions about mass profile and stellar anisotropy models.

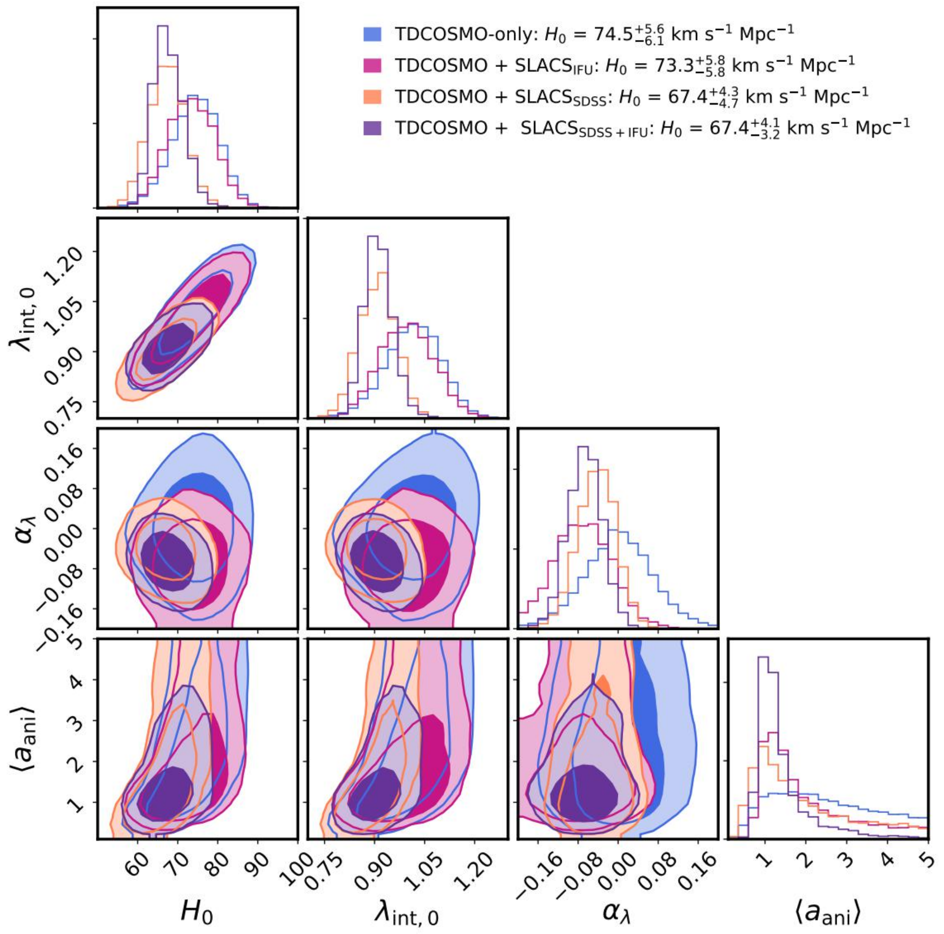
How to reduce precision?
add external information (add 33 SLACS lenses)



Hierarchical analysis of TDCOSMO+SLACS

Lense Sample: 6 lenses from TDCOSMO +33 lenses from SLACS (No time-delay!)
Precision: ~5%





Forecasts

Limiting factor

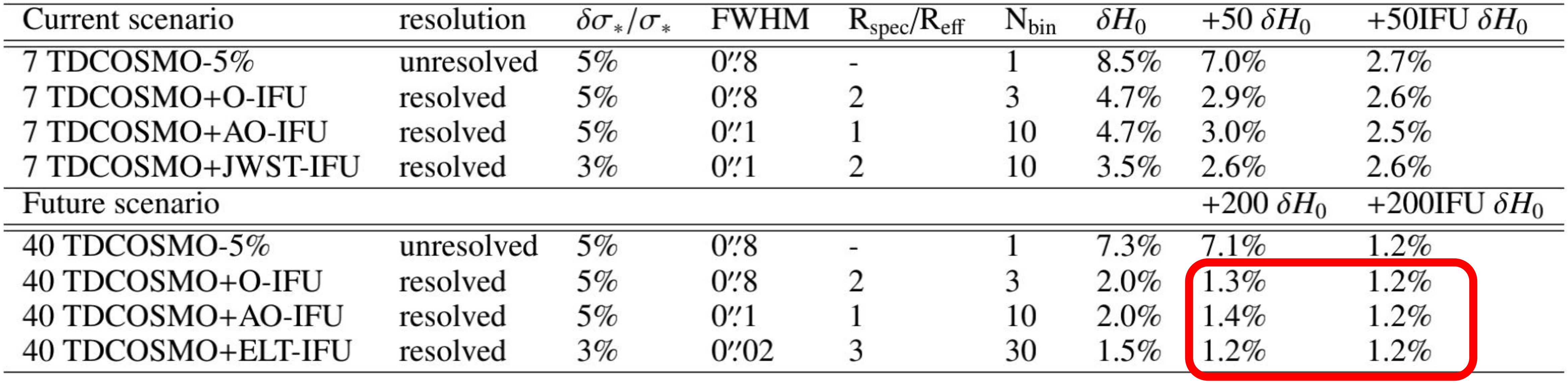
Timedelay lenses: Unresolved stellar velocity dispersion measurements

External lenses: the precision of aperture velocity dispersion measurements
the absolute calibration and sample size of integral field data
the overall sample size.

Two strategies

Spatially resolved stellar velocity dispersion of the TDCOSMO samples: IFU/ AO-IFU/JWST-IFU/ELT-IFU

