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Protoplanetary Disks: Gas Dynamics

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Topics to be covered

- Hydrodynamics and magnetohydrodynamics (MHD)
- Angular momentum transport in accretion disks
- Gravitational instability
- Magnetorotational instability
- Magnetocentrifugal wind
- Hydrodynamic instabilities
- Non-ideal MHD physics
- Summary: current understandings

Hydrodynamics

- Continuity equation: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

- Euler's equation: $$\rho \frac{d\boldsymbol{v}}{dt} = -\nabla P - \rho \nabla \Phi$$

  where the Lagrangian derivative: $$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla$$

- Energy/Enthalpy equation: (not needed for barotropic EoS)

- Equation of state: $$P = P(\rho, T)$$ (general, e.g., $P = \frac{\rho}{\mu m_p} kT$)

  $$P = P(\rho)$$ (barotropic, e.g., $P = \rho c_s^2$)

Hydrodynamics (conservation form)

- Continuity equation: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$
- Euler's equation: $$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v} + P\mathsf{I}) = 0$$

  (gravity ignored for the moment)
- Energy equation: $$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P)\boldsymbol{v}] = 0$$

where $E \equiv \frac{P}{\gamma - 1} + \frac{1}{2}\rho v^2$ for adiabatic equation of state.

Viscous flow (Navier-Stokes viscosity)

Euler's equation:

$$\frac{\partial(\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v} + P\mathsf{I}) = \nabla \cdot \boldsymbol{\sigma}$$

Viscous stress tensor: (Laundau & Lifshitz, 1959)

$$\sigma_{ij} = \rho\nu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla \cdot \boldsymbol{v}\right)$$

Physical interpretation:

"Diffusion" of momentum: momentum exchange across velocity gradient.

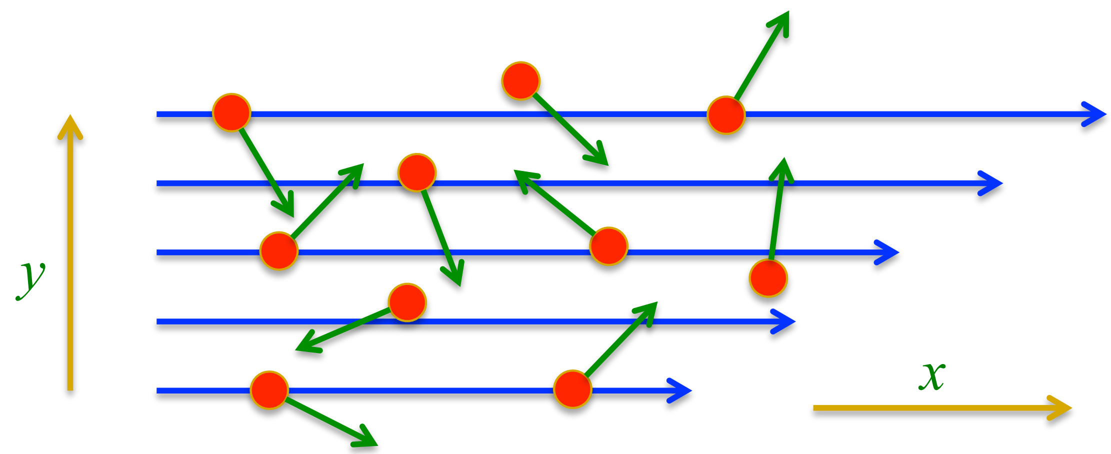
Reynolds number:

$$\mathrm{Re} \equiv \frac{VL}{\nu}$$

viscosity unimportant when Re>>1.

Microphysics of viscosity

Momentum flux: $$\sigma_{ij} = \rho\nu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$



Exchange of x momentum due to thermal motion in y and molecular collisions

$$\sigma_{xy} \sim \rho v_{\rm th}\left(\frac{dv_x}{dy}\lambda_{\rm mfp}\right)$$

$$\Rightarrow \quad \nu \sim \frac{1}{3}\lambda_{\rm mfp} v_{\rm th}$$

Most astrophysical flows are inviscid:

$$\mathrm{Re} = \frac{VL}{\nu} \sim \frac{V}{v_{\rm th}}\frac{L}{\lambda_{\rm mfp}}$$

$$V \gtrsim v_{\rm th}$$

$$L \gg \lambda_{\rm mfp}$$

Exercise

Using the minimum-mass solar nebular (MMSN) disk model, estimate the molecular mean free path at 1 AU in the disk midplane, and compare it with the disk scale height H.

Hint: the cross section for molecular collisions is typically on the order of 10^{-15} cm^2.

Magnetohydrodynamics (MHD)

- Hydrodynamics + Lorentz force ($\boldsymbol{J}\times\boldsymbol{B}$)
- Need one more equation to evolve magnetic field

Induction equation: $$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E}$$

- Ideal MHD: gas is a perfect electric conductor

In the co-moving frame: $$\boldsymbol{J}' = \sigma \boldsymbol{E}' \xrightarrow{\sigma \to \infty} \boldsymbol{E}' = 0$$

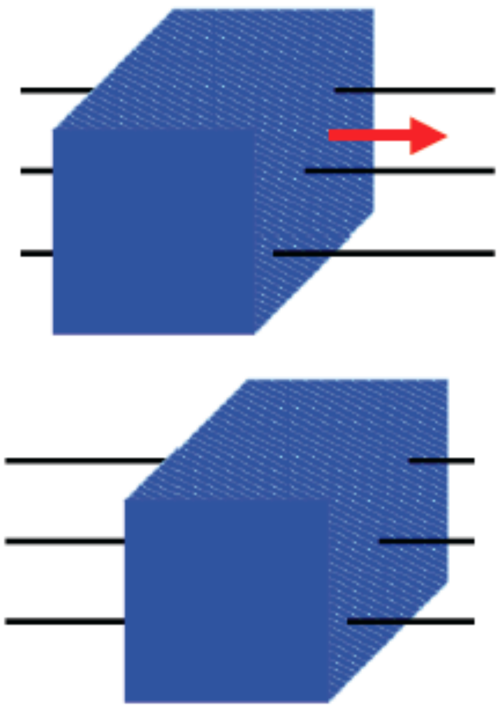
Transforming to the lab frame: $$\boldsymbol{E} = \boldsymbol{E}' - \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B} = -\frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

Implication: flux freezing

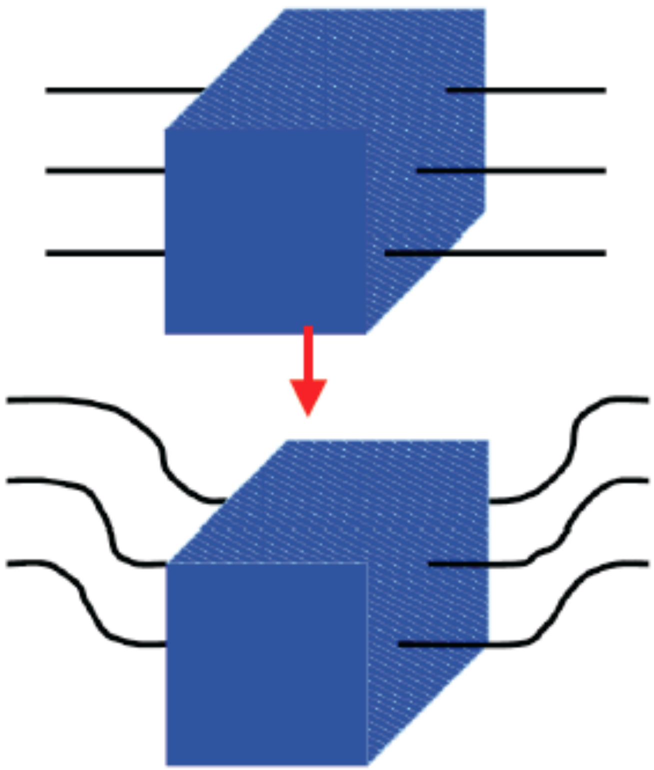
Ideal MHD

Strong field: matter move along field lines (beads on a wire).



$$\frac{|B|^2}{8\pi} >> P_{\text{gas}} + \rho|\mathbf{v}|^2$$

Weak field: field lines are forced to move with the gas.



$$\frac{|B|^2}{8\pi} << P_{\text{gas}} + \rho|\mathbf{v}|^2$$

When is flux freezing applicable?

