Fuzzy Dark Matter

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✓ Background introduction

☐ Cosmology

☐ Galaxy

☐ Experimental studies

Outline of Background introduction

- What is Fuzzy Dark Matter (FDM)
- Particle physics motivation
- FDM numerical simulation
- Phenomenology

Before Fuzzy Dark Matter… What is Dark Matter?

- Existence of DM was first inferred by Zwicky in 1933, who discovered the “missing mass”
- Rich evidence for the existence of dark matter
- Range of possible mass is yet inconclusive

We are ignorant about the properties of DM

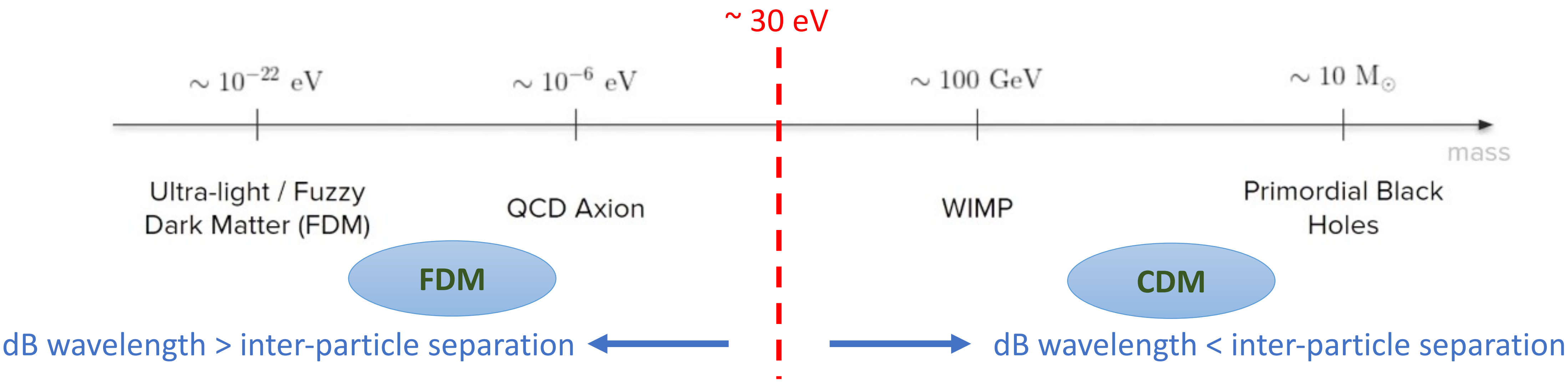
- Unknow range of possible mass & supporting theories

- Where should we consider DM as a wave instead of particle?

$$\lambda_{dB} = \frac{2\pi}{mv}$$

We are ignorant about the properties of DM

- Unknow range of possible mass & supporting theories



~ 30 eV

FDM

CDM

dB wavelength > inter-particle separation

dB wavelength < inter-particle separation

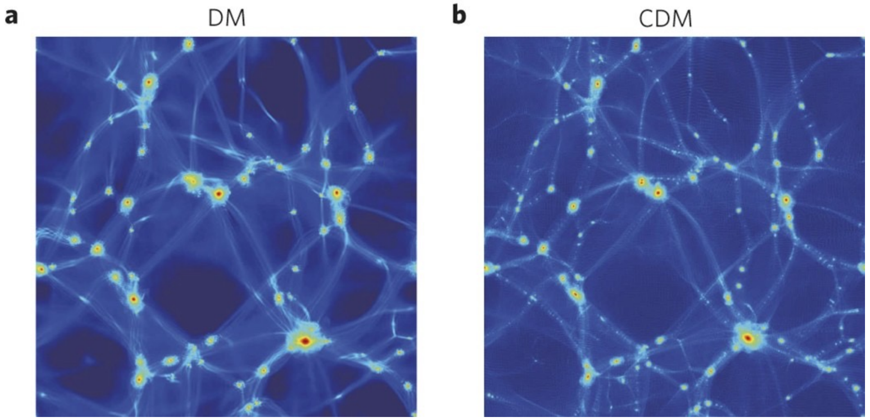
- Where should we consider DM as a wave instead of particle?

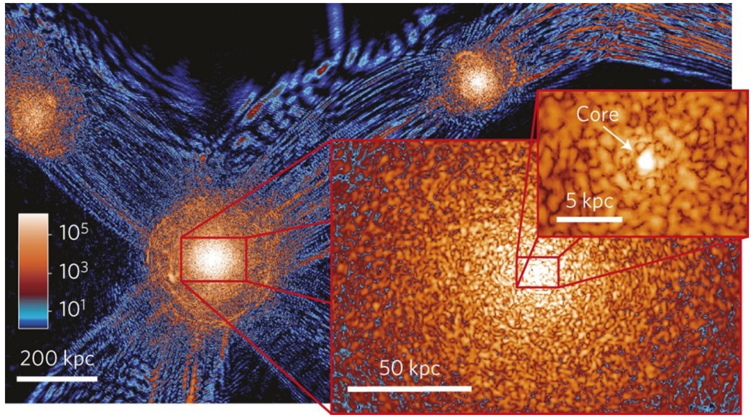
$$\lambda_{dB} = \frac{2\pi}{mv}$$

gives m ~ 30 eV

FDM and CDM are similar in large scales

- FDM is similar to CDM in large scale





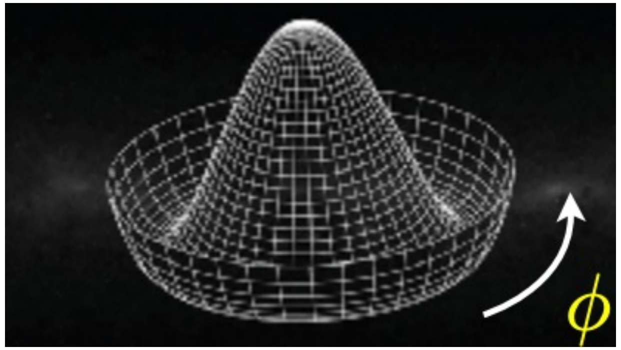
Schive, Chiueh, Broadhurst 2014

- In small scale FDM shows interference pattern, which motivates scientist on the cosmological formation of dwarf galaxy halos
- Get rid of the bothering issues of CDM simulation: smaller satellites problem, cusp-or-core problem, and “too big to fail” problem

Inconvenience of ultra-light particle

- Extremely hard to simulate
- CDM: adaptive mesh refinement (AMR) or smoothed particle hydrodynamics (SPH) techniques.
- FDM: you'll always need to resolve down to de Broglie wavelength in every cell of your box.

Particle physics motivations

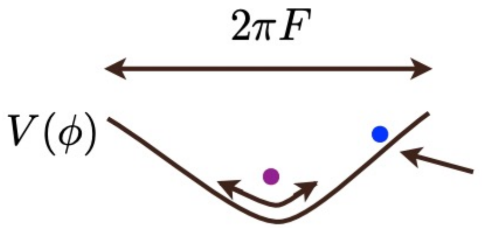


- Pseudo-Nambu-Goldstone boson

- Relic abundance sets the mass value
- Axion-like field with potential from non-perturbative effects

$$\mathcal{L} \sim -\frac{1}{2}(\partial\phi)^2 - \Lambda^4(1 - \cos[\phi/F]) \qquad\qquad m \sim \Lambda^2/F$$

- Mis-alignment mechanism: mass range is relatively large



$\phi \sim F$ at early times until $H \sim m$

$$\Omega_{\text{matter}} \sim 0.1 \left(\frac{F}{10^{17}\,\text{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\text{eV}}\right)^{1/2}$$

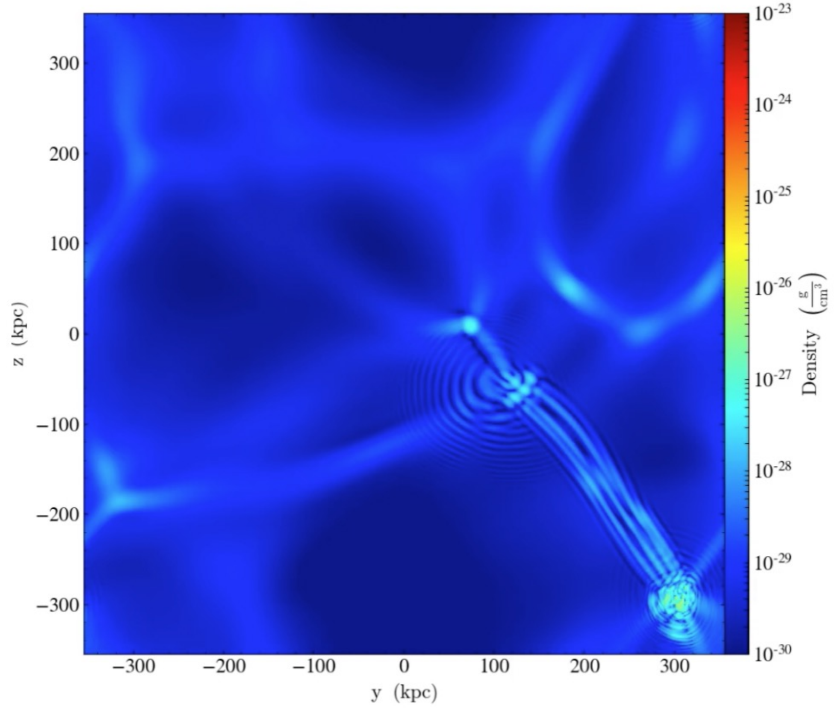
Construction of wave halos

- Schrodinger-Poisson system
- $i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m_a}\nabla^2 + m_a V\right)\psi$
- $\nabla^2 V = 4\pi G\rho = 4\pi G m_a |\psi|^2$

Construction of wave halos -- fluid description

- Consider ψ as a classical fluid
- $\psi = \sqrt{\rho/m}\, e^{i\theta} \longrightarrow \rho = m|\psi|^2$
- Mass conservation: $\frac{\partial \rho}{\partial t} + \nabla \cdot \rho v = 0,$ where $v = \frac{1}{m}\nabla\theta$
- Euler equation: $\frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla V + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$

Numerical simulations



Schive, Chiueh, Broadhurst 2014;
Li et al. 2019

- Wave effects in a cosmological simulation.

$$\psi(t,\vec{x}) = \sum_{\vec{k}} A_{\vec{k}} e^{iB_{\vec{k}}} e^{i\vec{k}\cdot\vec{x} - i\omega_k t}$$

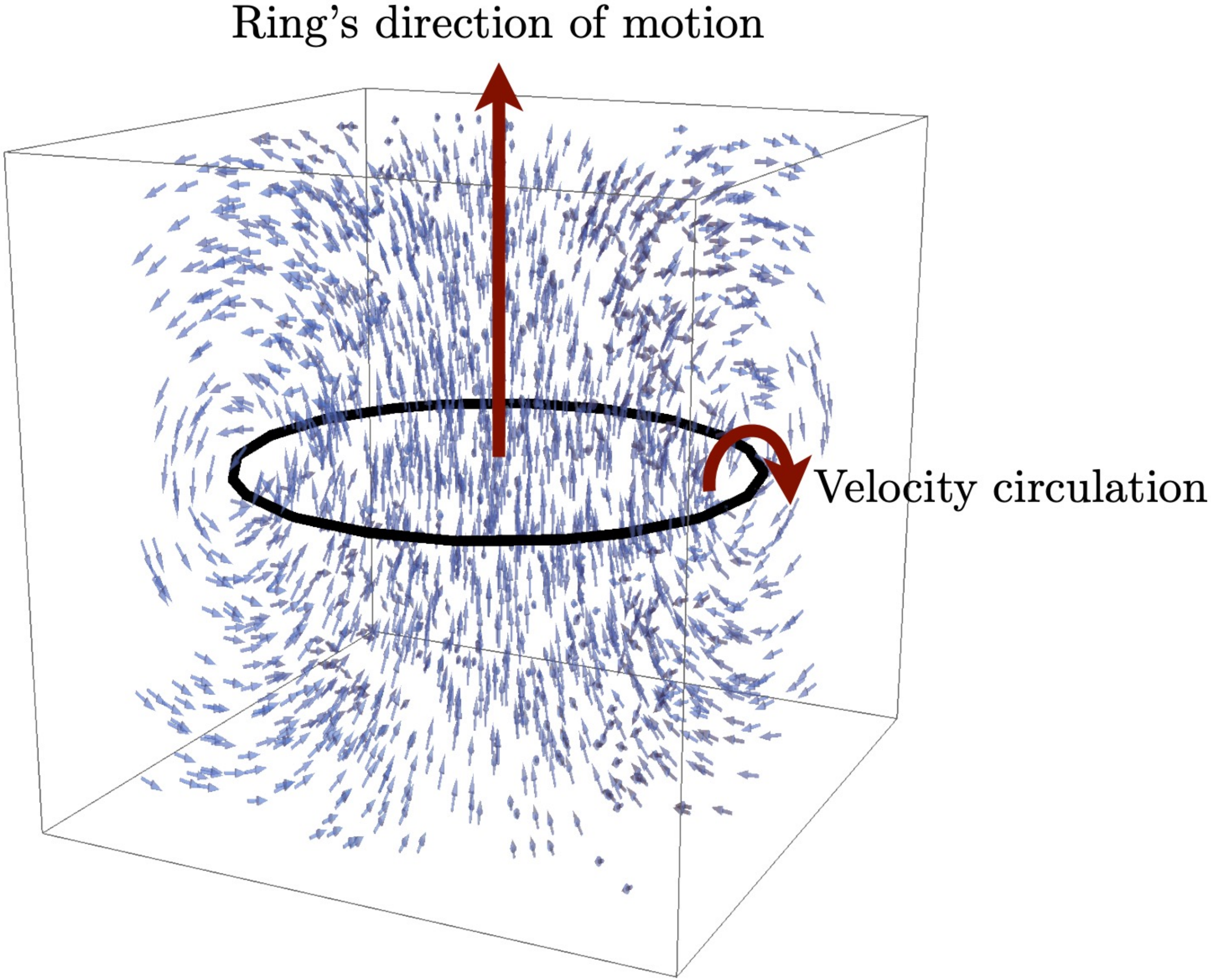
- Figure: a z = 5 snapshot of the dark matter density in a cosmological simulation of ultra-light dark matter with m = 10^{-22} eV