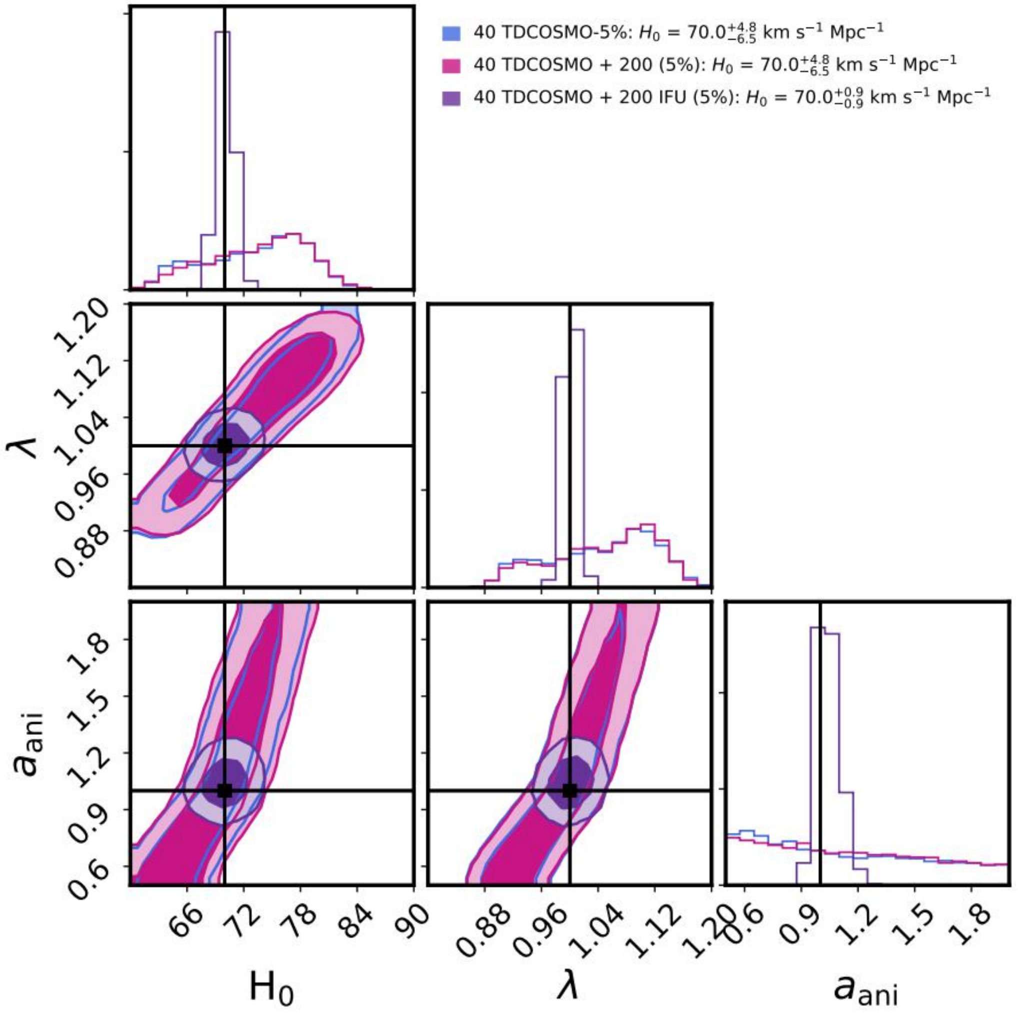
Add external lenses with/without IFU

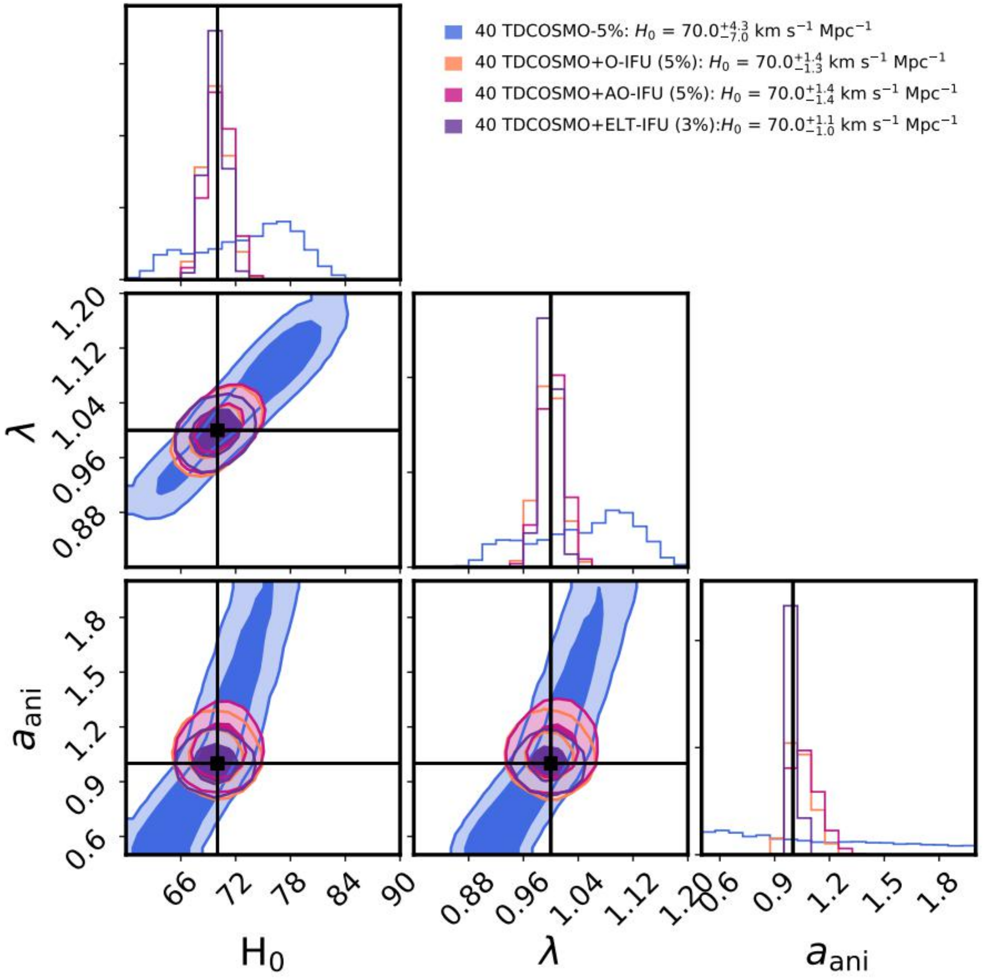


Current scenario	resolution	$\delta\sigma_*/\sigma_*$	FWHM	R_{spec}/R_{eff}	N_{bin}	δH_0	+50 δH_0	+50IFU δH_0
7 TDCOSMO-5%	unresolved	5%	0″.8	-	1	8.5%	7.0%	2.7%
7 TDCOSMO+O-IFU	resolved	5%	0″.8	2	3	4.7%	2.9%	2.6%
7 TDCOSMO+AO-IFU	resolved	5%	0″.1	1	10	4.7%	3.0%	2.5%
7 TDCOSMO+JWST-IFU	resolved	3%	0″.1	2	10	3.5%	2.6%	2.6%
Future scenario							+200 δH_0	+200IFU δH_0
40 TDCOSMO-5%	unresolved	5%	0″.8	-	1	7.3%	7.1%	1.2%
40 TDCOSMO+O-IFU	resolved	5%	0″.8	2	3	2.0%	1.3%	1.2%
40 TDCOSMO+AO-IFU	resolved	5%	0″.1	1	10	2.0%	1.4%	1.2%
40 TDCOSMO+ELT-IFU	resolved	3%	0″.02	3	30	1.5%	1.2%	1.2%

Forecasts

Future dataset
40 time-delay and 200 nontime-delay   Presicion: 1.2-1.5%





Later time-delay results

Modification

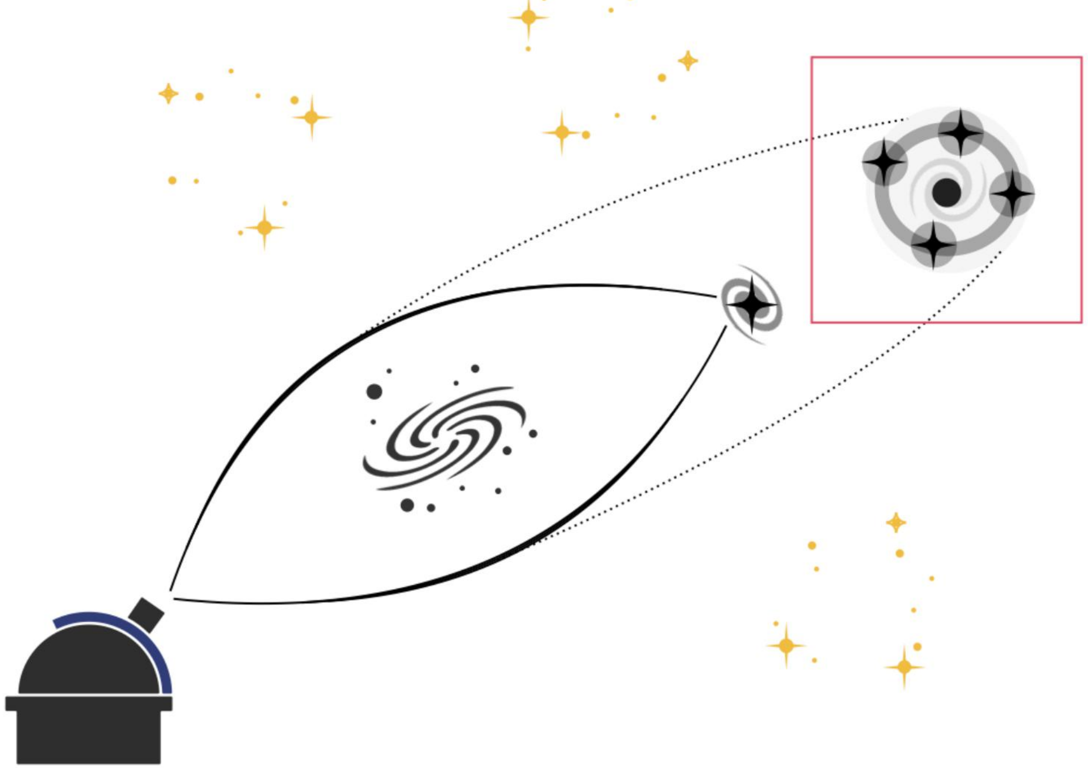
- Model the Residual uncertainty: Encode the mass sheet transform (MST):
- enlarge sample size: add 33 strong gravitational lenses from the Sloan Lens ACS (SLACS)