Ideal MHD applies widely in most astrophysical plasma:

Magnetic Reynolds number: $$\mathrm{Re}_M \equiv \frac{VL}{\eta} \gg 1$$

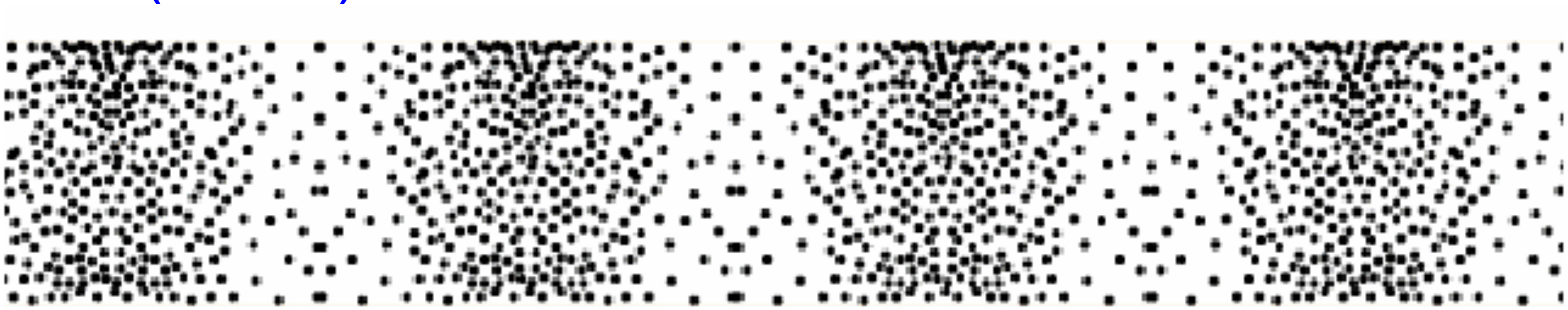
η: microscopic resistivity

When can the flux freezing condition be broken?

- Microscopic scale (MHD no longer applicable due to plasma effects)
- Magnetic reconnection (localized plasma phenomenon)
- Turbulence (rapid reconnection thanks to turbulence)
- Weakly ionized gas => non-ideal MHD (most relevant in PPDs)

MHD waves

Acoustic (sound) waves:

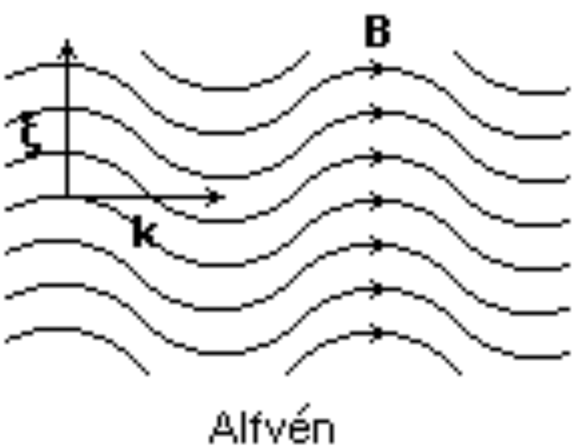


direction of propagation

Properties modified by magnetic field:

Fast and slow magnetosonic waves.

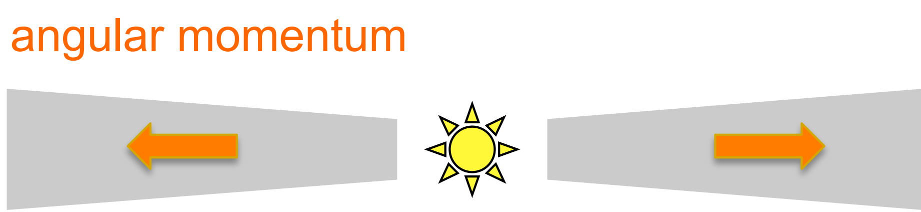
Alfvén waves:

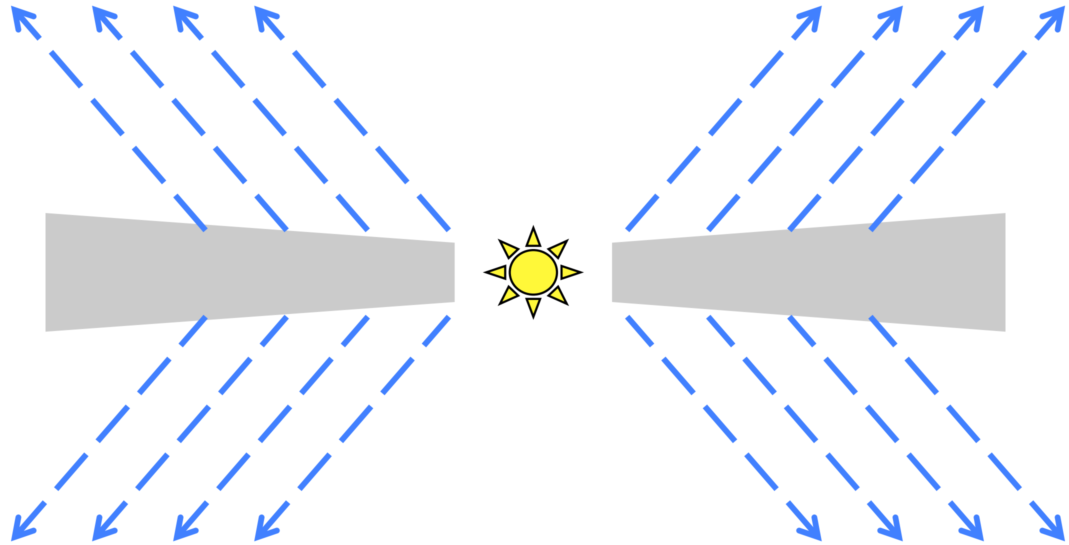


Incompressible, transverse wave; restoring force is magnetic tension.

$$v_A = \frac{B}{\sqrt{4\pi\rho}}$$

Angular momentum transport





Radial transport by:

"viscosity"

Inner disk falls in,
outer disk expands

Vertical transport by:

disk wind

The entire disk falls in.