- Presence of interference fringes: a characteristic prediction of wave dark matter

Wave interference – vortices

- Naively, vorticity cannot exist because the velocity field is gradient flow.
- Consider fluid formation $\psi = \sqrt{\rho/m}\, e^{i\theta}$
- The loophole: when ρ=0
- No vortices in early universe
- Vortex generally takes the form of a loop i.e. vortex ring

Hui 2021



Hui 2021

Take-away messages

- FDM represents for the ultra-light DM, where we should consider as a wave instead of particles.
- FDM is more convenient in small-scale simulation compared to CDM, where you can see interference fringes pattern.
- The particle physics approach sets a range for FDM mass.
- There are 2 ways of wave construction of FDM: Schrodinger-Poisson system and fluid description.
- Existence of vortices is a important phenomenology of FDM.

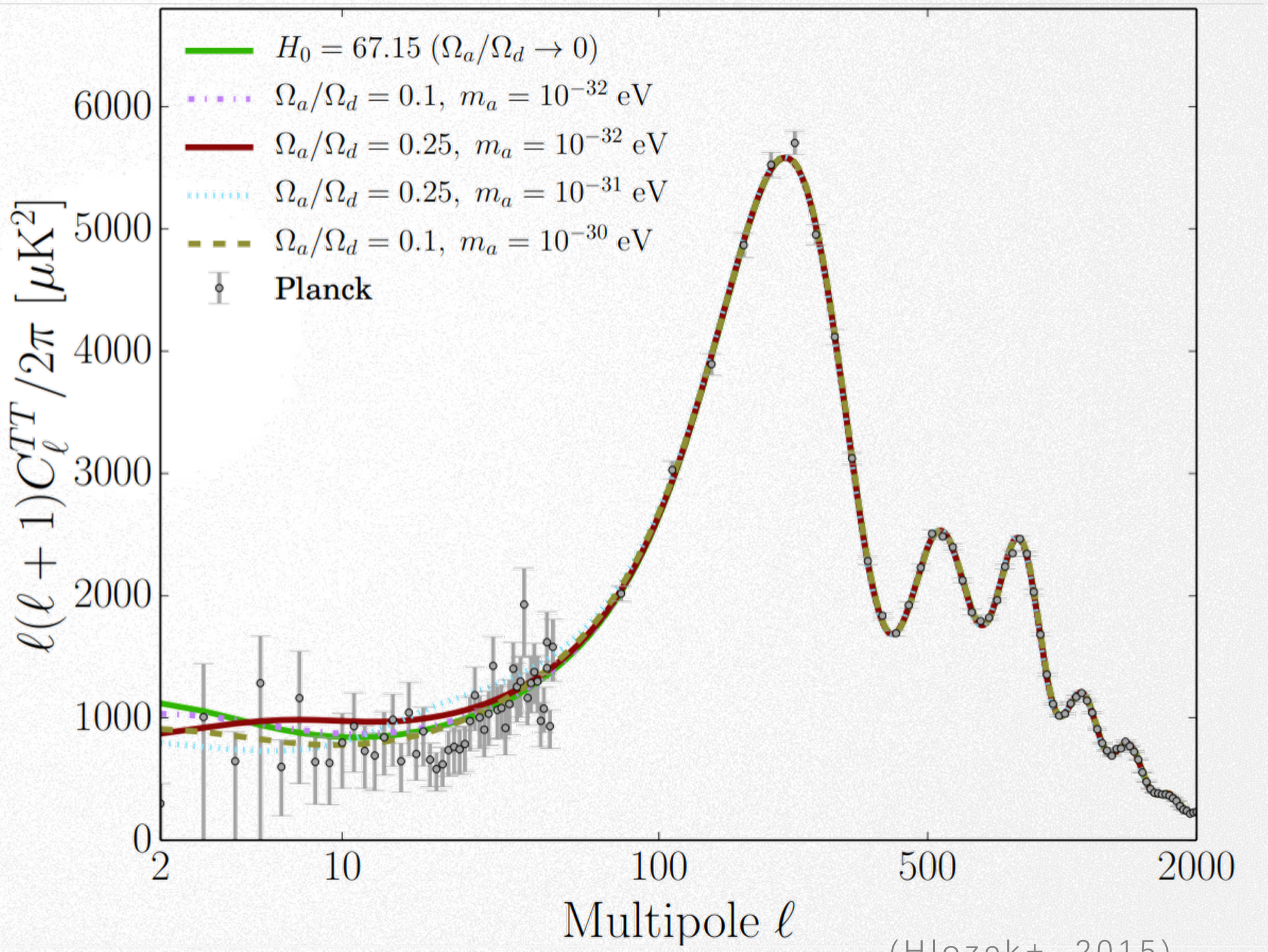
Now let’s talk about observational / experimental implications and constraints

- ❑ Cosmology by Xiaochen Sun
- ❑ Galaxy by Ruizhe Feng
- ❑ Experimental studies by Jiejia Liu

Cosmology Implications & Constraints

$z{\sim}1100$: CMB

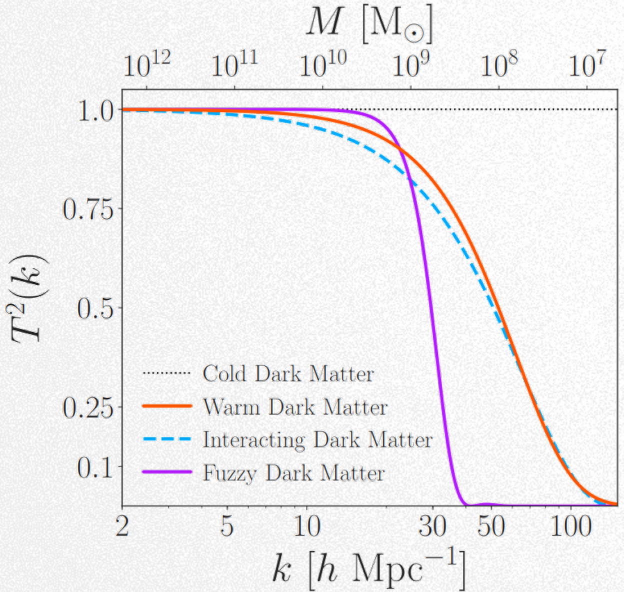
- How:
  - EoS: $w_a \equiv \frac{p_a}{\rho_a} = \begin{cases} -1, \text{early} \\ 0, \text{later} \end{cases}$
  - Sound speed $\frac{\delta p_a}{\delta \rho_a}$ depneds on wavenumber
- Others: Lensing deflection & E-mode polarization
- Uncertainty: FDM coupling w/ others?
- Constraint: $m_a > 10^{-24}\mathrm{eV}$ (Hlozek+, 2018)



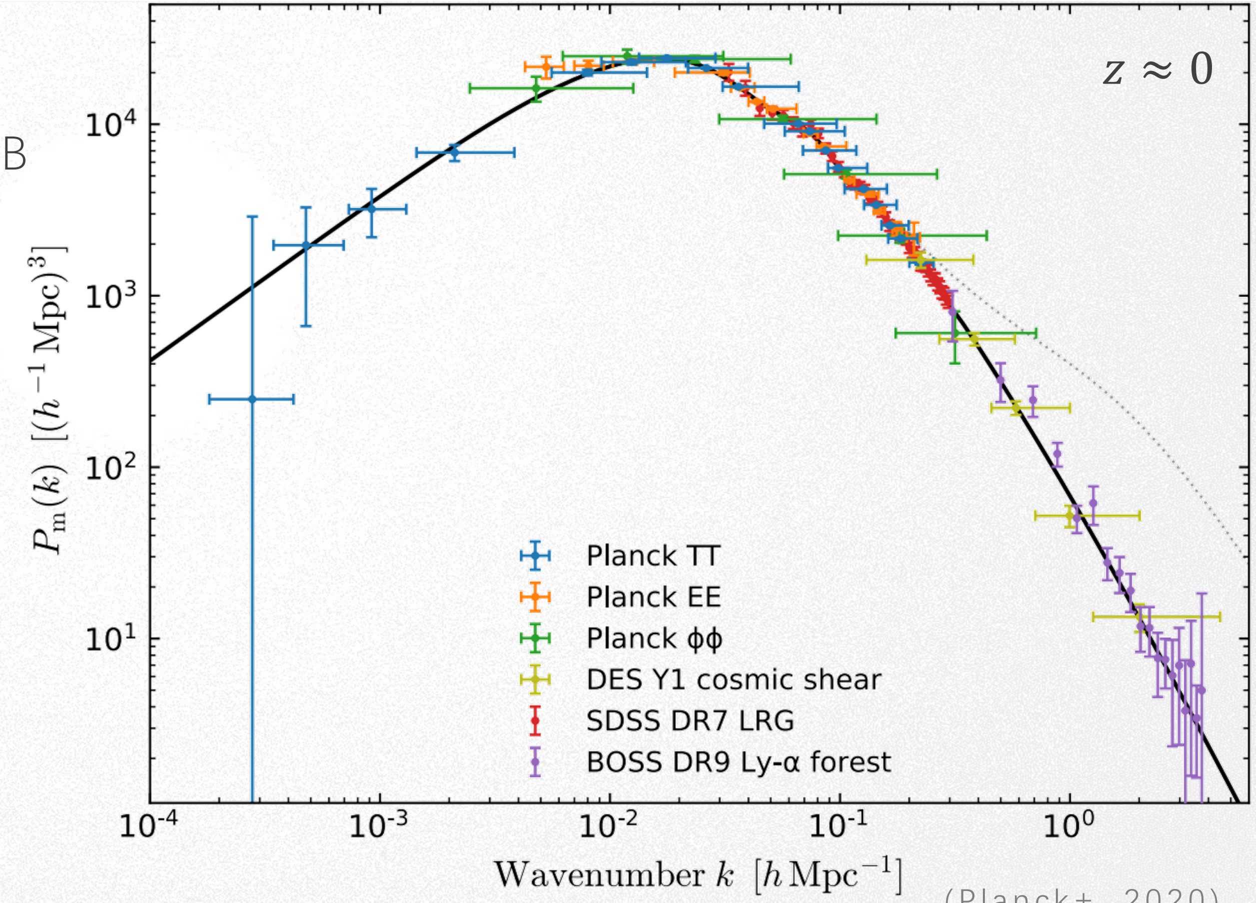
(Hlozek+, 2015)

$z\sim\mathcal{O}(1)$: (linear) matter power spectrum

- Fourier transfer from correlation function + Flux absorption from Lyα + Linear perturbation
- $P_m(k,z) = P_o(k)\times T^2(k,z)$
  - $P_o(k)$: primordial power spectrum from CMB
  - $T^2(k,z)$: transfer functions from DM