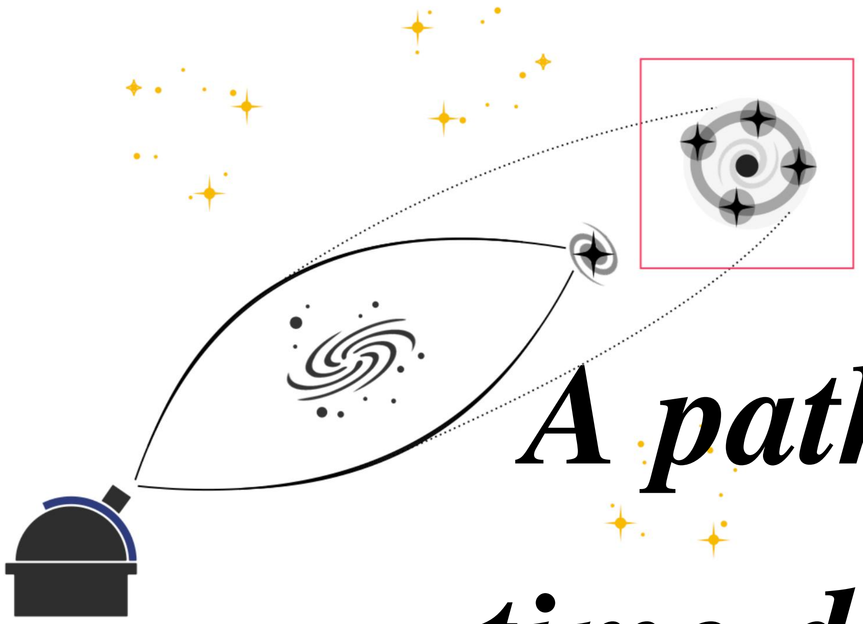
Results:

- ~5% measurement of H_0
- $H_0 = 67.4^{+4.1}_{-3.2} \text{km s}^{-1}\ \text{Mpc}^{-1}$

Further improvements

- The spatial resolved kinematics
- The larger sample size

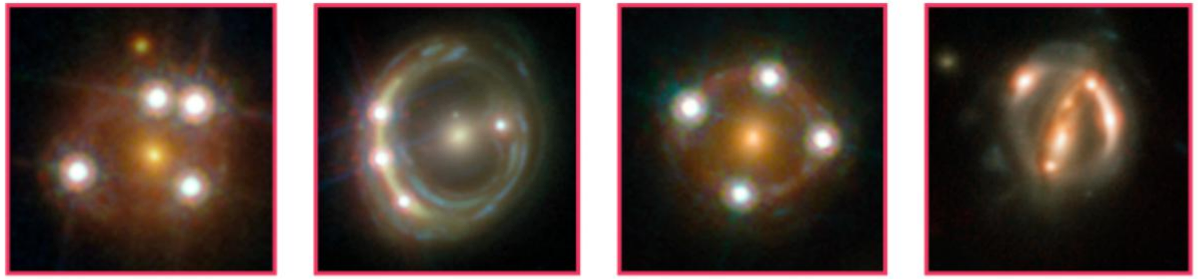




A pathway forward for time-delay cosmography

赵思逸，邹佳琪，程卓，郭彦汉

2022年4月29日,student seminar



Where do the uncertainties come from?

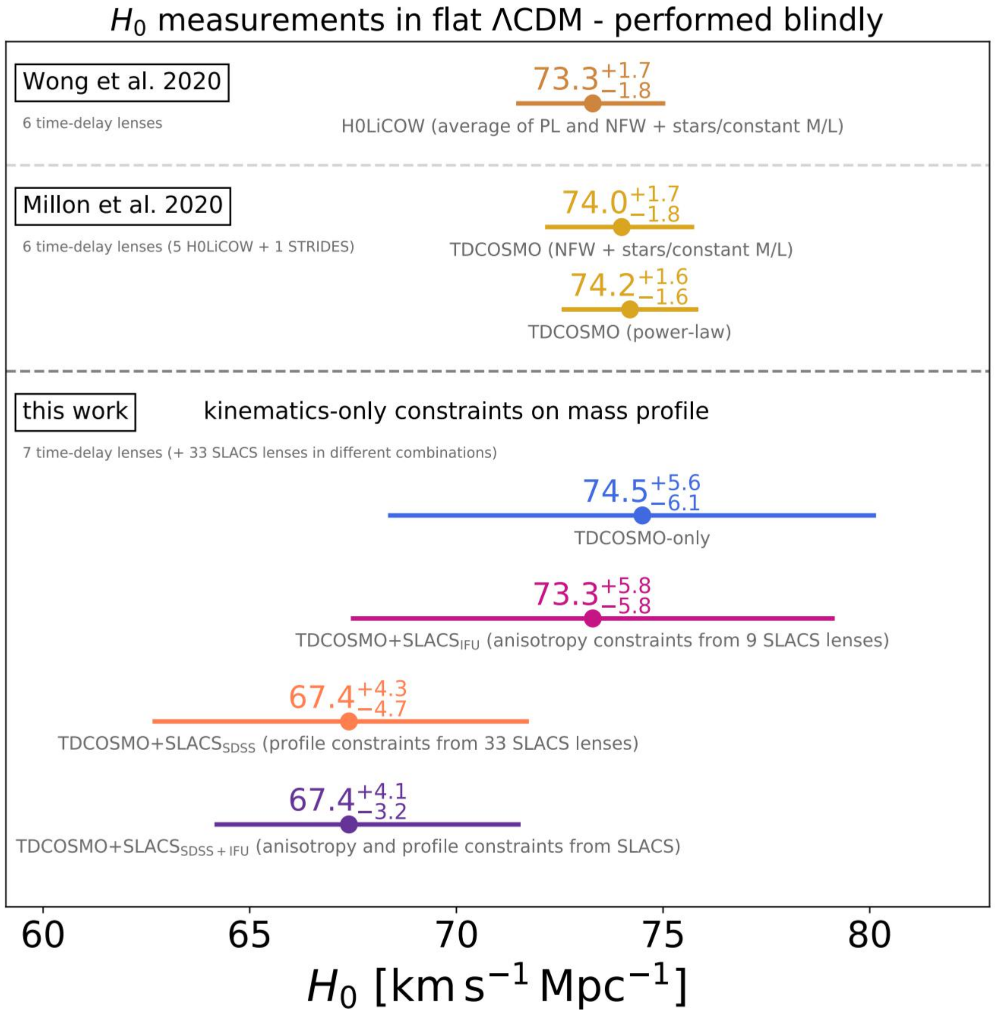
- Time delay measurement: ~1% (Guo yanhan's introduction)
- Mass Sheet Degeneracy(Zhao siyi and Zou jiaqi's talk)

$$\kappa_\lambda(\theta) = \lambda\,\kappa(\theta) + (1 - \lambda)$$

The image positions and image shapes will be preserved under this transformation.

  - Contributions of MST:~4.5%

- ***How to break the degeneracy?***



(S. Birrer et al. 2020)

How to break the Mass-Sheet degeneracy?