Angular momentum transport

- Using from the MHD equations in cylindrical coordinate, the ϕ-momentum equation can be written in conservation form:

$$\frac{\partial J_z}{\partial t} + \frac{\partial}{\partial R}\left(-\dot{M}_a j_z + 2\pi R^2 \int_{-\infty}^{\infty} T_{R\phi} dz \right) + 2\pi R^2 T_{z\phi}\Big|_{z=-z_s}^{z_s} = 0$$

AM flux due to accretion

AM transport (radial)

AM extraction (vertical)

where $J_z \equiv 2\pi R \Sigma j_z \ , \quad j_z \equiv R v_{\phi,0}$ (specific AM)

Driving force of angular momentum transport:

$$T_{R\phi} \equiv -\sigma_{R\phi} + \overline{\rho v_R v_\phi} - \frac{\overline{B_R B_\phi}}{4\pi} + \frac{g_R g_\phi}{4\pi G}$$

$$T_{z\phi} \equiv \overline{\rho \delta v_z \delta v_\phi} - \frac{\overline{B_z B_\phi}}{4\pi}$$

Accretion rate (steady state)

- Using from the MHD equations in cylindrical coordinate, the ϕ-momentum equation can be written in conservation form:

$$\frac{\partial J_z}{\partial t} + \frac{\partial}{\partial R}\left(-\dot{M}_a j_z + 2\pi R^2 \int_{-\infty}^{\infty} T_{R\phi} dz \right) + 2\pi R^2 T_{z\phi}\Big|_{z=-z_s}^{z_s} = 0$$

AM flux due to accretion

AM transport (radial)

AM extraction (vertical)

Radial transport

$$\dot{M}_a \approx \frac{2\pi}{\Omega} \int_{-\infty}^{\infty} T_{R\phi} dz$$

Vertical transport

$$\dot{M}_a \approx \frac{8\pi R}{\Omega} |T_{z\phi}|_{z_s}$$

(Assuming $T_{z\phi}(z_s)=-T_{z\phi}(-z_s)$).

If $T_{R\phi}$ and $T_{z\phi}$ are similar, then vertical transport (by wind) is more efficient than radial transport by a factor of ~R/H >>1.

Energy dissipation

Radial transport of angular momentum:

$$T_{R\phi} \equiv -\sigma_{R\phi} + \overline{\rho v_R v_\phi} - \frac{\overline{B_R B_\phi}}{4\pi} + \frac{g_R g_\phi}{4\pi G}$$

Energy dissipation rate:

$$Q^+ = -\frac{d\Omega}{d\ln R} T_{R\phi} = \frac{3}{2}\Omega T_{R\phi}$$

Radial transport of angular momentum is accompanied by (local) heating.

α-disk model

- Radial transport of angular momentum by viscosity:

$$T_{R\phi} = \alpha P$$ (Shakura & Sunyaev, 1973)

For N-S viscosity:

$$T_{R\phi} = -\sigma_{R\phi} = -\rho\nu R\frac{d\Omega}{dR} = \frac{3}{2}\rho\nu\Omega = \left(\frac{3}{2}\frac{\nu}{c_s H}\right)P$$

With microscopic viscosity: $\alpha \sim \lambda_{\rm mfp}/H \ll 1$

Required value of α to explain PPD accretion rate: ~10^{-3}-10^{-2}

(Exercise: show this result based on typical PPD accretion rate and MMSN disk model)

Need anomalous viscosity (e.g., turbulence) to boost α.

Viscous evolution based on the α-disk model

Angular momentum conservation (ignore wind):

$$\frac{\partial J_z}{\partial t} + \frac{\partial}{\partial R}\left(-\dot{M}_a j_z + 2\pi R^2 \int_{-\infty}^{\infty} T_{R\phi} dz \right) + 2\pi R^2 T_{z\phi}\Big|_{z=-z_s}^{z_s} = 0$$

$$\Rightarrow \quad 2\pi R j_z \frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}\left(-\dot{M}_a j_z + 2\pi R^2 \alpha c_s^2 \Sigma \right) = 0$$

Mass conservation: $$2\pi R \frac{\partial \Sigma}{\partial t} - \frac{\partial \dot{M}_a}{\partial R} = 0$$

Viscous evolution equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{dR}{dj_z}\frac{\partial}{\partial R}\left(\alpha c_s^2 \Sigma R^2 \right) \right]$$

Note: $j_z(R) = \Omega R^2$

Gas is assumed to be locally isothermal.

Viscous evolution based on the α-disk model

Viscous evolution equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{dR}{dj_z}\frac{\partial}{\partial R}\left(\alpha c_s^2 \Sigma R^2\right)\right]$$

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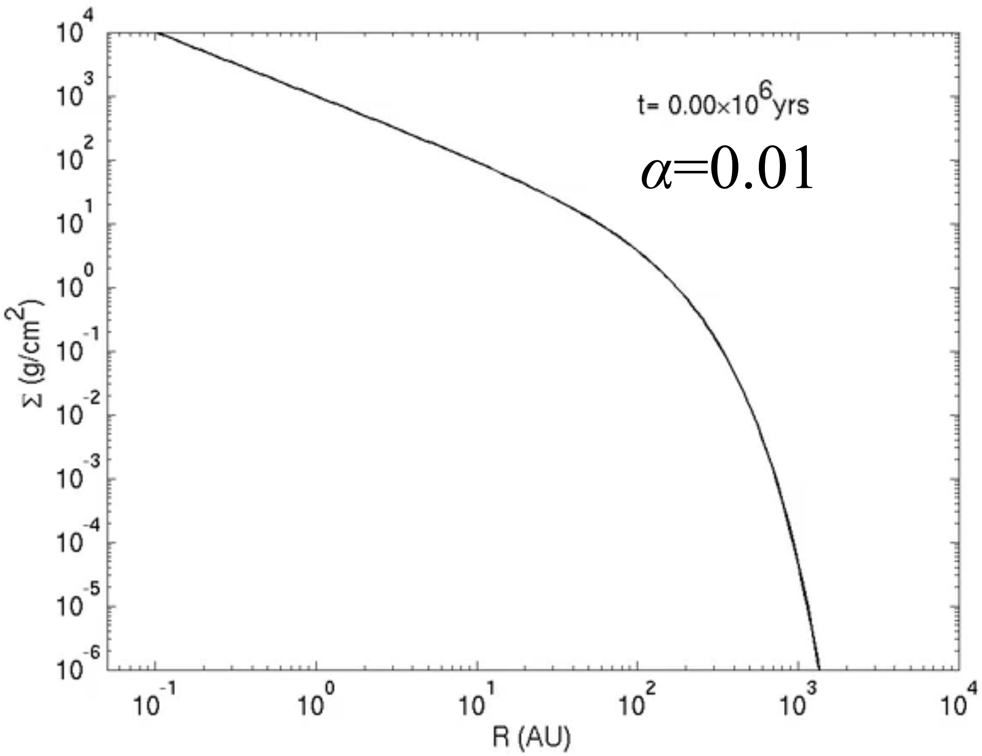