(Nadler+, 2020)



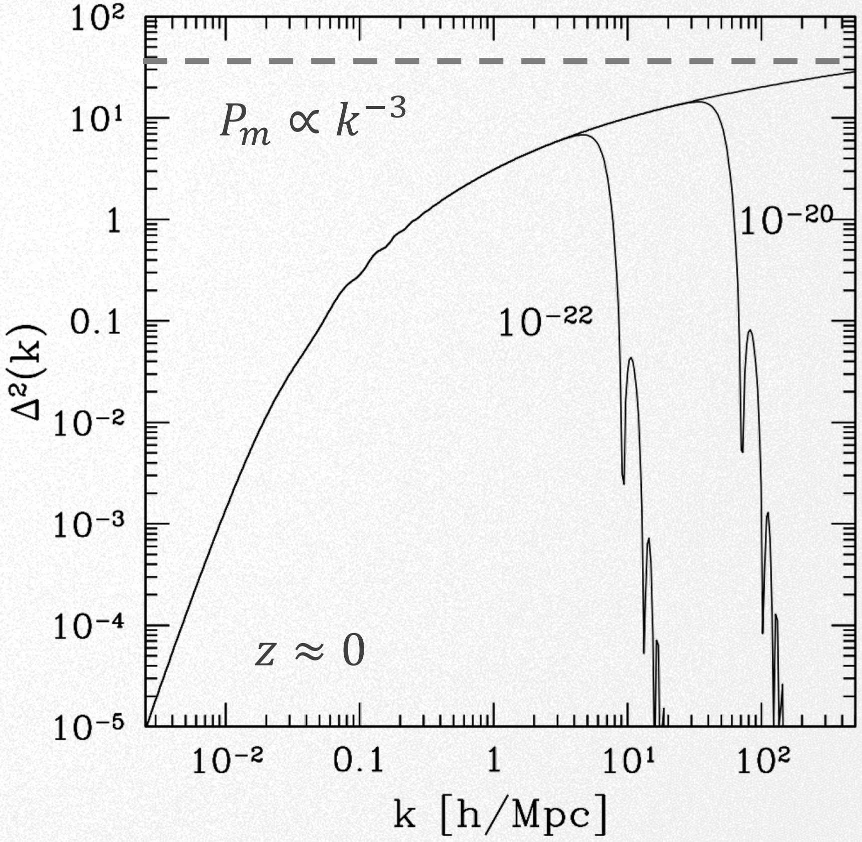
(Planck+, 2020)

$z\sim\mathcal{O}(1)$: (linear) matter power spectrum

- Fourier transfer from correlation function + Flux absorption from Lyα + Linear perturbation
- $P_m(k,z) = P_o(k)\times T^2(k,z)$
- How: quantum pressure
    - $p_a \propto -k^2\rho\,\ln\rho/m_a$

      ↓
    - pressure wins gravity on small scales

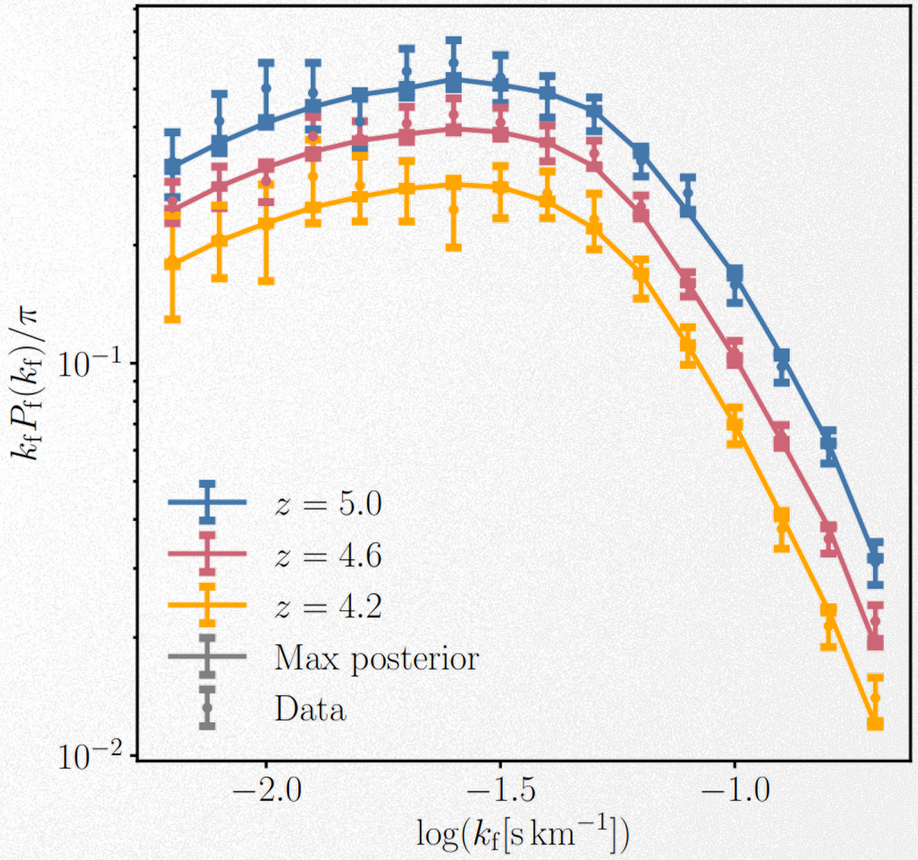
      ↓
    - suppress fluctuations



(Hu+, 2000)

$z \sim \mathcal{O}(1)$: (linear) matter power spectrum

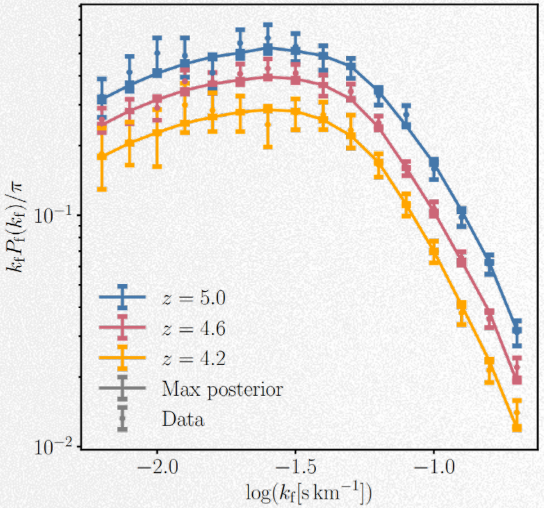
- Fourier transfer from correlation function + Flux absorption from Lyα + Linear perturbation
- $P_m(k,z) = P_o(k) \times T^2(k,z)$
- How: quantum pressure
- Uncertainty: IGM model & ionizing background
- Constraint: $m_a > 2\times10^{-20}\mathrm{eV}$



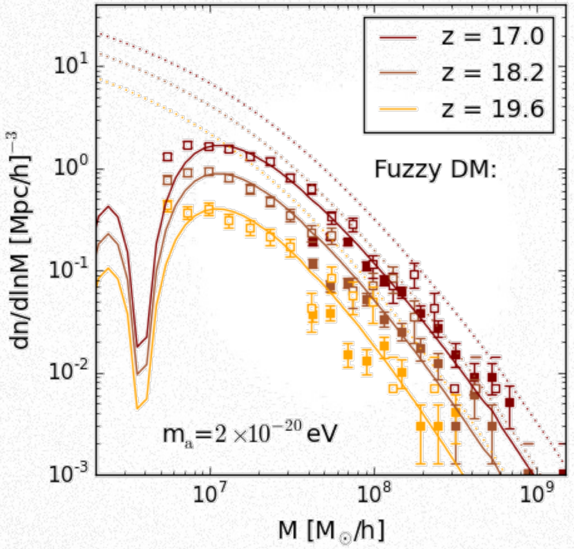
(Rogers & Peiris, 2021)

$z\sim\mathcal{O}(10)$: 21cm lines

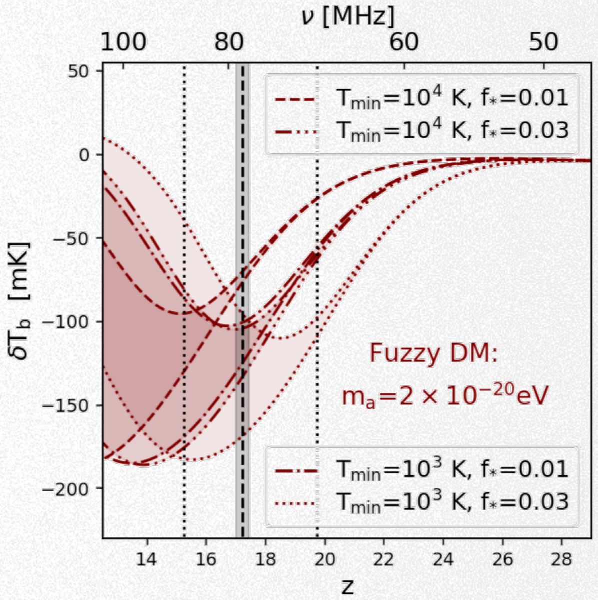
- Matter power spectrum $\xrightarrow{\text{Integrate}}$ Halo mass function $\xrightarrow{\text{Accrete \& Reionize}}$ 21cm absorption signal



(Rogers & Peiris, 2021)



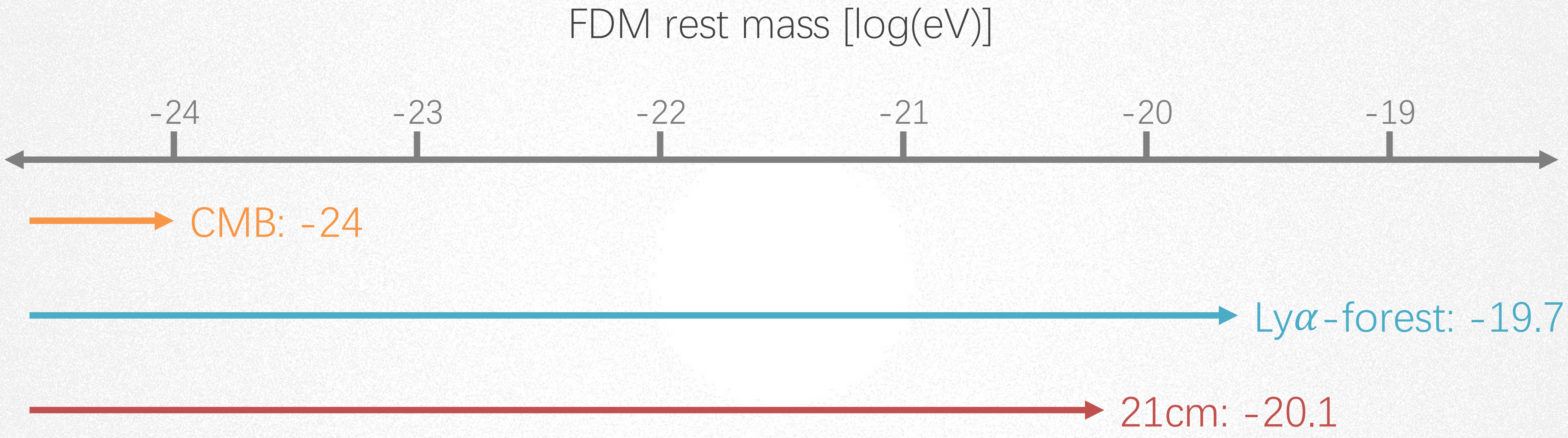
(Schneider, 2018)



(Schneider, 2018)

- Uncertainty: stellar-to baryon fraction, minimum virial temperature & gas heating processes
- Constraint: $m_a > 8\times10^{-21}\mathrm{eV}$ from EDGES

Mini Summary



FDM rest mass [log(eV)]

CMB: -24

Lyα-forest: -19.7

21cm: -20.1

- FDM always suppress small scale clustering due to quantum pressure.