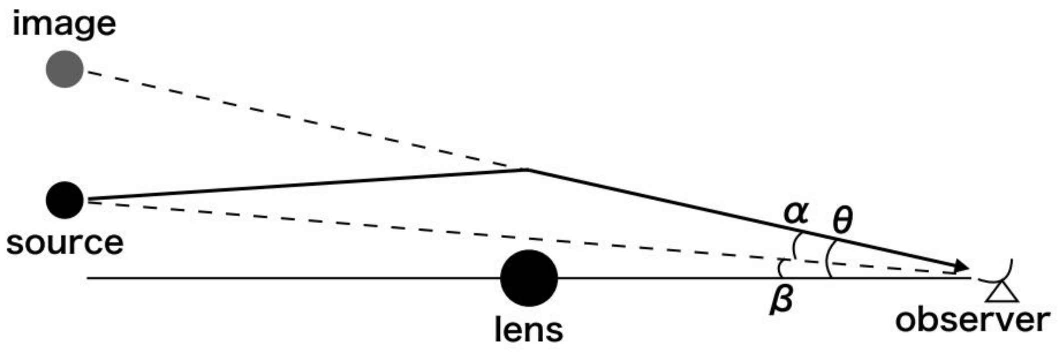
- **The change of magnification**

$$\theta_{\lambda} = \theta$$

$$\beta_{\lambda} = \lambda\beta$$

$$\Rightarrow \quad \mu_{\lambda} = \frac{\mu}{\lambda^2}$$

$$\Downarrow$$

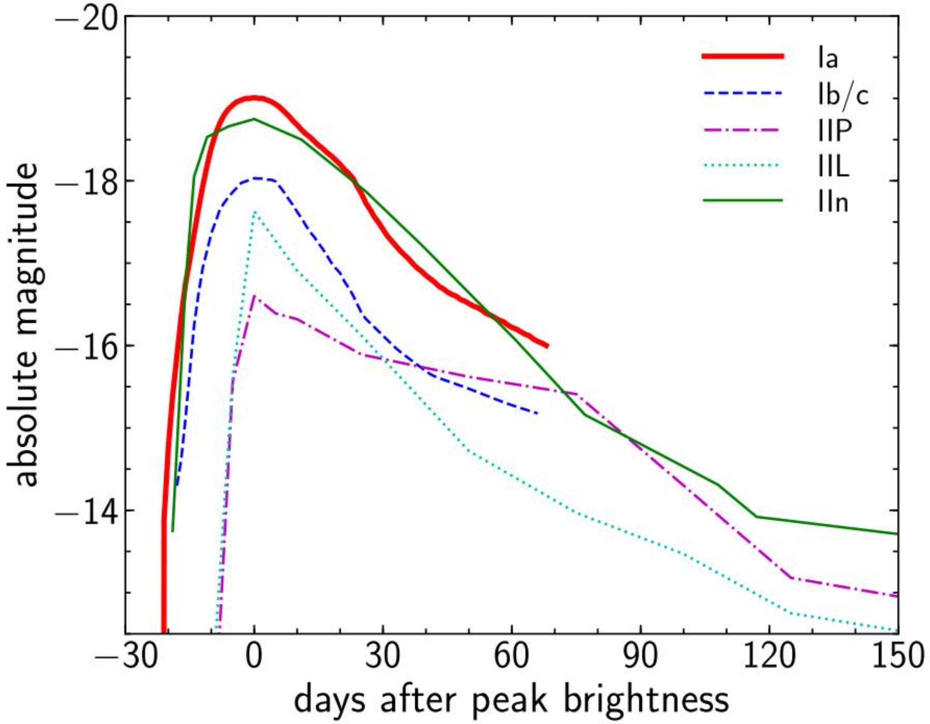


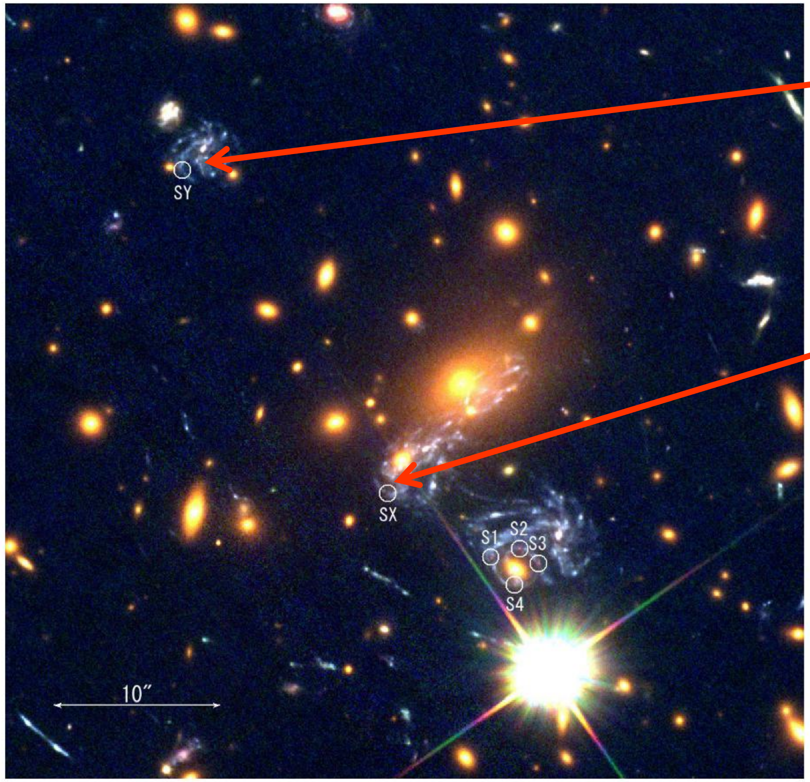
(Oguri 2019)

If we know the intrinsic luminosity of source, the value of λ can be constrained.

Strong lensing of Type Ia supernovae!

- Peak luminosities of Type Ia supernovae are quite similar
- Standard candle





predicted to have appeared ~10 years before

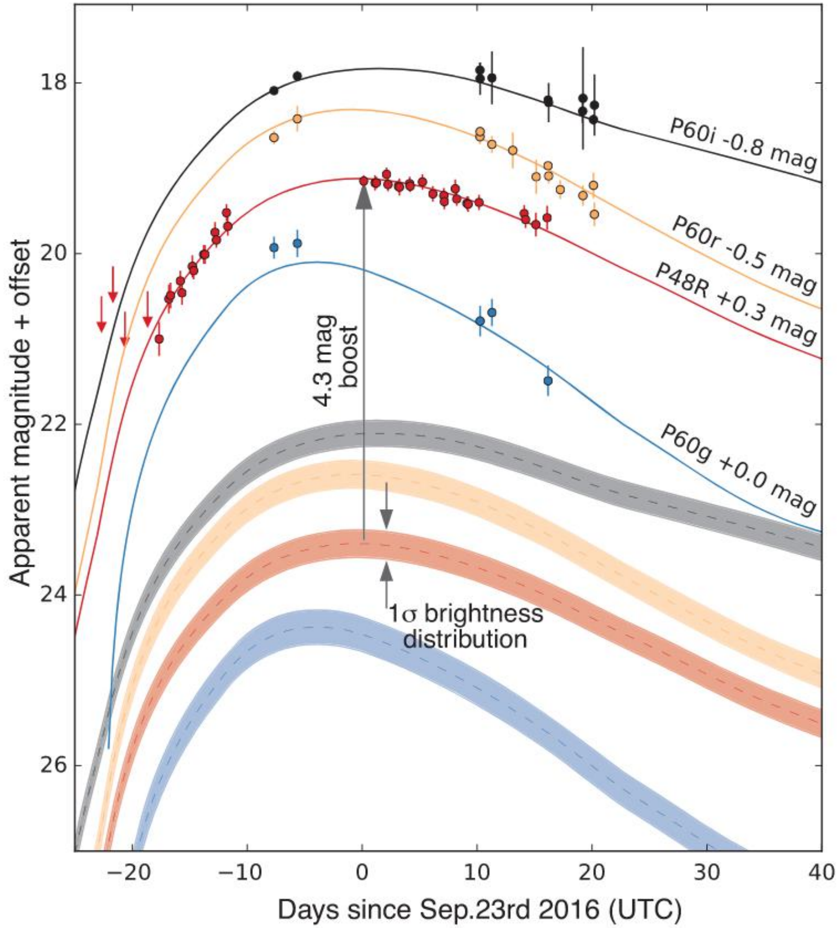
1 year after the discoveries of S1–S4

SN Refsdal: the first strongly lensed supernova discovered with resolved multiple images.

(Kelly et al. 2015)

The first strongly lensed Type Ia supernova with resolved multiple images!

The supernova is 4.3 magnitudes (30 standard deviations) brighter than expected.



(Goobar et al. 2017)