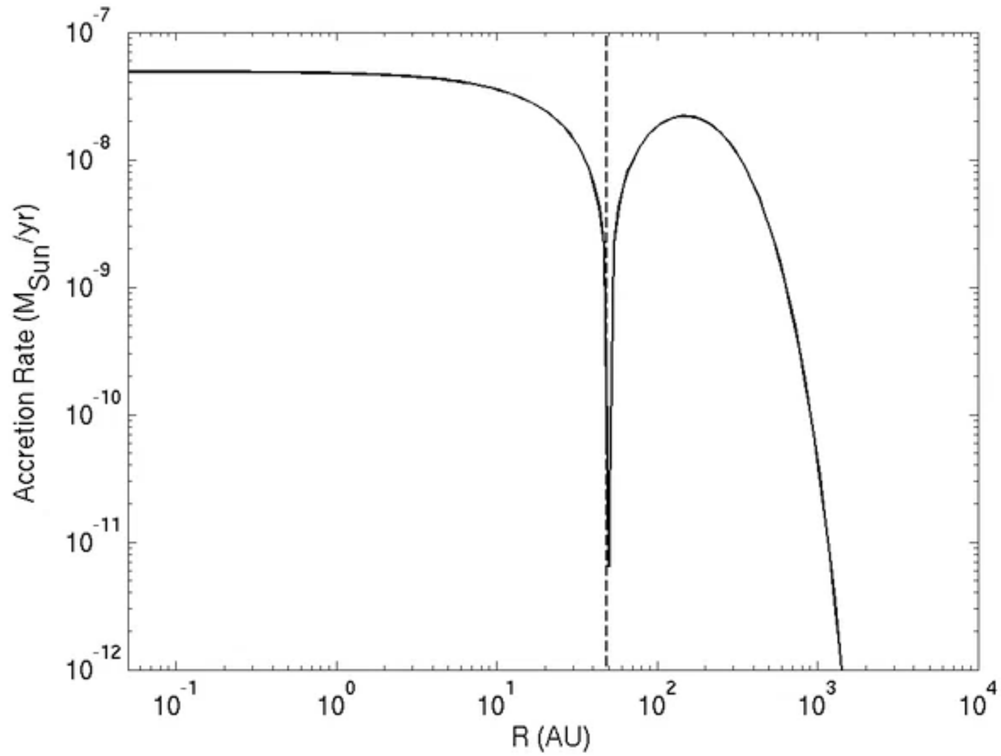

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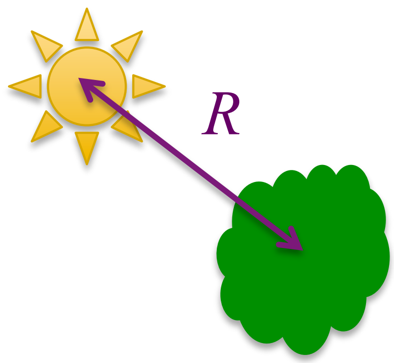
Gravitational instability

Self-gravity, tidal force, pressure

Consider a clump of gas in the disk with size L.



$$M_{\rm clump} \sim \Sigma \pi L^2$$

Rotation (tidal force) stabilizes against gravitational collapse if:

$$\frac{GM_*}{R^2}\frac{L}{R} > \frac{GM_{\rm clump}}{L^2}$$

$$\Rightarrow \quad L > \frac{\pi G \Sigma}{\Omega^2}$$

Gas pressure stabilizes against gravitational collapse if:

$$\frac{1}{\rho}\frac{P}{L} > \frac{GM_{\rm clump}}{L^2} \quad \Rightarrow \quad L < \frac{c_s^2}{\pi G \Sigma}$$

Gravitational instability

Gas pressure stabilizes at small scale:

$$L < \frac{c_s^2}{\pi G \Sigma}$$

Rotation stabilizes at large scale:

$$L > \frac{\pi G \Sigma}{\Omega^2}$$

System is stable at all scales if they overlap, otherwise, gravitational instability develops at intermediate scale.

Toomre's Q parameter: (Toomre, 1964)

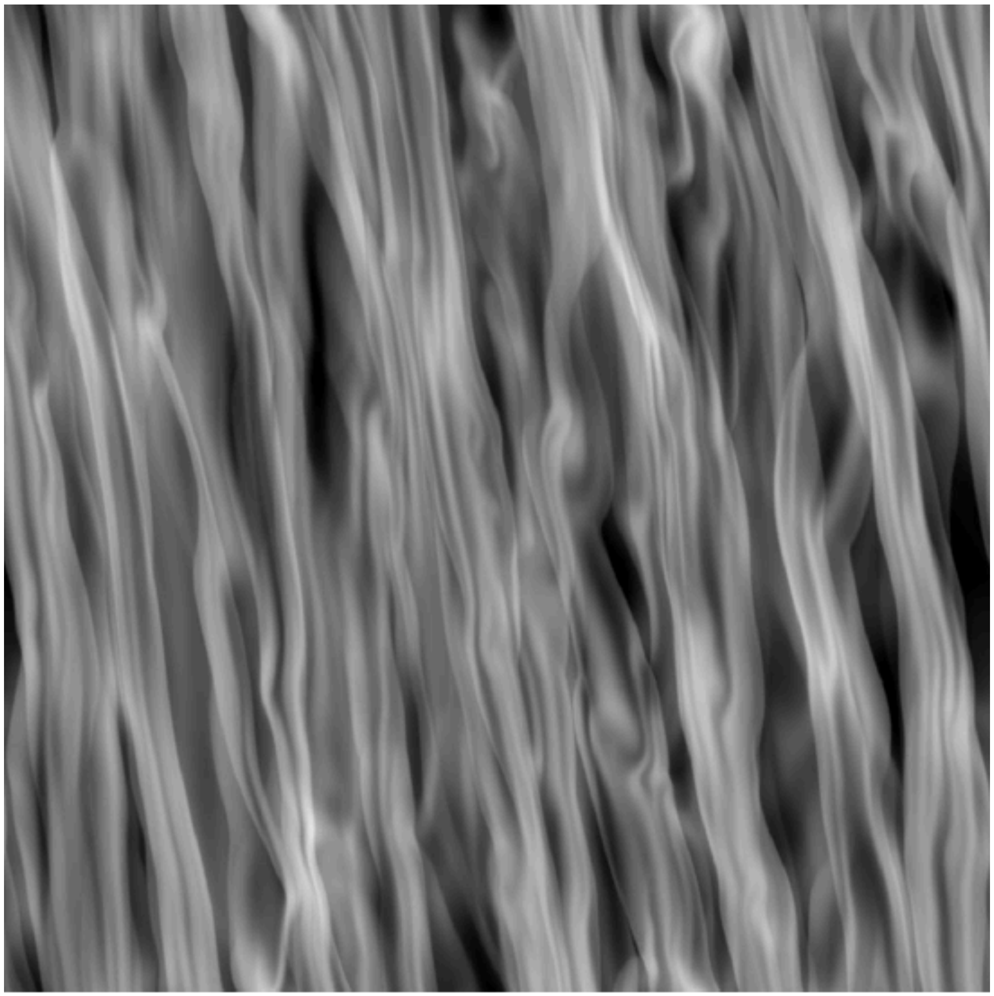
$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma}$$

Disk is gravitationally unstable if $Q<1$.

Can also be derived from rigorous analysis.

Gravitational instability: outcome

Slow cooling: gravito-turbulence



Ωt_{cool}=10 (Gammie, 2001)

Cooling time: $$\frac{dT}{dt} \sim \frac{T - T_0}{t_{\mathrm{cool}}}$$

Self-regulation:

Heat the disk until Q~1.

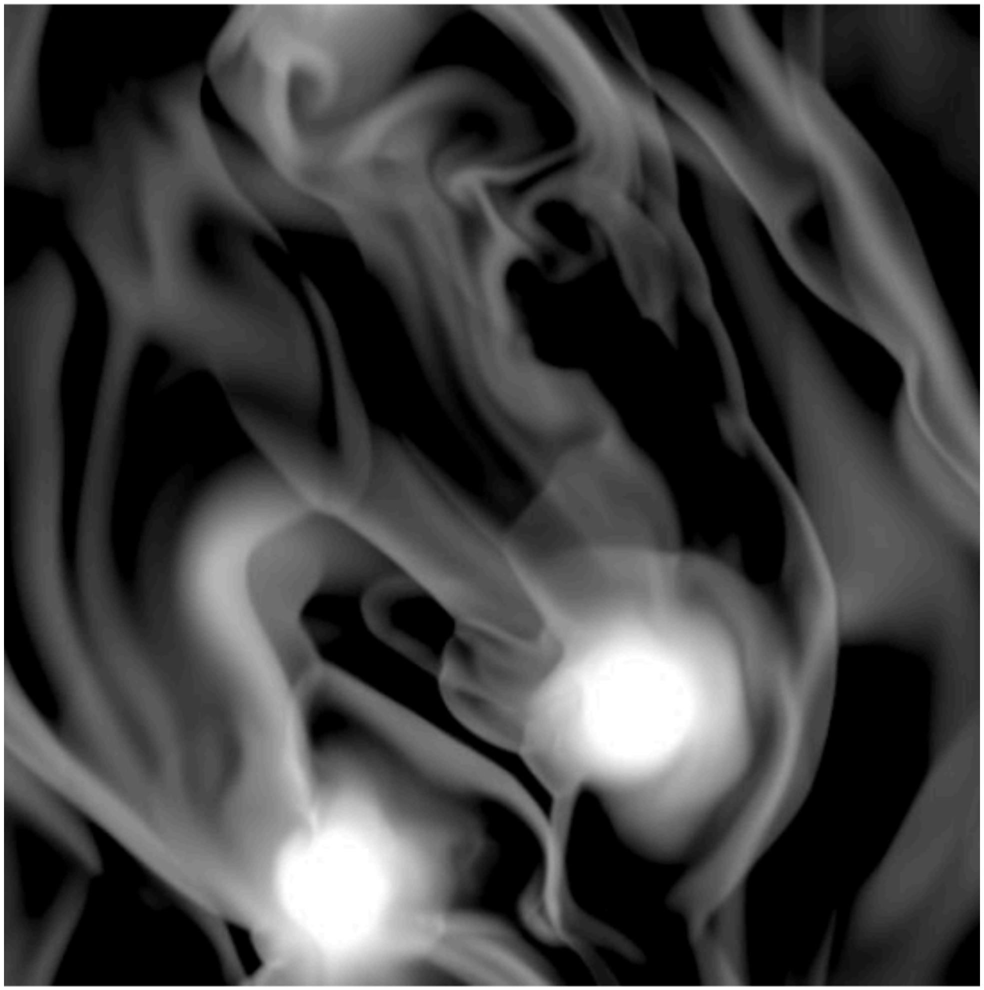
Heating by GI turbulence is balanced by cooling, resulting in viscous transport of AM:

$$\alpha \sim \frac{4}{9\gamma(\gamma - 1)} \frac{1}{\Omega t_{\mathrm{cool}}}$$

$$\sim \frac{0.4}{\Omega t_{\mathrm{cool}}}$$

Gravitational instability: outcome

Fast cooling: fragmentation



$\Omega t_{\mathrm{cool}}=1$ (Gammie, 2001)

Pathway for giant planet formation?

Not so easy!

Heating of (outer) PPDs is dominated by irradiation.

More difficult to fragment.

(Kratter & Murray-Clay, 2011, Rafikov, 2009)

When fragments, could suffer from:

Type-I migration, tidal disruption

Too large mass -> brown dwarfs

(Kratter+ 2010, Zhu+ 2012)

Exercise

1. Using the minimum-mass solar nebular (MMSN) disk model, estimate the Toomre Q parameter as a function of radius.

2. Given the disk temperature profile, estimate the maximum disk surface density to be marginally gravitationally unstable (Q=1).

Magnetorotational Instability (MRI)

- Rayleigh criterion for unmagnetized rotating disks:

Unstable if: $$\frac{d(\Omega R^2)}{dR} < 0$$ (Rayleigh, 1916)

Confirmed experimentally (Ji et al. 2006).

All astrophysical disks should be stable against this criterion.

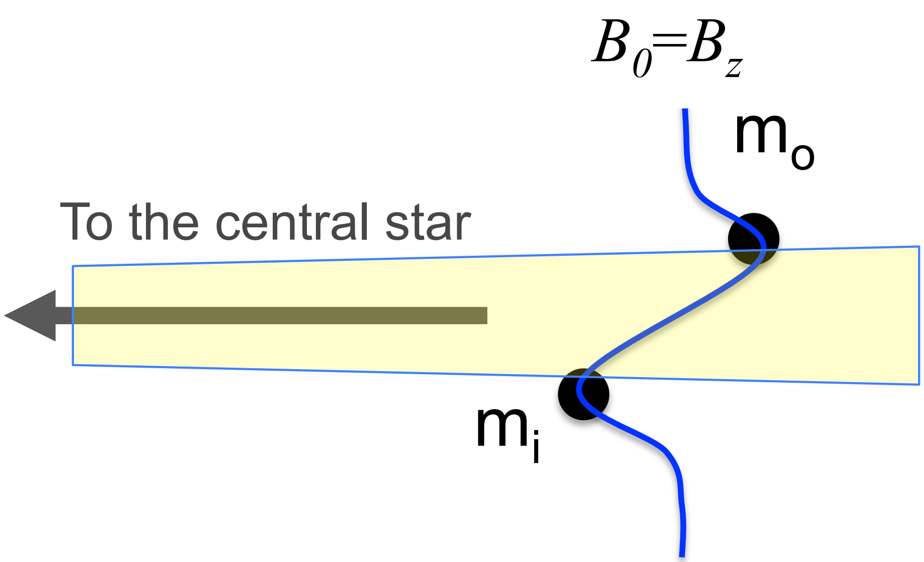
- Including (a vertical, well-coupled) magnetic field qualitatively changes the criterion (even as B->0):

Unstable if: $$\frac{d\Omega}{dR} < 0$$ Velikhov (1959), Chandrasekhar (1960), Balbus & Hawley (1991)

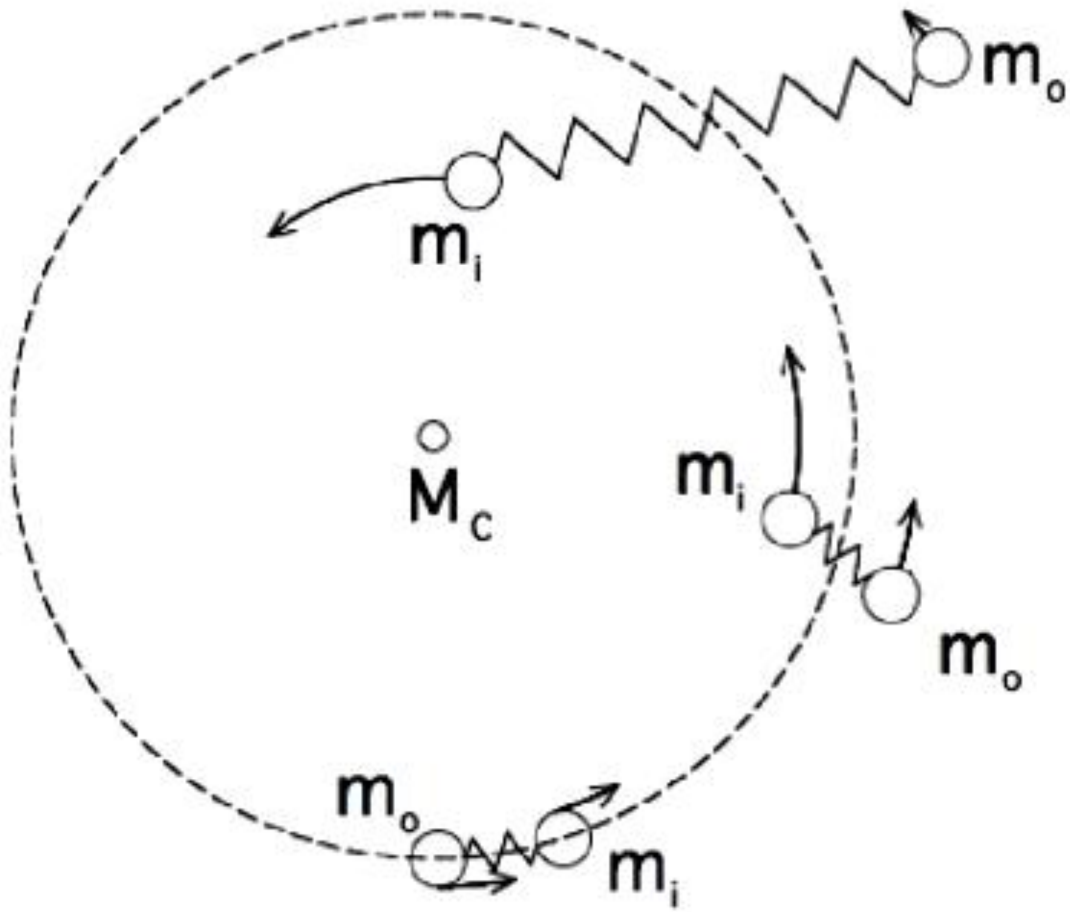
All astrophysical disks should be unstable!