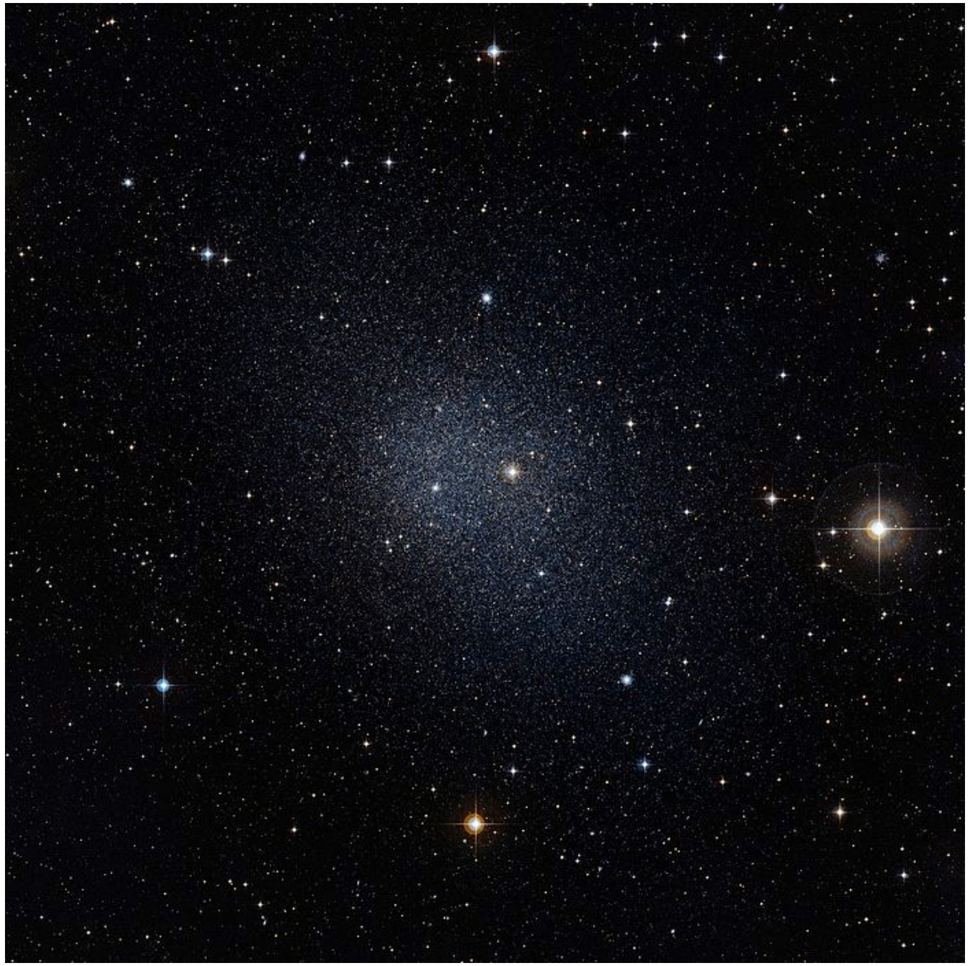
Fuzzy dark matter: observational implications and constraints related to galaxies

Outline

- Explanation: Fornax dwarf galaxy
- Constraint: Subhalo mass function from stellar streams
- Prediction: Interference substructures

Fornax dwarf galaxy

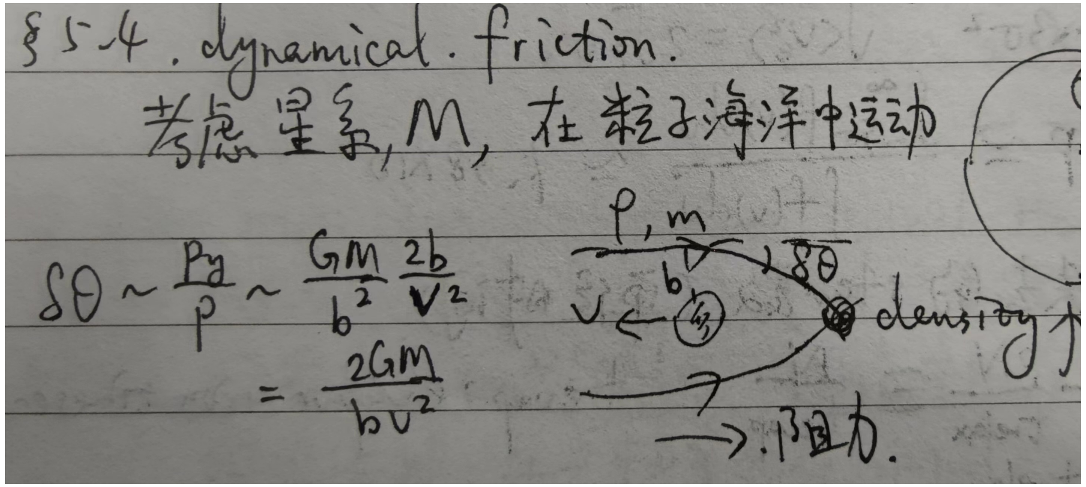
- One of the Milky Way's neighboring dwarf galaxies
- It contains five globular clusters
- Dynamical friction should have caused
  - most of the clusters spiral to the center
  - merge to form a prominent nucleus
- BUT, this is NOT seen



Credit: ESO/Digitized Sky Survey 2

Dynamical friction

- Consider a subject body traveling through a population of field stars
- The field stars will be focused behind the subject body due to gravity, leading to a drag which slows down the subject body
- Dynamical friction will cause **orbital decay** of globular clusters



§5-4. dynamical. friction.

考虑星系, M, 在粒子海洋中运动

$$\delta\theta \sim \frac{P_y}{p} \sim \frac{GM}{b^2}\frac{2b}{V^2} = \frac{2GM}{bv^2}$$

Credit: notes from Shude's course

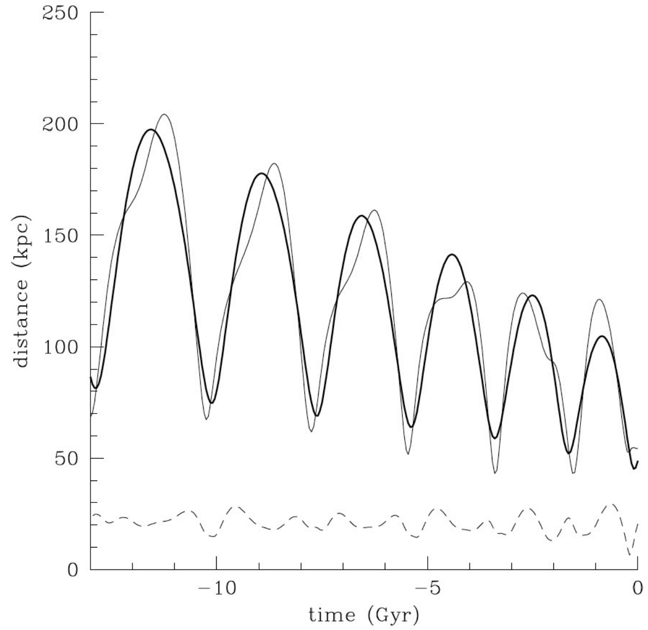


Figure 8.3 The decay of the orbits of the Magellanic Clouds around our Galaxy.

From Binney & Tremaine, Galactic Dynamics

If the dark matter in Fornax is FDM rather than CDM …

- The dynamical friction is suppressed
  - the wave nature of FDM is expected to suppress the overdensity, reducing the drag
  - standard estimates of the drag from dynamical friction must be modified
- Comparison between frictional decay times in FDM and CDM for the five Fornax clusters shows
  - Substantial increase in the time scale for dynamical friction
  - The orbital decay times are longer in a FDM halo than in a CDM halo
  - The shortest decay time in the FDM halo exceeds 2 Gyr

$$\tau = \frac{37.5\ \mathrm{Gyr}}{C}\left(\frac{\mathfrak{M}(r)}{10^8\ M_\odot}\,\frac{1\ \mathrm{kpc}}{r}\right)^{3/2}\frac{10^5\ M_\odot}{m_{\mathrm{cl}}}\,\frac{0.01\ M_\odot\,\mathrm{pc}^{-3}}{\rho(r)}$$

$m = 3\times10^{-22}$ eV

τ (Gyr)	τ (Gyr)
112	215
9.7	12
0.62	2.2
0.37	10
21.3	31
CDM	FDM

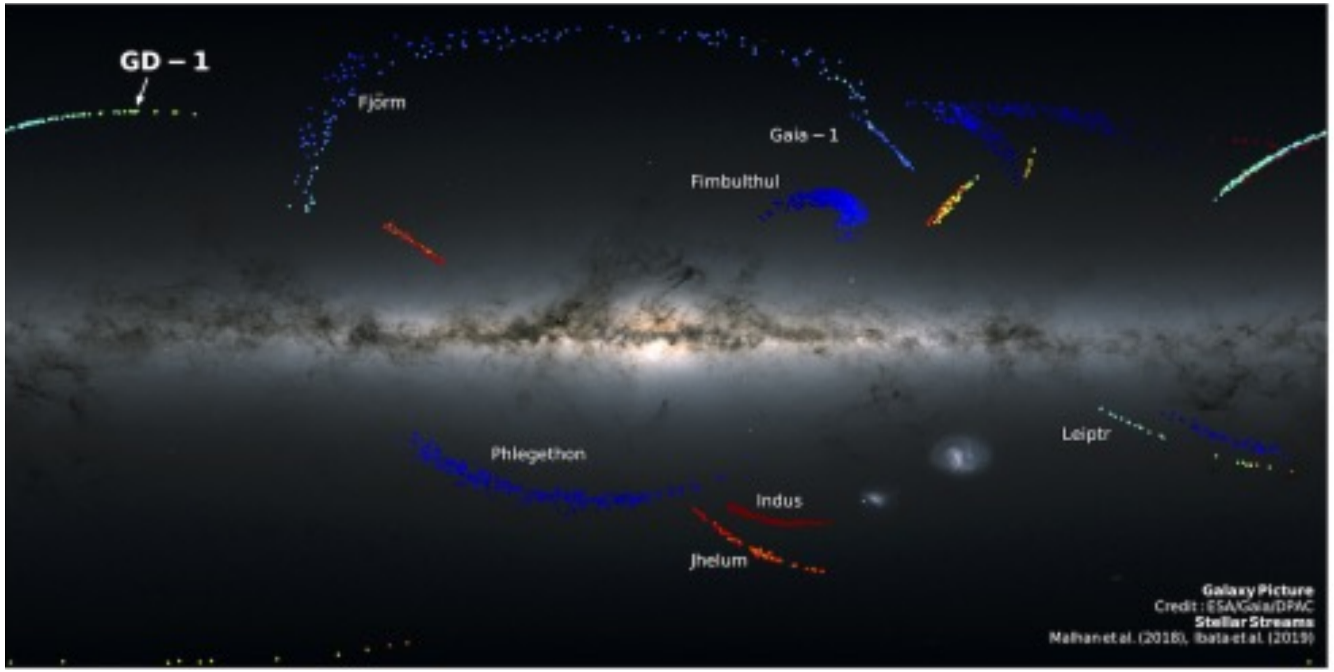
(Hui et al. 2017)

Subhalo mass function

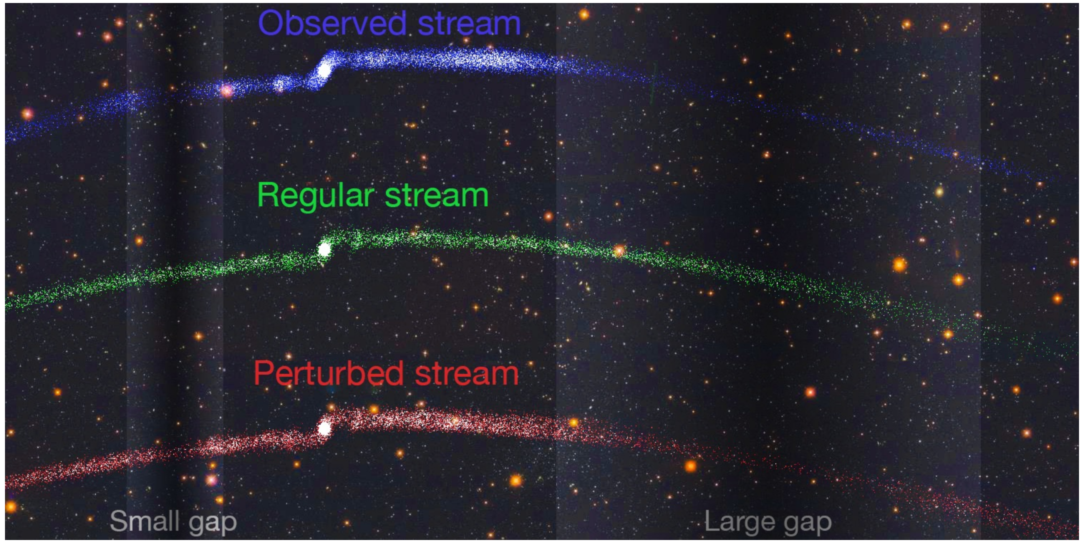
- The cold nature of DM causes it to form structure hierarchically starting from small gravitationally-bound halos that merge together to form larger halos
- The small halos are dense and highly concentrated, many of them survive tidal stripping and should exist as subhalos in galaxies today
- If the initial conditions of DM are warm (e.g. sterile neutrino models) or if DM is an ultra-light axion, then the abundance of lower-mass subhalos is significantly reduced