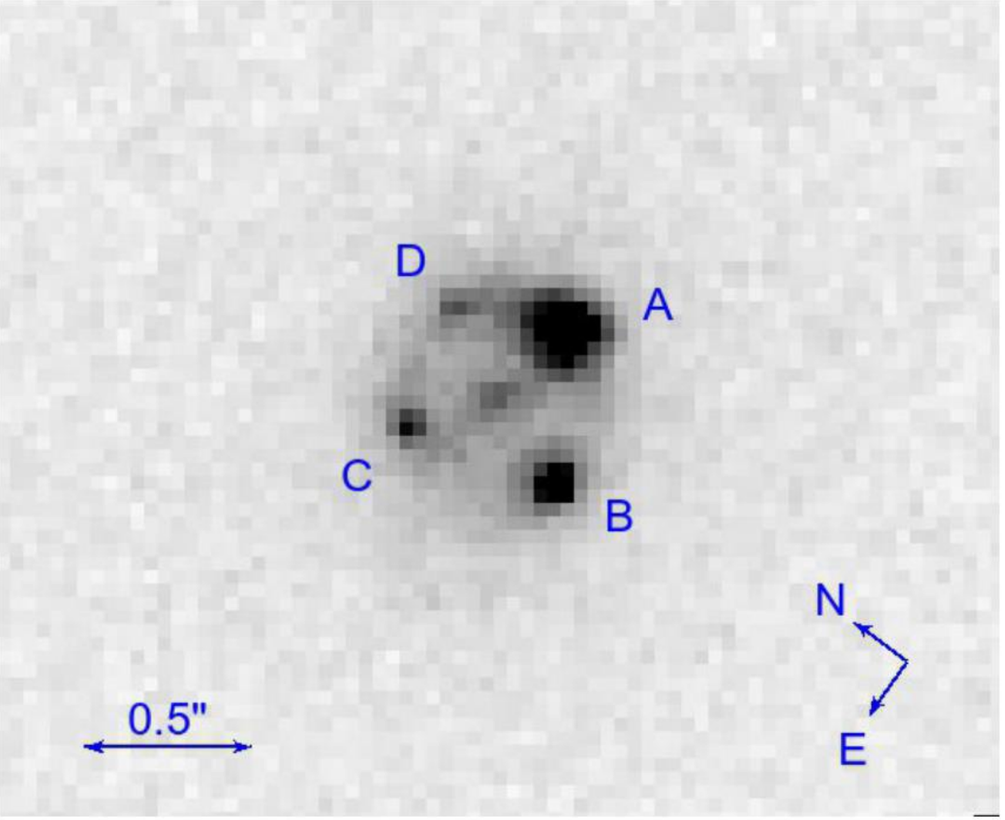


Figure 15. The *Hubble Space Telescope* F814W image of the strongly lensed Type Ia supernova iPTF16geu [368]. The 4 supernova images are marked by A–D.

Gaining a Time-Delay? or Losing a Standard Candle?

Model Prediction

Model Profile	Δt (days) A	B	C	D
GLAFIC SIE	$0.40^{+0.02}_{-0.02}$	$\equiv 0$	$0.47^{+0.01}_{-0.02}$	$0.25^{+0.01}_{-0.01}$
GLEE SIE	$0.52^{+0.08}_{-0.05}$	$\equiv 0$	$0.65^{+0.07}_{-0.07}$	$0.35^{+0.05}_{-0.05}$
GLEE PL	$0.56^{+0.06}_{-0.06}$	$\equiv 0$	$0.70^{+0.06}_{-0.07}$	$0.37^{+0.03}_{-0.04}$
GLEE PL+$\gamma_{\rm ext}$	0.6 ± 0.1	$\equiv 0$	0.7 ± 0.1	0.4 ± 0.1

- The magnification factors from the model are too small.
- The time-delay is very short.

(*More* et al. 2017)

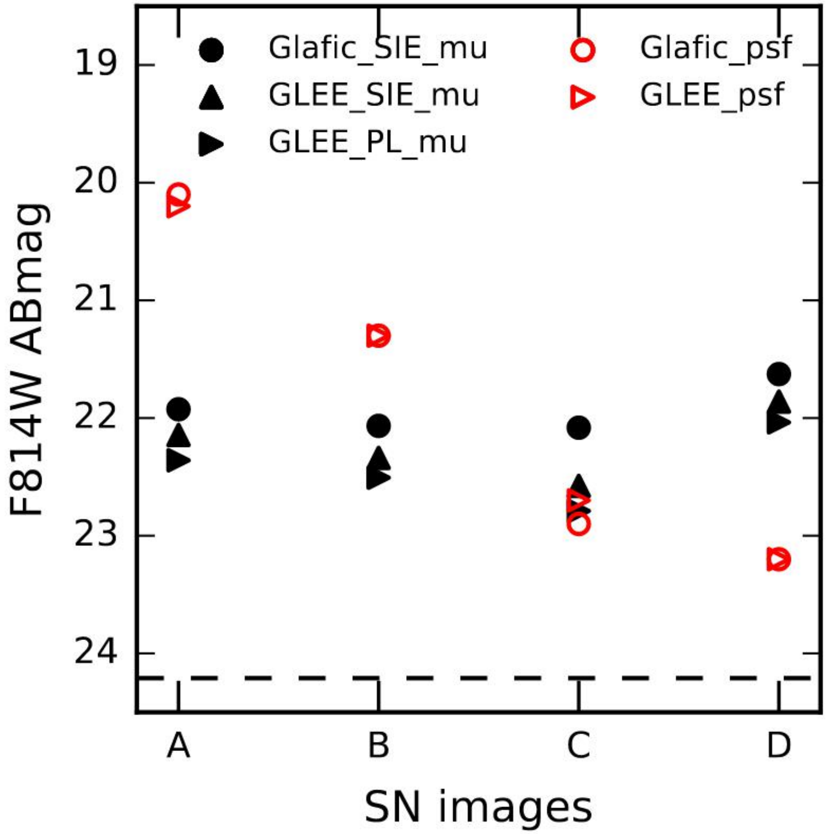
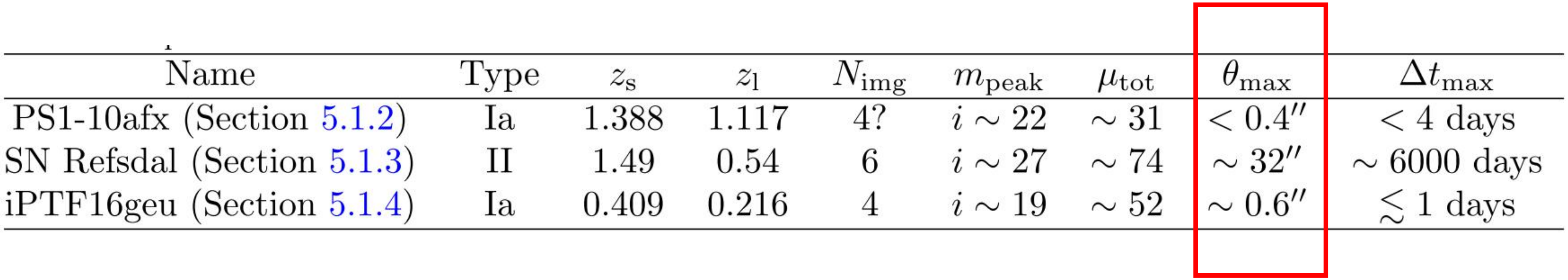


FIG. 2.— Fluxes of SN images A, B, C and D. Expected fluxes after scaling the intrinsic SN flux (24.21 ABmag, dashed line) by the lens-model magnification (μ) factors (filled symbols) are compared with PSF model fluxes fit to the *HST* image. Relative magnifications are more robust than the absolute values across different models. Fluxes of most of the images depart from predictions. Image A is the most magnified and image D appears to be suppressed (see text in Section 4 for further discussion).

Current strongly lensed supernovae



Name	Type	z_s	z_l	N_{img}	m_{peak}	μ_{tot}	θ_{max}	Δt_{max}
PS1-10afx (Section 5.1.2)	Ia	1.388	1.117	4?	$i \sim 22$	~ 31	$< 0.4''$	< 4 days
SN Refsdal (Section 5.1.3)	II	1.49	0.54	6	$i \sim 27$	~ 74	$\sim 32''$	~ 6000 days
iPTF16geu (Section 5.1.4)	Ia	0.409	0.216	4	$i \sim 19$	~ 52	$\sim 0.6''$	$\lesssim 1$ days

(Oguri 2019)

Limitations:

- Small image separations such that they are barely resolved.
- The time scale of their light curves is ~30 days. (*still too long*)

A possible ideal condidate for time delay measurement---*Fast Radio Burst(FRB)*

Advantages:

- *Precise time delay measurements*: $\frac{T_{delay}}{T_{FRB}} \sim 10^9$
- Repeating fast radio bursts

Complication:

- The accurate dispersion measure(MD)

$$\Delta T_{FRB} \propto \frac{DM}{\nu^2}$$

- Plasma effect(local enciornment), intergalactic medium (IGM)...