Magnetorotational Instability (MRI)

Edge on view:



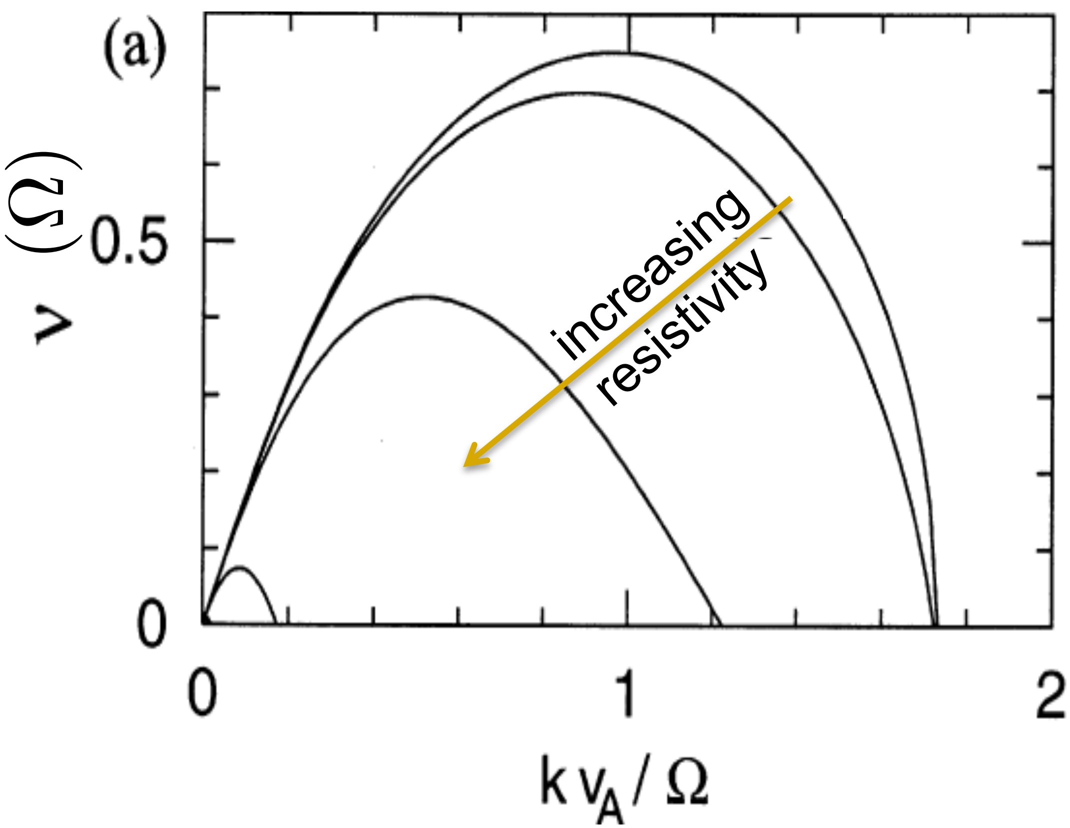
Face on view:



Magnetic tension force behaves like a spring.

Magnetorotational Instability (MRI)

Dispersion relation:



Fastest growth rate: $\sim \Omega$

Very rapid growth

Most unstable wavelength:

$$\lambda \sim \frac{2\pi v_A}{\Omega}$$

To be fit into the disk, require:

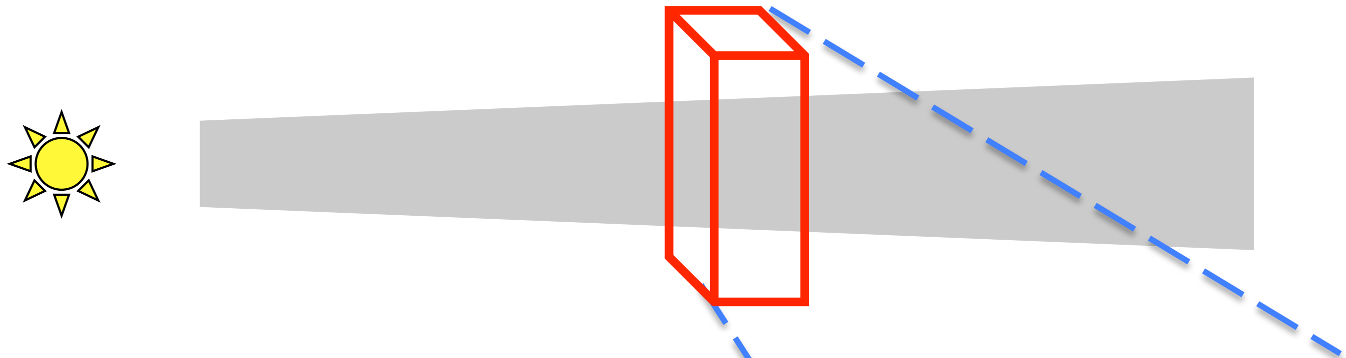
$$\lambda \lesssim H = \frac{c_s}{\Omega}$$

$$\Rightarrow \quad v_A \lesssim c_s/2\pi$$

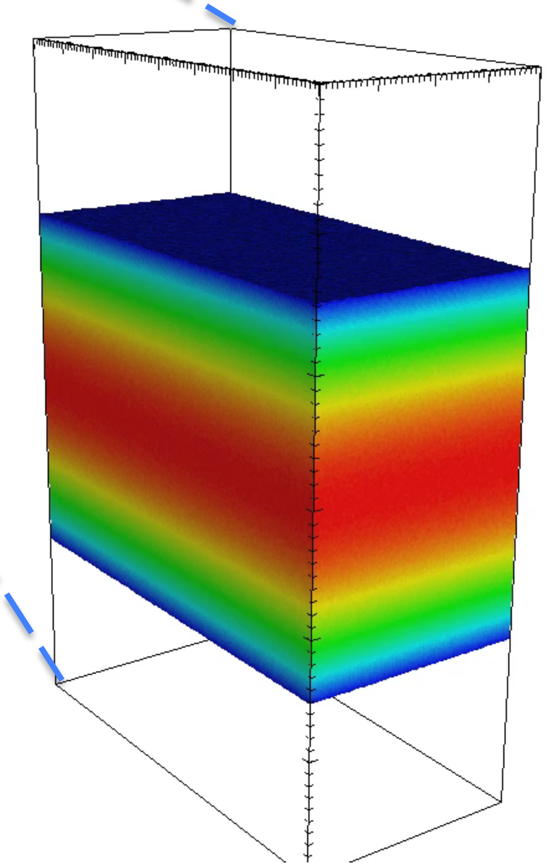
$$\Rightarrow \quad \beta_0 \equiv \frac{P_{\rm gas}}{P_{\rm mag}} \gtrsim 8\pi^2$$

Weak field instability

The local shearing-box framework



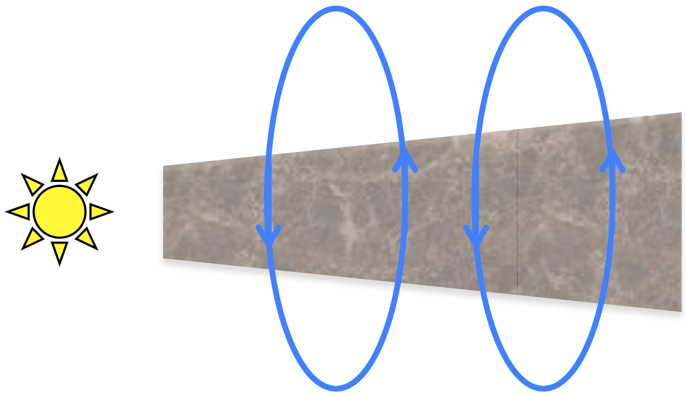
- Take a local patch of the disk.
- Work in the co-rotating frame with Cartesian coordinate.
- Use shearing-periodic radial boundary conditions (account for differential rotation).