Stellar stream

- Elongated, almost one-dimensional structures produced by the tidal disruption of globular clusters or dwarf galaxies merging into the Milky Way
- They are highly sensitive to perturbations from passing DM subhalos
- Thus provide a means of measuring subhalo mass function



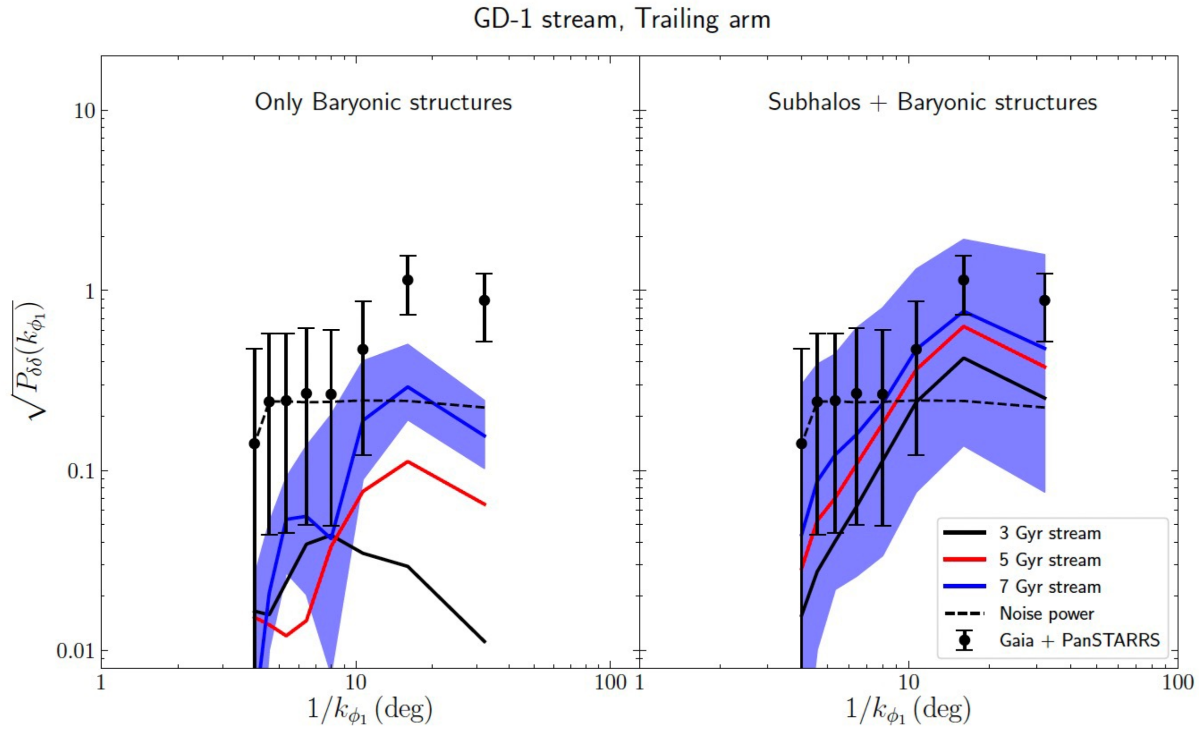
Stellar streams found in Gaia DR2



Observations of Pal 5 compared with simulations
From: https://people.ast.cam.ac.uk/~derkal/files/pal5_pr/

Power spectrum analysis

- 1D power spectrum of the density contrast from the normalized linear density
  - which encodes the correlation in density contrast as a function of the angular scale
- Observations: GD-1 stream and Pal 5 stream
- Compare the observed power spectrum with the one predicted with mock realizations
  - Mock streams (baryonic substructures)
  - Mock streams + subhaloes (baryonic plus DM substructures from parameterized subhalo mass function)

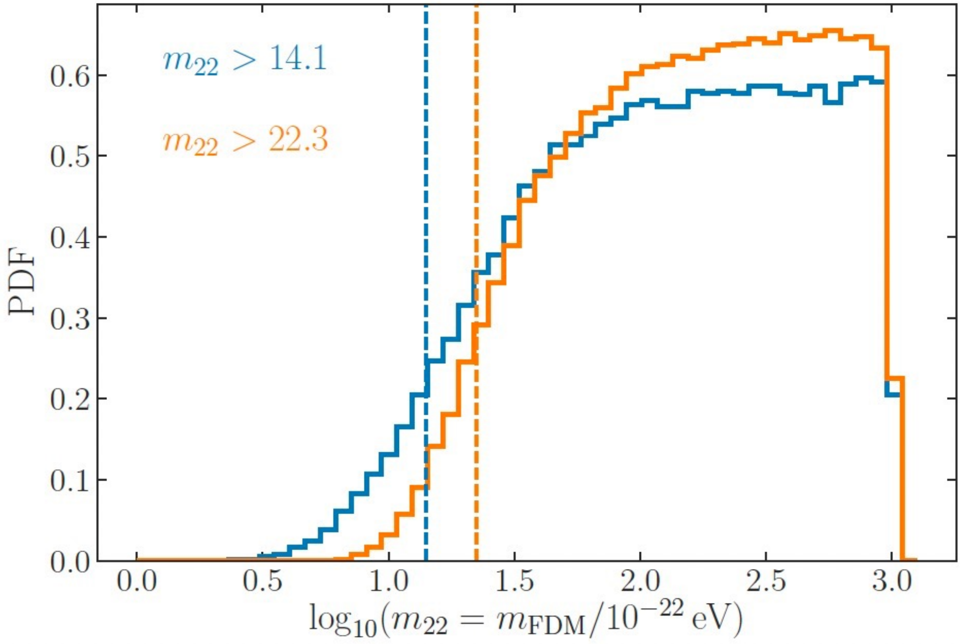


(Banik et al. 2021)

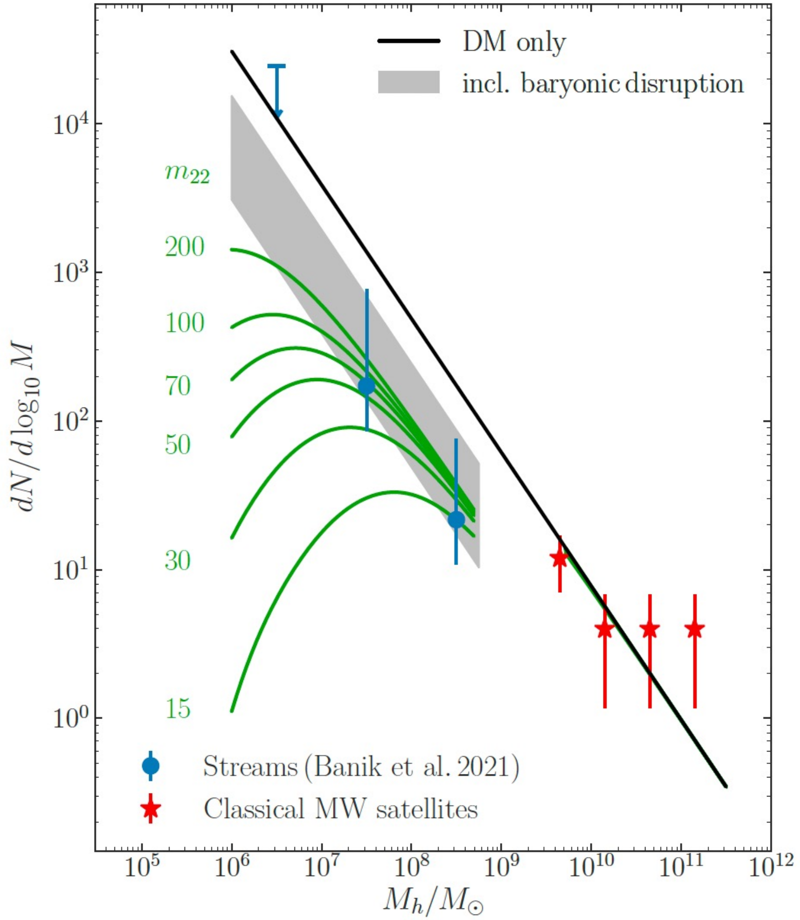
- FDM subhalo mass function is related to the particle mass

$$\left(\frac{dn}{dM}\right)_{\rm FDM} = \left[1+\left(\frac{M}{2\,m_{22}^{-1.6}\times 10^{8}\,M_\odot}\right)^{-0.72}\right]^{-13.9}\left(\frac{dn}{dM}\right)_{\rm CDM} + \frac{0.014\,m_{22}^{1.5}}{M}\exp\left[-\left(\ln\left\{\frac{M}{4.7\,m_{22}^{-1.5}\times 10^{8}\,M_\odot}\right\}\right)^{2}/1.4\right]$$

- Fitting to get posterior PDF for particle mass
  - Constraint: $m > 1.4\times10^{-21}$ eV



(Banik et al. 2021)



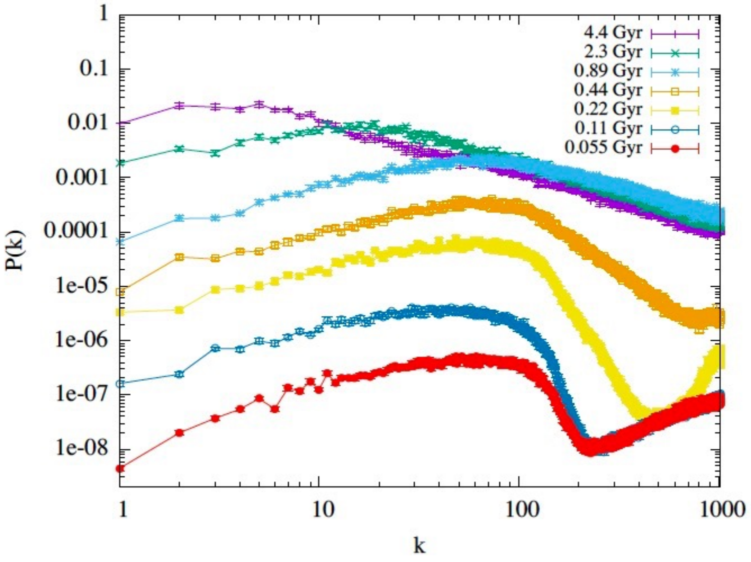
Interference substructures

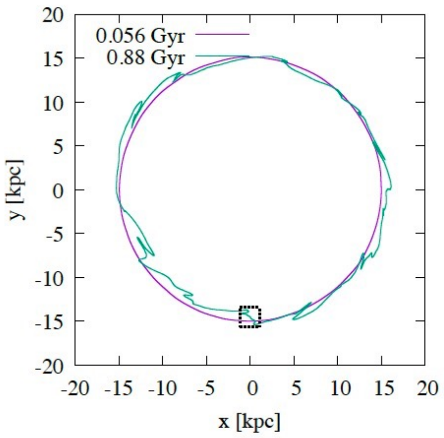
- A general feature of FDM models is the presence of ubiquitous density fluctuations
  - order unity density fluctuations on the scale of the de Broglie wavelength
  - can take the density all the way to zero (complete destructive interference, i.e., vortices)
  - distinct from subhalos as a form of halo substructure
  - these granular fluctuations occur because of the interference between bound waves in halos,
- These time-varying fluctuations can disturb the motions of stars, leading to potentially observable signatures in cold thin tidal streams in Galaxy
  - FDM model can generate significant small-scale structure in tidal streams

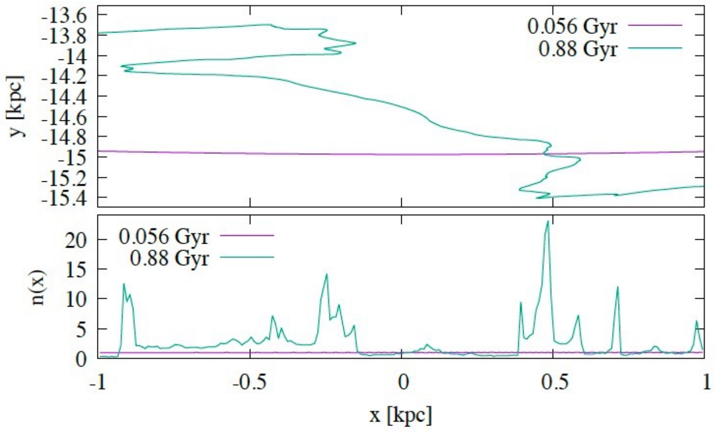
Calculation of FDM perturbations on tidal streams