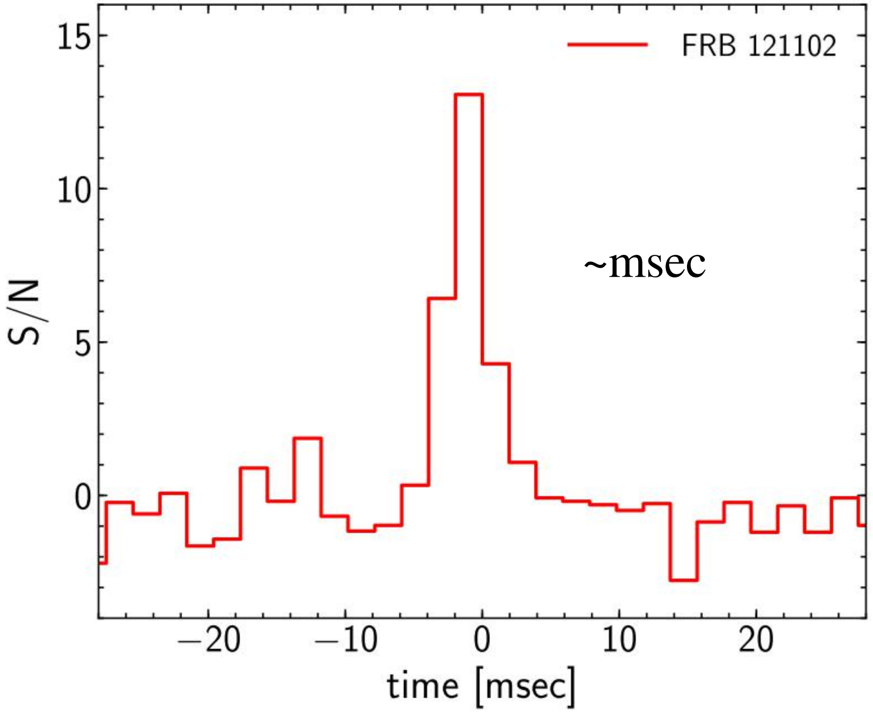


Figure 9. An example of light curves of fast radio bursts. Here we show a dedispersed, averaged pulse profile of FRB 121102 detected with the Arecibo Observatory [212].

(L. G. Spitler et al. 2014)

Another interesting system---Doubling Strong Lensing(DSL)

Ratio of time delay:

$$\frac{D_{\Delta t_1}}{D_{\Delta t_2}} = \frac{D_{s_1}\cancel{D_l(1+z_l)}}{D_{ls_1}}\frac{D_{ls_2}}{D_{s_2}\cancel{D_l(1+z_l)}}$$

- *The ratio is independent of H_0*
- *Break the MSD*

Eye of Horus

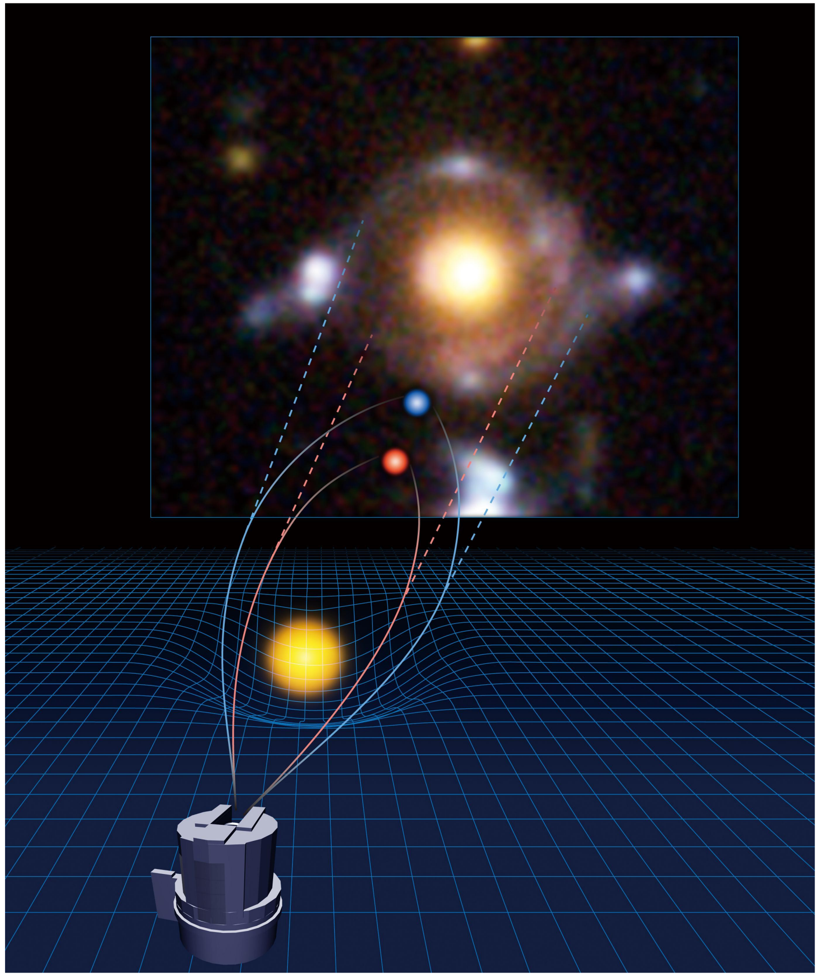
$z_{lens} = 0.795$

$z_{s_1} = 1.302$

$z_{s_2} = 1.988$

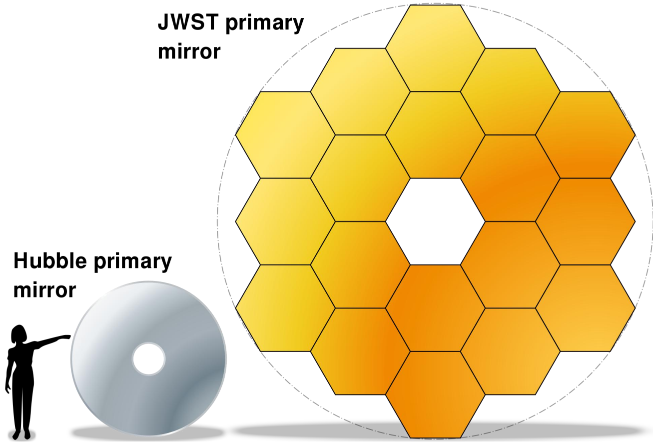
(Masayuki Tanaka et al. 2016)