- Mock stream: test particles placed uniformly along a circular ring at radius $r_0 = 15\ \mathrm{kpc}$
- The power spectra exhibit a sharp cutoff corresponding to the de Broglie wavelength of the FDM potential fluctuations
    - the gravitational forces are smooth on small scales, so the displacement perturbations and density perturbations are also smooth on small scales
- When stream perturbations become nonlinear, fold caustics generically arise
    - the cutoff disappears, and instead the power spectrum becomes a power-law
    - saturation in amplitude

$$m = 10^{-22}\ \mathrm{eV}$$
$$v_c = 200\ \mathrm{km/s}$$
$$\rightarrow \lambda = 0.6\ \mathrm{kpc}$$
$$\rightarrow \frac{2\pi r_0}{\lambda} \approx 157$$







(Dalal et al. 2021)

Take-home message

- FDM model can explain the survival of globular clusters against orbital decay in Fornax dwarf galaxy
- The subhalo mass function can be measured from perturbations of stellar streams, and put constraints on the particle mass of FDM
- The interference substructures predicted by FDM model can be probed by power spectrum of tidal streams, which will show a cutoff and fold caustics

Observational/Experimental Detection of FDM

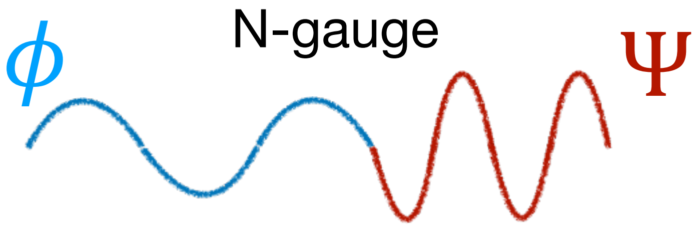
Outlines

- Astrophysical: pulsar timing array
- Experimental:
  - CAPSEr-Wind
  - Atomic clock

Time dependent gravitational oscillations

Scalar field of FDM

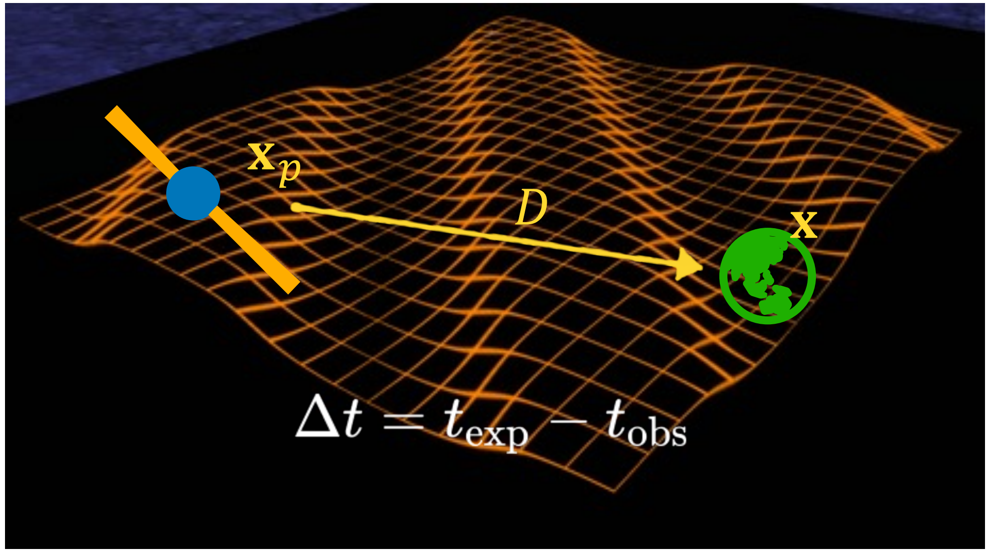
$$\phi(\mathbf{x},t) = A(\mathbf{x})\,\cos(mt + \alpha(\mathbf{x}))$$



Oscillating gravitational field

$$\Psi(\mathbf{x},t) \simeq \Psi_0 + \frac{\pi G \rho_{\mathrm{DM}}(\mathbf{x})}{m^2}\,\cos(2mt + 2\alpha(\mathbf{x}))$$

$$ds^2 = (1 + 2\Phi(\mathbf{x},t))dt^2 - (1 - 2\Psi(\mathbf{x},t))\delta_{ij}dx^i dx^j$$



Measure the time residual

$$\Delta t(t) = \Delta t_{\mathrm{DM}} \cos(2mt + \alpha(\mathbf{x}) + \alpha(\mathbf{x}_p) - mD)$$

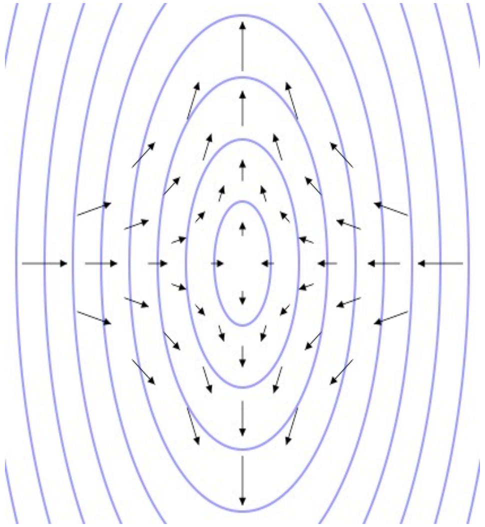
$$\Delta t_{DM} = \frac{\pi G \rho_{\mathrm{DM}}(\mathbf{x})}{m^3} \sin(mD + \alpha(\mathbf{x}) - \alpha(\mathbf{x}_p))$$

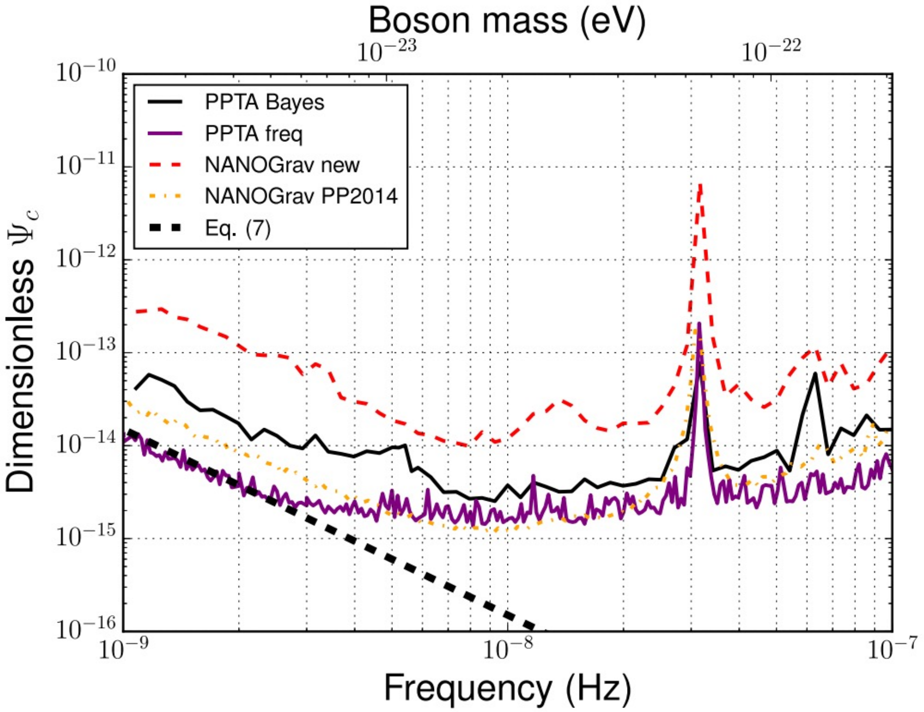
Sensitivity

similar to a gravitational wave background with characteristic strain

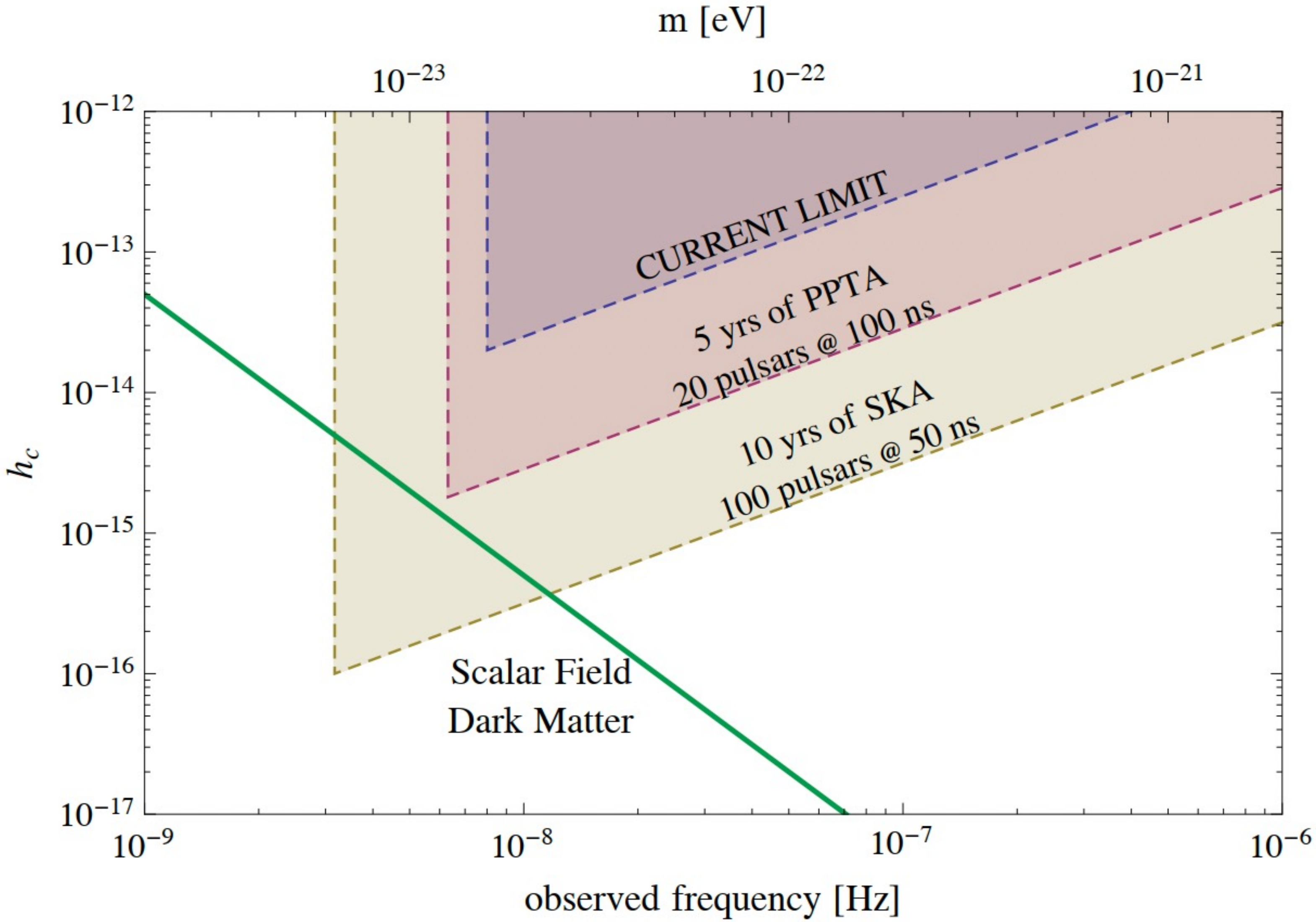
$$h_c = 2\sqrt{3}\Psi_c = 2 \cdot 10^{-15}\left(\frac{\rho_{\rm DM}}{0.3{\rm GeV/cm^3}}\right)\left(\frac{10^{-23}{\rm eV}}{m}\right)^2$$

$$f \equiv 2\pi\omega = 5 \cdot 10^{-9}\ {\rm Hz}\left(\frac{m}{10^{-23}{\rm eV}}\right)$$