HSC J142449−005322

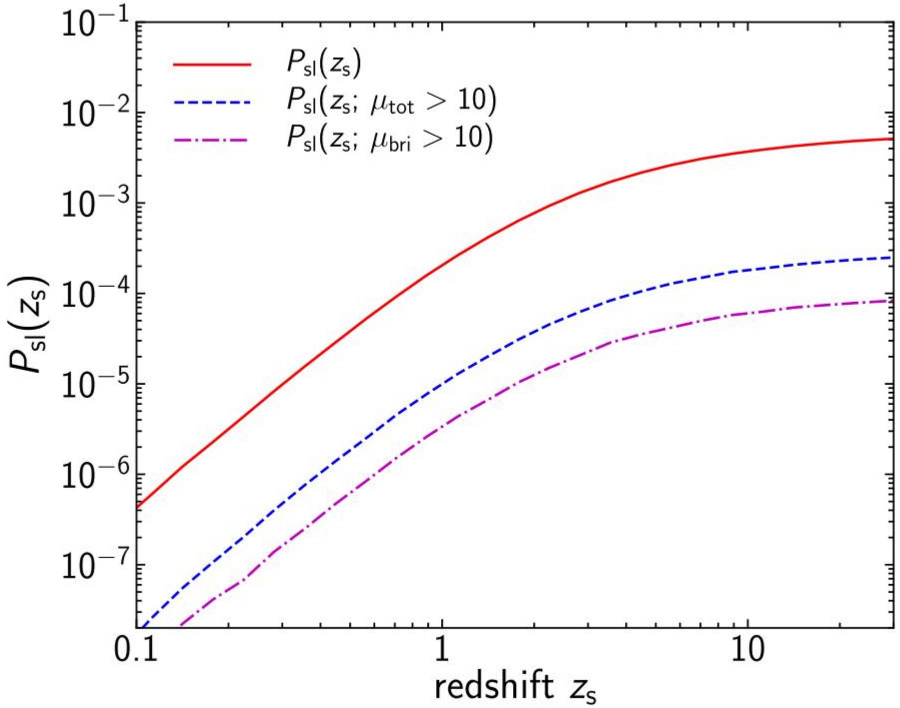


The future telescope/survey

- JWST

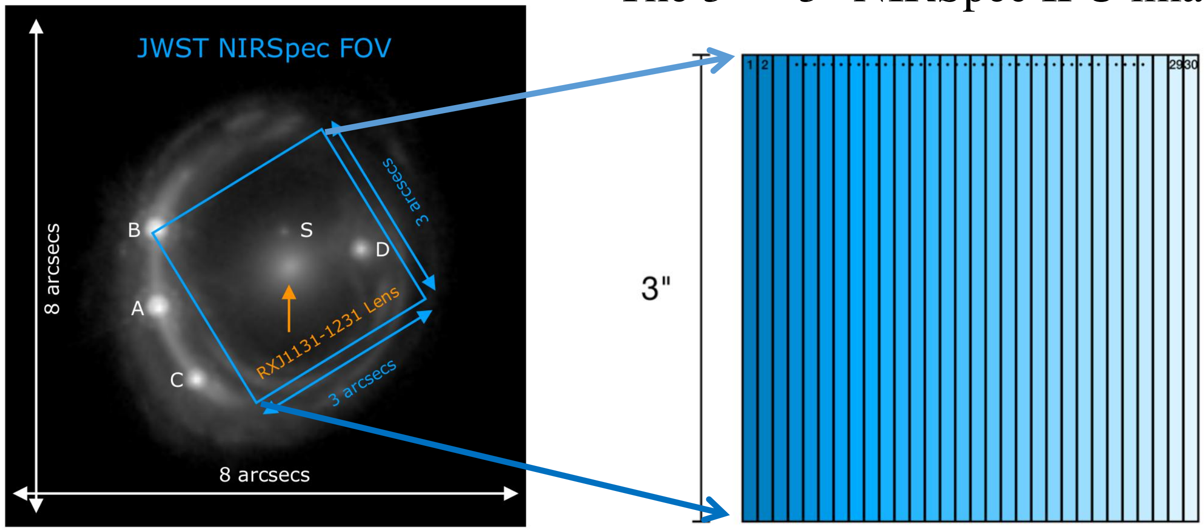


- Higher redshift, higer posibility



(Oguri 2019)

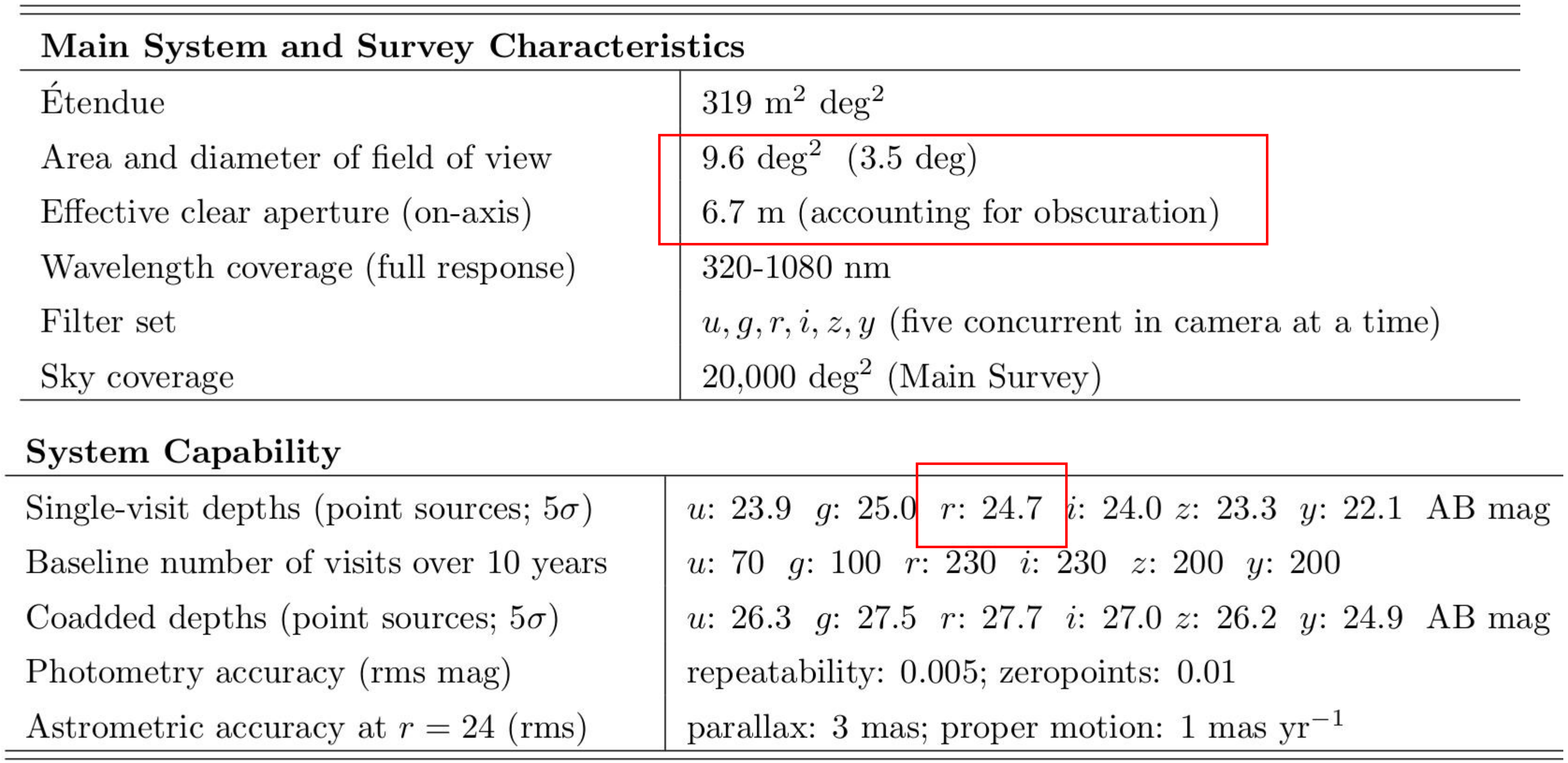
The 3" × 3" NIRSpec IFU image slices (0.1")



- Quasars are very bright so that they outshine their host galaxies and sometimes lensing galaxies as well.
- IFU Spectroscopy will be important!

The future telescope/survey

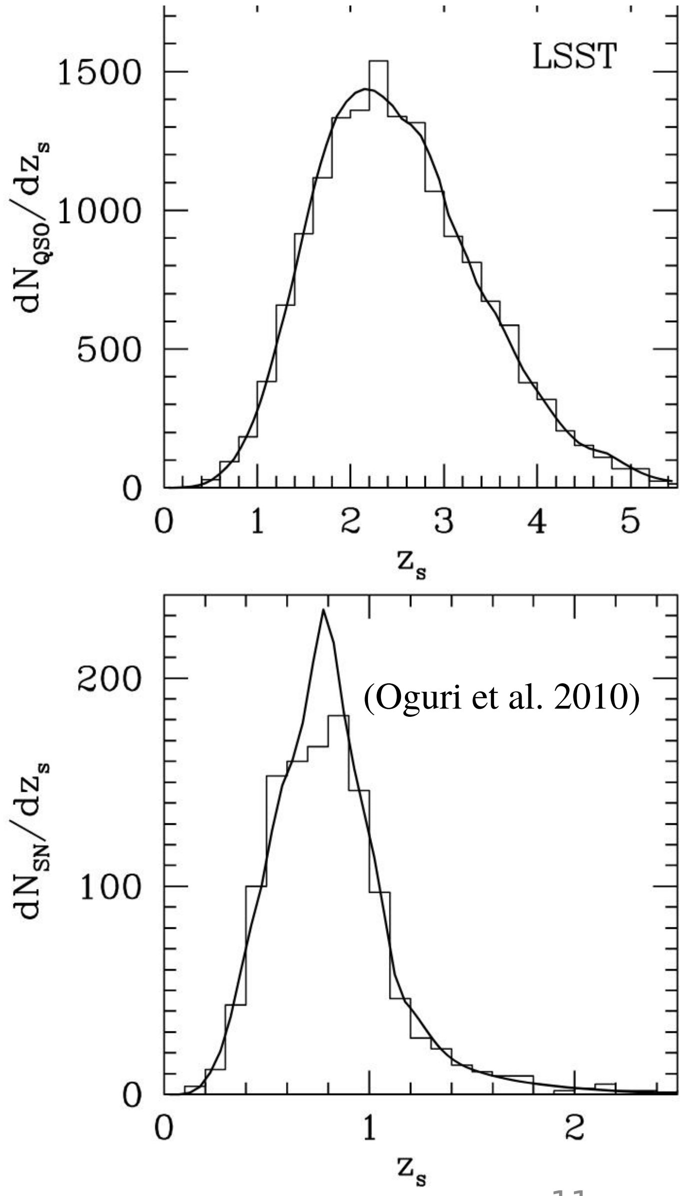
- Large Synoptic Survey Telescope(LSST)