Sensitivity



Take home message

- Scalar field of FDM causes time delay of pulsar signal
- It is similar to signals caused by GW but has 2 major differences:
  - FDM signal is independent on the direction to pulsar
  - It is monochromatic and can appear as an excess in the signal at particular frequency $\propto m$
- Currently ❎, future SKA ✅

Axial nuclear moment

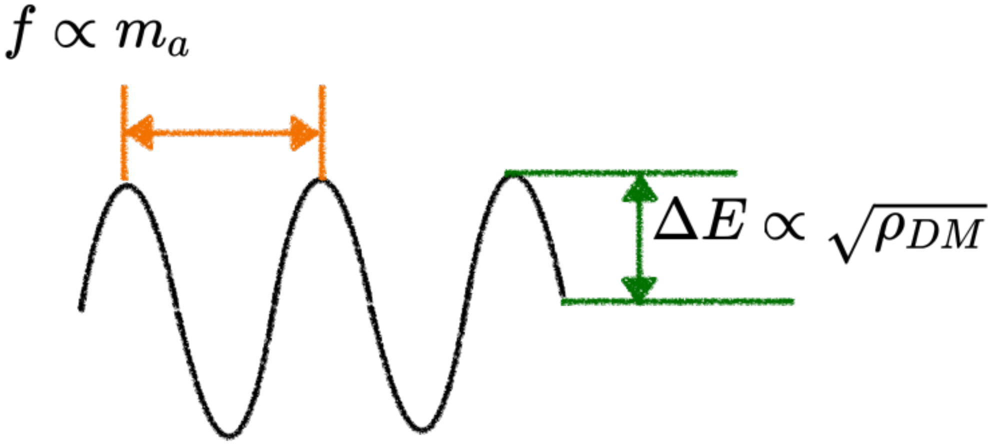
APL interaction with SM fermion:

$$\frac{\partial_\mu a}{f_a}\bar{\Psi}_f\gamma^\mu\gamma_5\Psi_f$$

APL nuclear current coupling

additional perturbation term in Hamiltonian

$$\begin{aligned} H_N &\supset g_{aNN}\nabla a\cdot\boldsymbol{\sigma}_N \\ &\supset g_{aNN}m_a a_0\cos(m_a t)\mathbf{v}\cdot\boldsymbol{\sigma}_N \end{aligned}$$



Energy oscillation amplitude

$$\Delta E \sim g_{aNN}\sqrt{\rho_{DM}}v$$

CAPSEr-Wind

CAPSEr (Cosmic Axion Spin Precession Experiment)

The lab has a relative motion with respect to the FDM wind

↓

The FDM-nucleon interaction causes the spin to precession around *v*

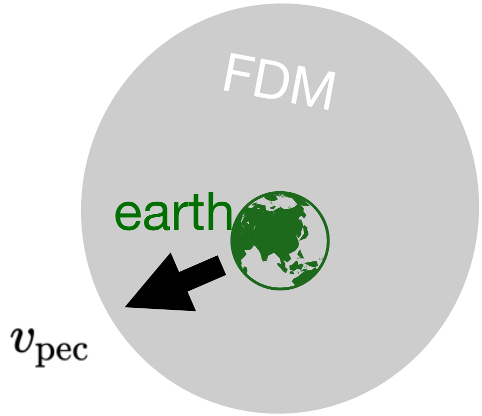
↓

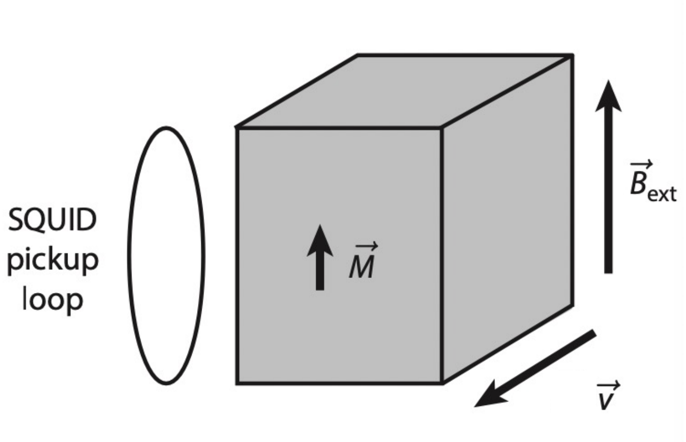
The spin precession both around *v* and *B*

↓

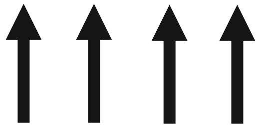
Transverse magnetization

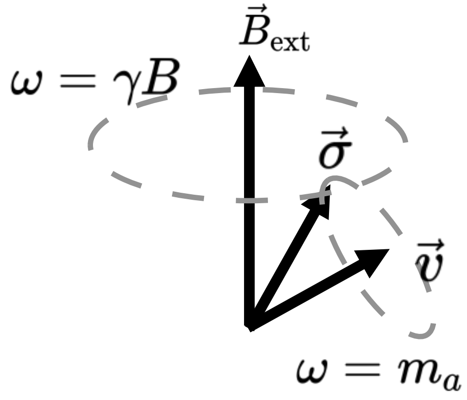
$$M(t) \propto \sqrt{\rho_{DM}}\, \frac{\sin[(2\mu B_{\mathrm{ext}} - m_a)t]}{2\mu B_{\mathrm{ext}} - m_a}$$





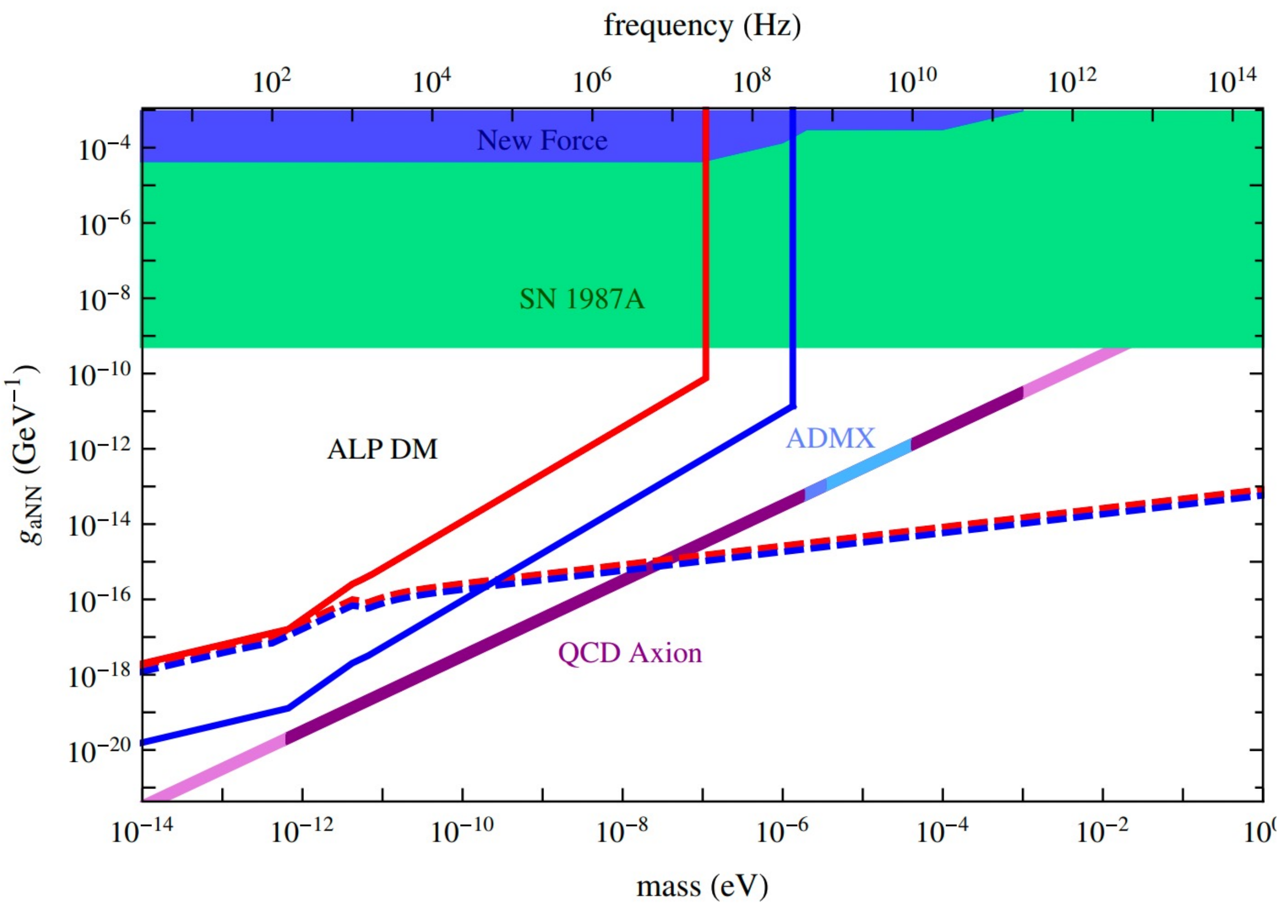
spin polarized



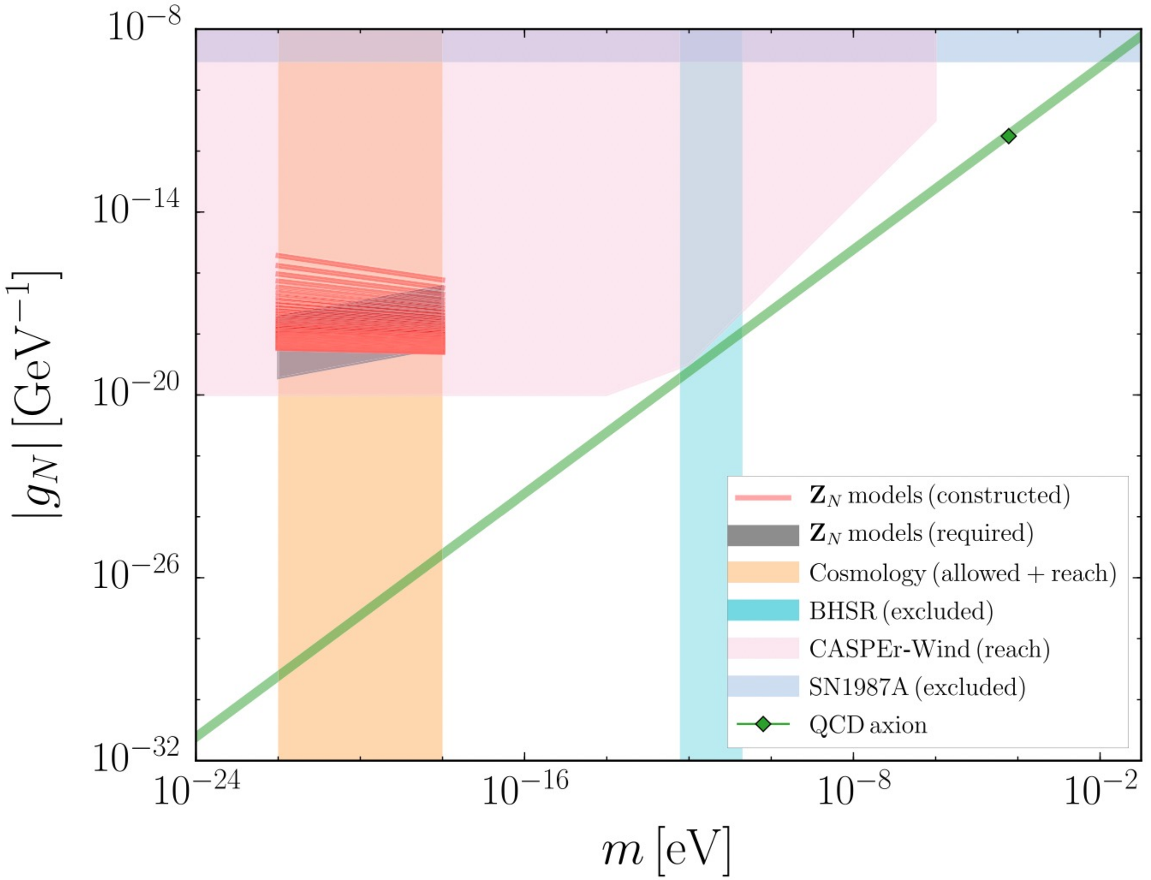


sensitivity issues

The detection in particle physics is model dependent, experiment sensitivity may reach the detection limit when FDM model is well-constructed