Main System and Survey Characteristics	
Étendue	319 m^2 deg^2
Area and diameter of field of view	9.6 deg^2 (3.5 deg)
Effective clear aperture (on-axis)	6.7 m (accounting for obscuration)
Wavelength coverage (full response)	320-1080 nm
Filter set	u, g, r, i, z, y (five concurrent in camera at a time)
Sky coverage	20,000 deg^2 (Main Survey)

System Capability	
Single-visit depths (point sources; 5σ)	u: 23.9 g: 25.0 r: 24.7 i: 24.0 z: 23.3 y: 22.1 AB mag
Baseline number of visits over 10 years	u: 70 g: 100 r: 230 i: 230 z: 200 y: 200
Coadded depths (point sources; 5σ)	u: 26.3 g: 27.5 r: 27.7 i: 27.0 z: 26.2 y: 24.9 AB mag
Photometry accuracy (rms mag)	repeatability: 0.005; zeropoints: 0.01
Astrometric accuracy at $r = 24$ (rms)	parallax: 3 mas; proper motion: 1 mas yr^{-1}

1000 visits (summed over all six bands) during the anticipated 10 years



(Oguri et al. 2010)

Time delay cosmography

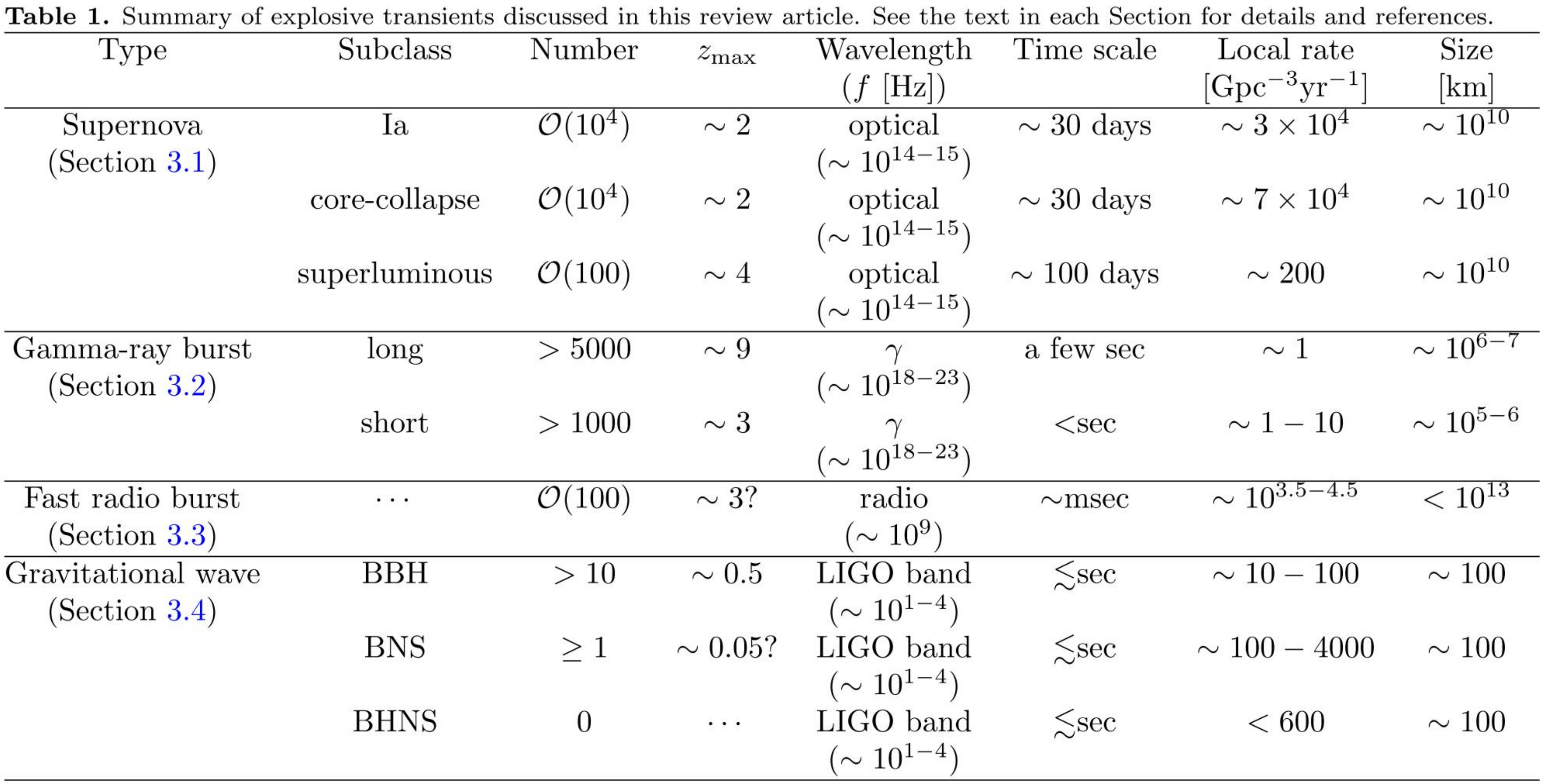
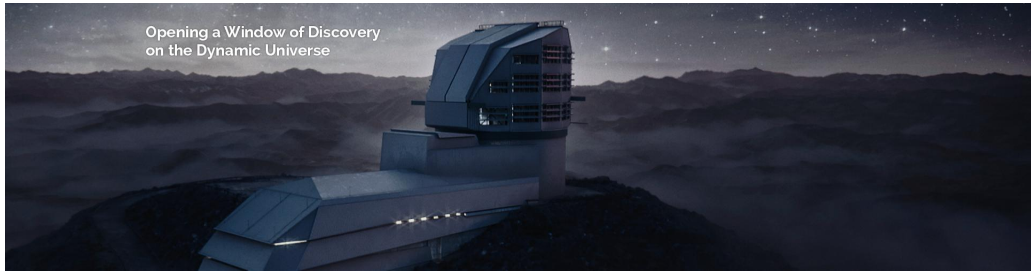


Table 1. Summary of explosive transients discussed in this review article. See the text in each Section for details and references.

Type	Subclass	Number	$z_{\max}$	Wavelength (f [Hz])	Time scale	Local rate [$\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$]	Size [km]
Supernova (Section 3.1)	Ia	$\mathcal{O}(10^4)$	~ 2	optical ($\sim 10^{14-15}$)	~ 30 days	$\sim 3\times10^4$	$\sim 10^{10}$
	core-collapse	$\mathcal{O}(10^4)$	~ 2	optical ($\sim 10^{14-15}$)	~ 30 days	$\sim 7\times10^4$	$\sim 10^{10}$
	superluminous	$\mathcal{O}(100)$	~ 4	optical ($\sim 10^{14-15}$)	~ 100 days	~ 200	$\sim 10^{10}$
Gamma-ray burst (Section 3.2)	long	> 5000	~ 9	γ ($\sim 10^{18-23}$)	a few sec	~ 1	$\sim 10^{6-7}$
	short	> 1000	~ 3	γ ($\sim 10^{18-23}$)	$<$sec	$\sim 1-10$	$\sim 10^{5-6}$
Fast radio burst (Section 3.3)	$\cdots$	$\mathcal{O}(100)$	$\sim 3?$	radio ($\sim 10^9$)	$\sim$msec	$\sim 10^{3.5-4.5}$	$< 10^{13}$
Gravitational wave (Section 3.4)	BBH	> 10	~ 0.5	LIGO band ($\sim 10^{1-4}$)	$\lesssim$sec	$\sim 10-100$	~ 100
	BNS	≥ 1	$\sim 0.05?$	LIGO band ($\sim 10^{1-4}$)	$\lesssim$sec	$\sim 100-4000$	~ 100
	BHNS	0	$\cdots$	LIGO band ($\sim 10^{1-4}$)	$\lesssim$sec	< 600	~ 100

(Oguri 2019)

Summary



- The uncertainties of H_0 measurement in strong lensing is dominated by the Mass Sheet Degeneracy and error of time delay measurement.
- Explosive transients (like SN, FRB, and so on) will be powerful probes in time delay cosmology.
- The future telescope like JWST/LSST/Euclid will improve the accuracy of H_0 significantly(~2%).