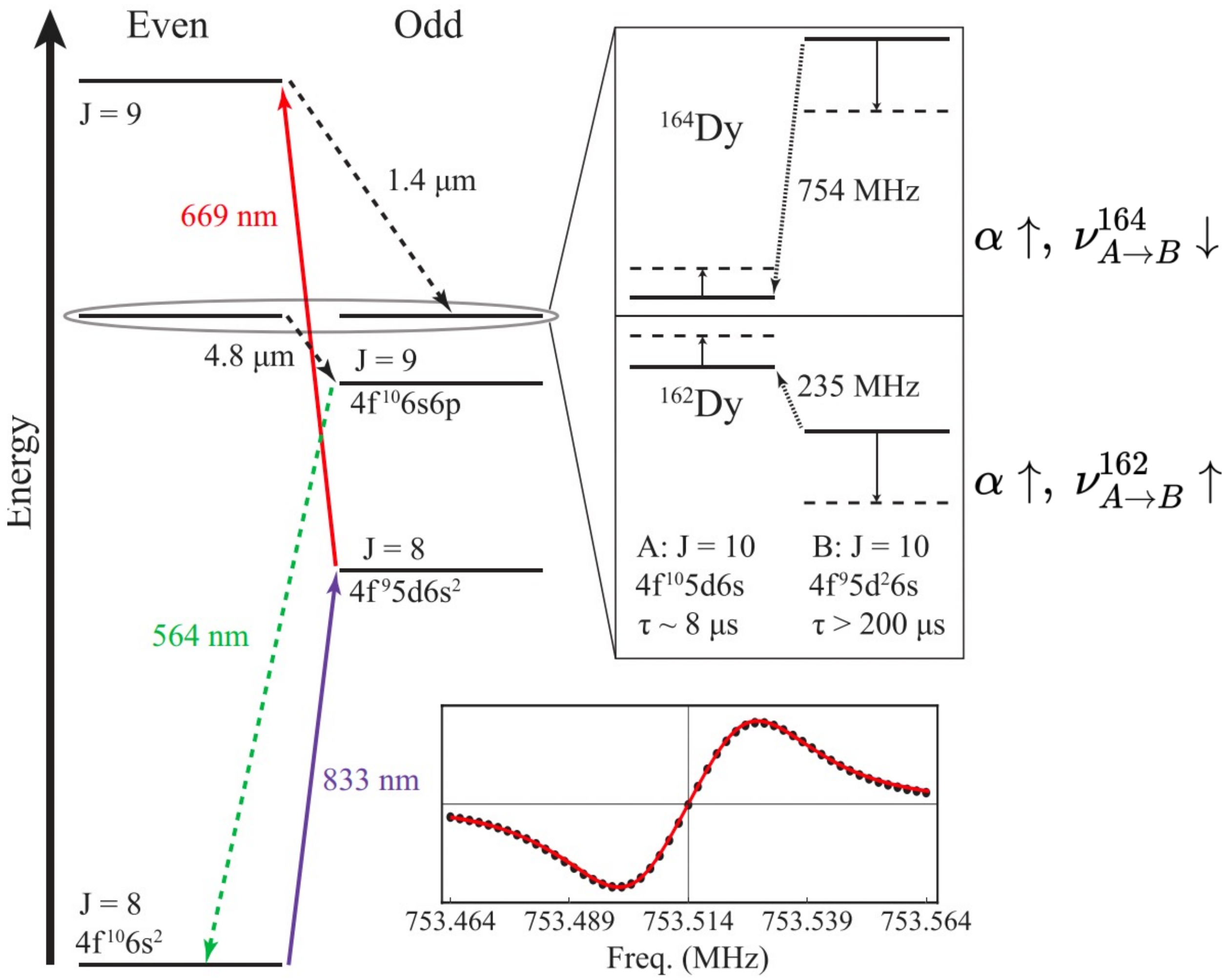
Yonatan Kahn et al., 2016: $m \sim 10^{-14}$eV



J. E. Kim and D. J. E. Marsh (2016)

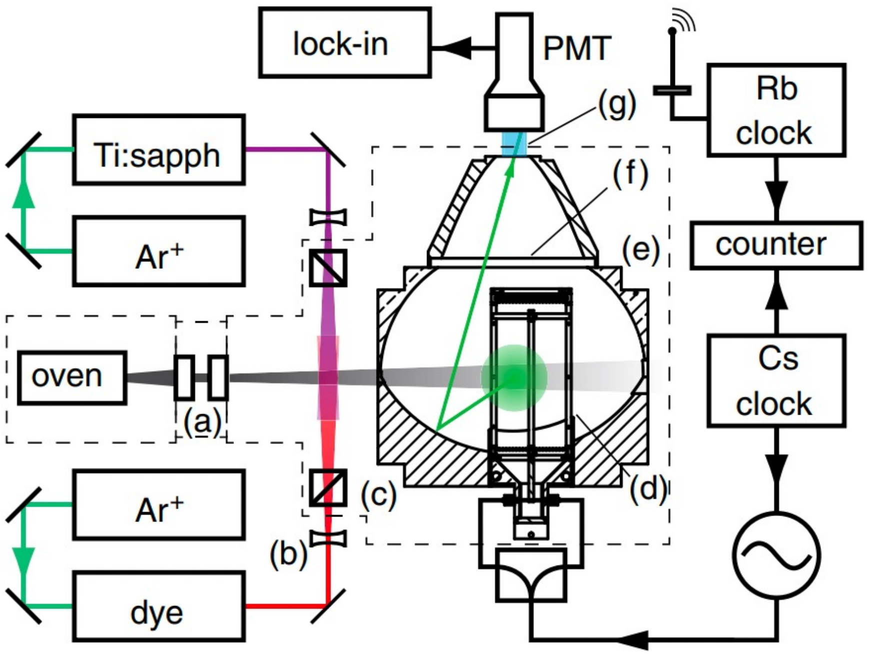
Atomic clock

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 + \frac{-1 + d_e \kappa \phi}{4e^2} F_{\mu\nu} F^{\mu\nu} \longrightarrow \alpha(t) \simeq \alpha[1 + d_e \kappa \phi_0 \cos(m_\phi t + \delta)]$$

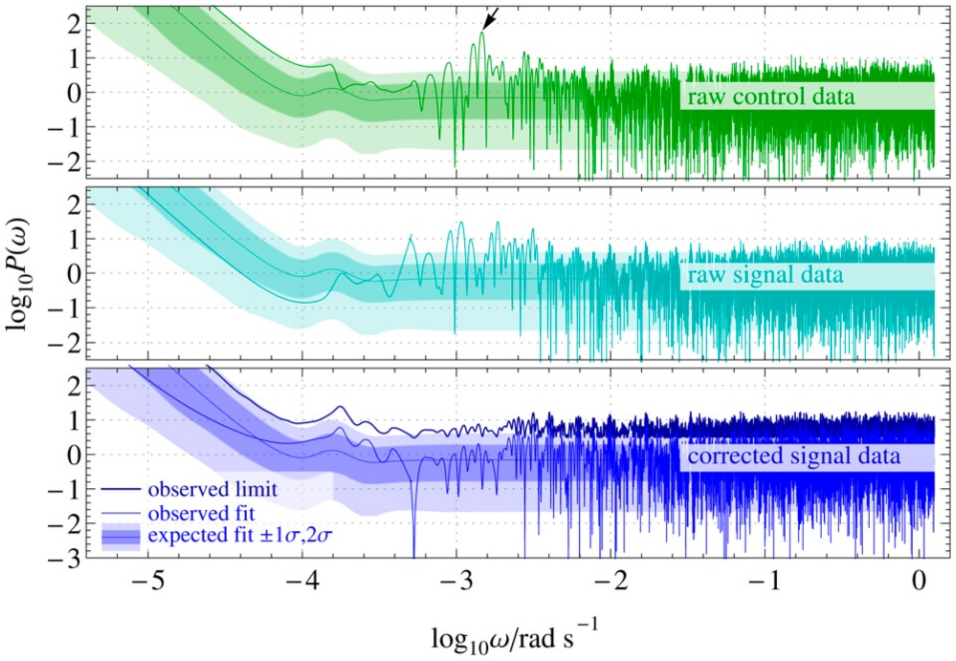


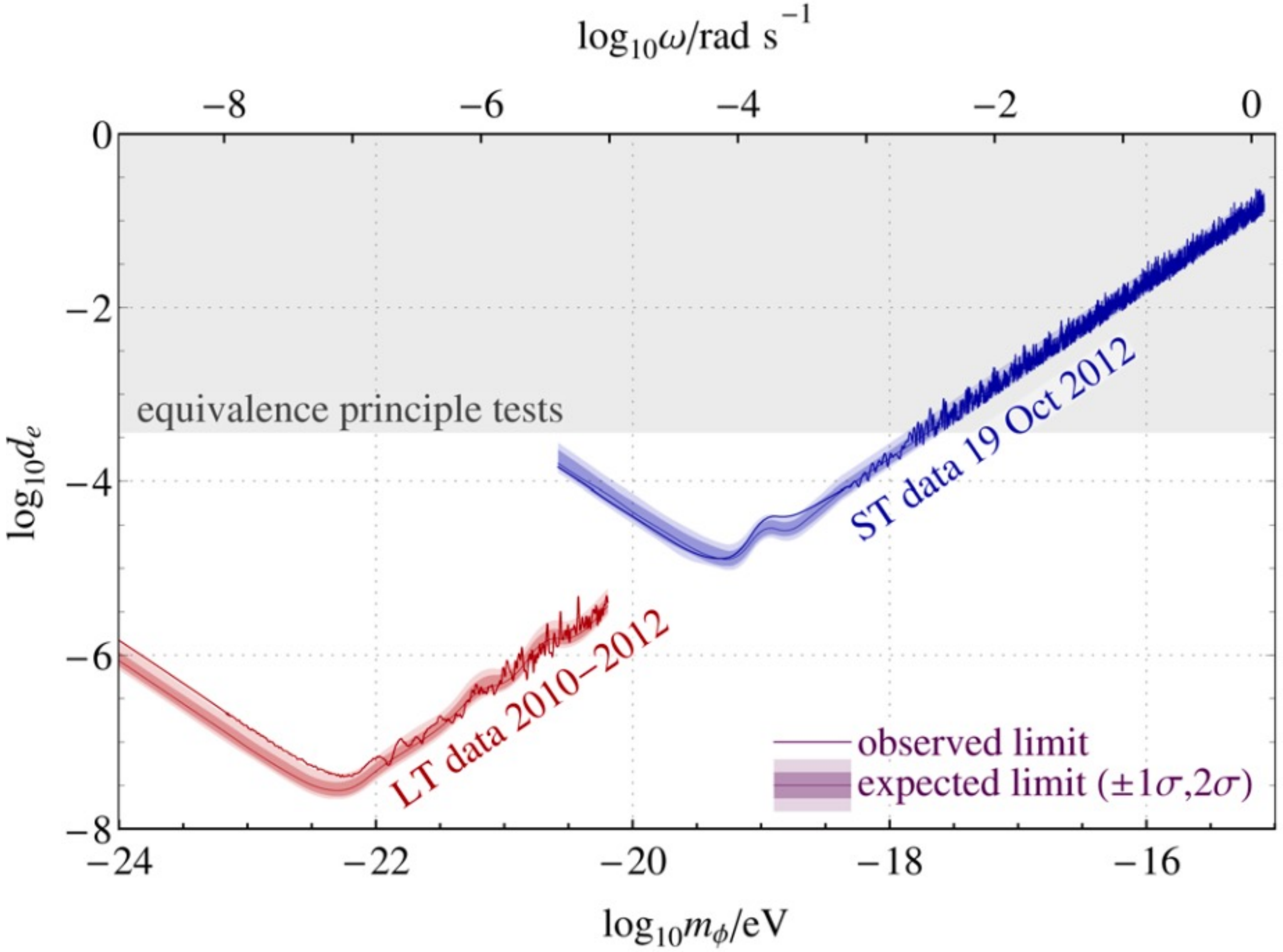
Transition frequency shift between A/B states:

- $v_{162} - v_{164}$: α sensitive, measures the signal
- $v_{162} - v_{164}$: α insensitive, correct systematic error



Atomic clock





Take home message

- Axion-nucleon interaction causes spin precession around the relative velocity
- transverse magnetization happens when applied magnetic field equals to the precession frequency, which $\propto m$
- To reach the limit of FDM is challenging using CAPSEr-Wind method (unless you change the model), atomic clock seems to have hope
- Some paper claims FDM can change the neutrino oscillation probability, and thus they can be detected in neutrino experiment.