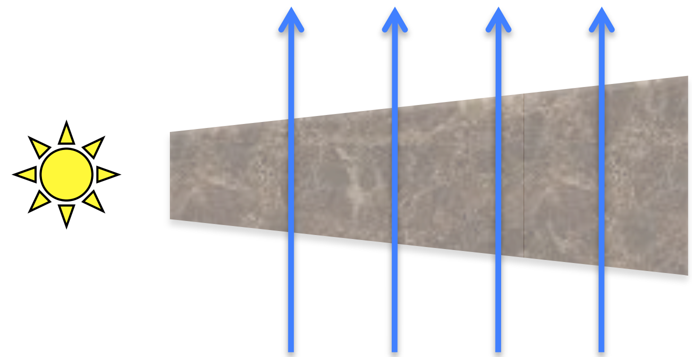


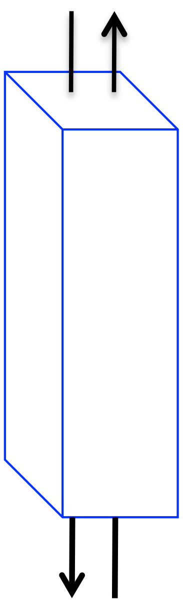
Isothermal EoS is assumed.

(Goldreich & Lynden-Bell, 1965
Hawley, Gammie & Balbus, 1995)

Magnetic field geometry

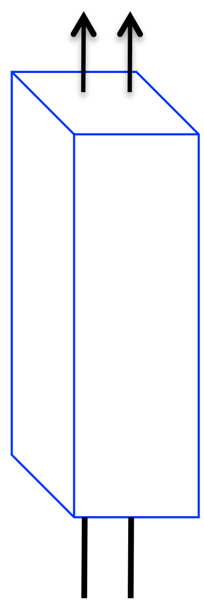






Zero net vertical magnetic flux

$\alpha \approx 0.01 - 0.02$

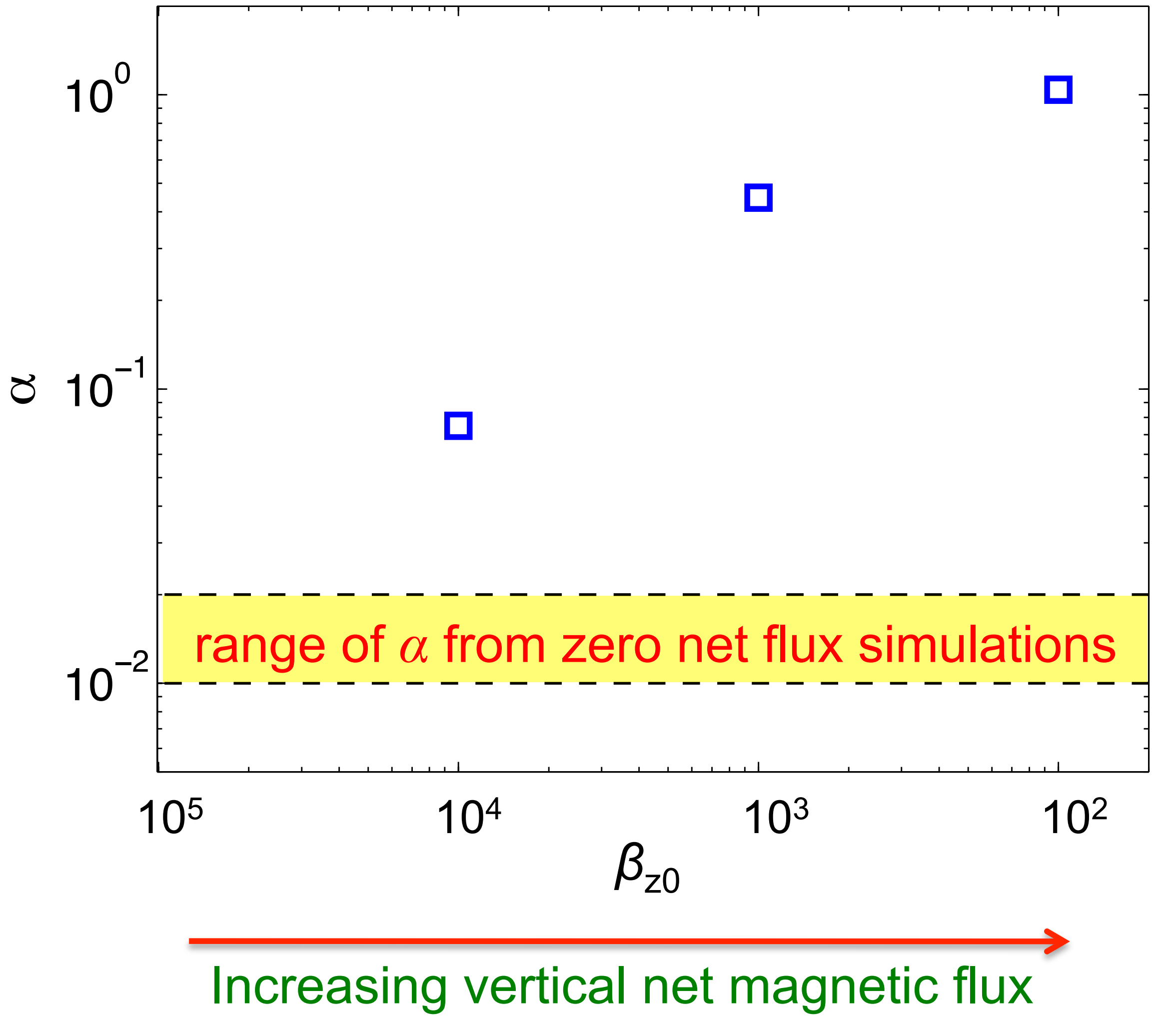


With net vertical magnetic flux

rarely studied until recently

$\beta_{z0} = P_{gas,mid}/P_{mag,net}$

How big is α ?



(Bai & Stone, 2013)

In the ideal MHD case:

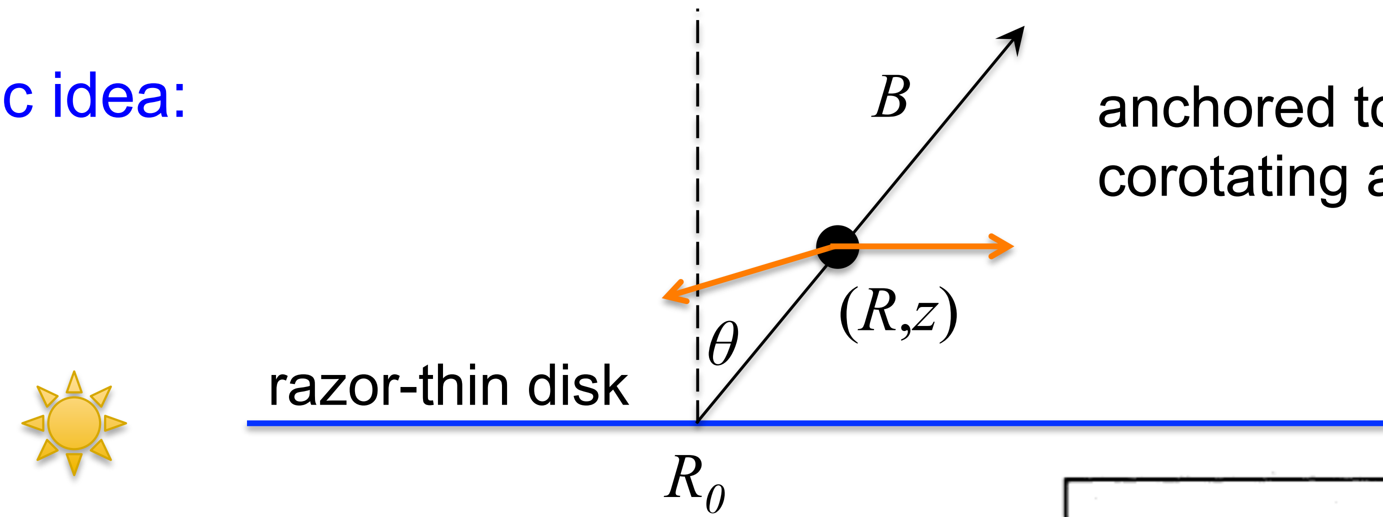
- α increases with B_{z0} until field becomes too strong.
- α spans a range between 0.01 and 1.

The bulk of PPDs are extremely weakly ionized => need non-ideal MHD (see later slides).

Magnetocentrifugal wind

(Blandford & Payne, 1982)

Basic idea:



anchored to the disk, corotating at $\Omega(R_0)$.

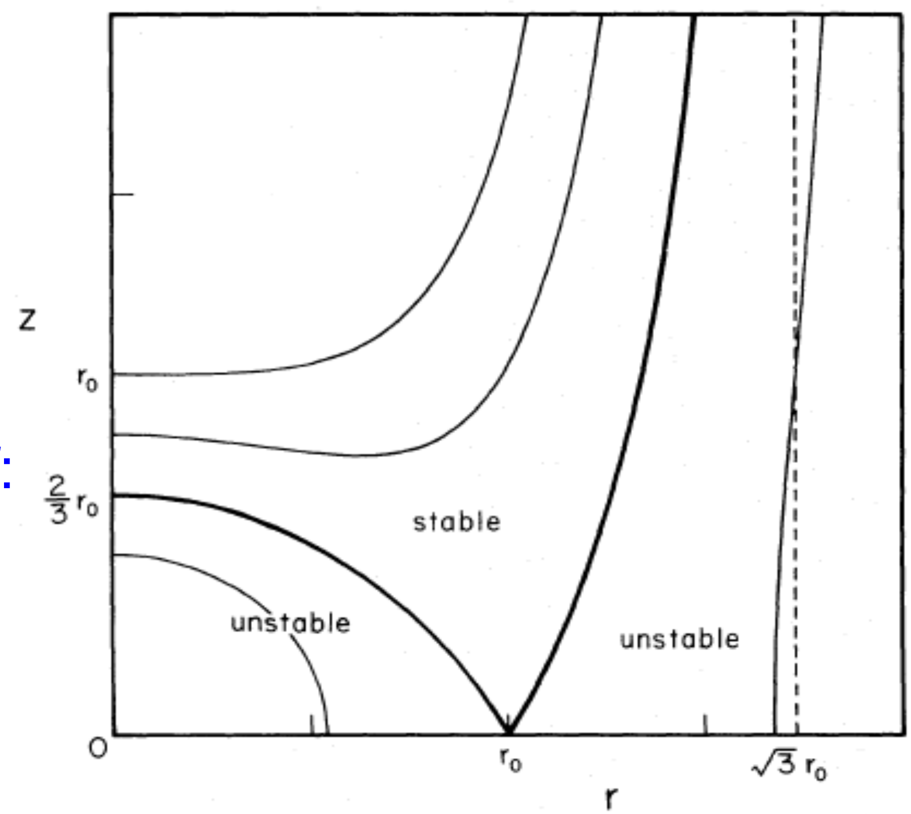
Effective potential for "bead on a wire":

$$\Phi_{\rm eff}(R,z) = -\frac{GM_*}{\sqrt{R^2+Z^2}} - \frac{1}{2}\Omega_0^2 R^2$$

The "bead" travels downhill the potential if:

$$|\theta| > 30^\circ$$

Centrifugal acceleration



Equations of the wind

Ideal MHD: gas travels along magnetic field lines.

Decompose ***B*** and ***v*** into poloidal and toroidal components:

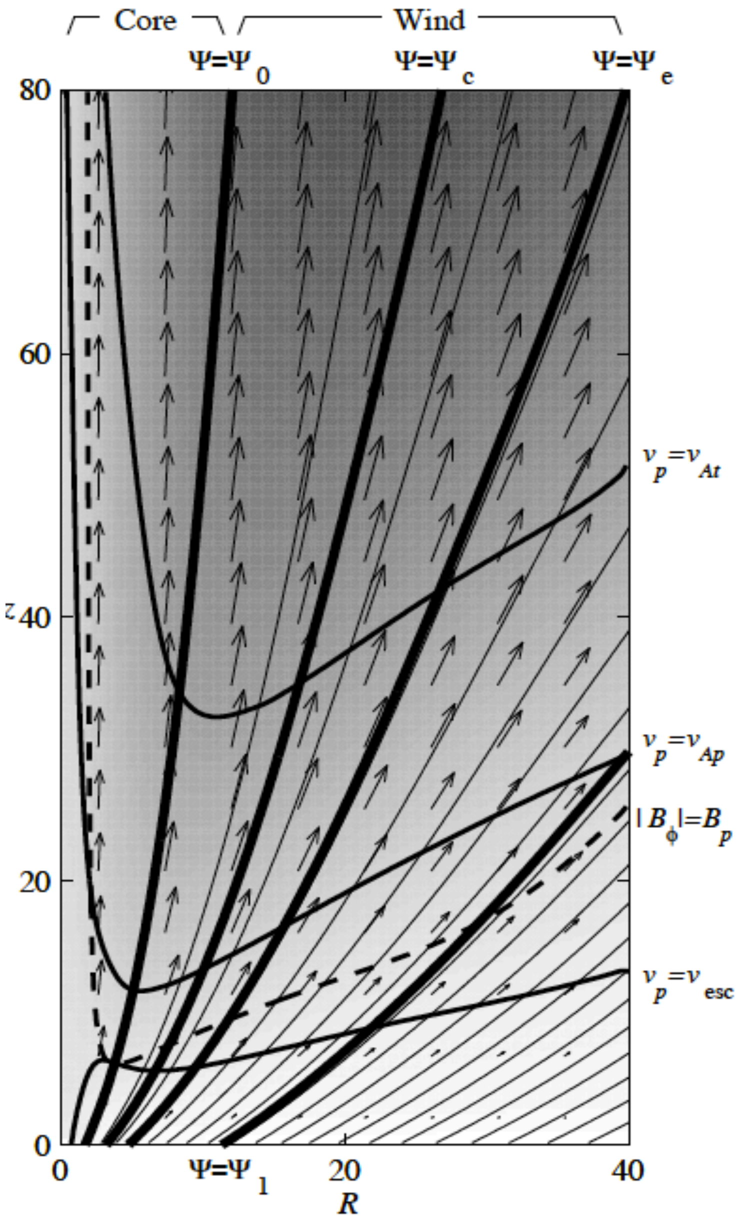
$$\boldsymbol{B} = \boldsymbol{B}_p + B_\phi \boldsymbol{e}_\phi \ , \qquad \boldsymbol{v} = \boldsymbol{v}_p + R\Omega(R)\boldsymbol{e}_\phi$$

From the simple assumptions of steady-state and axisymmetry, one can derive a series of conservation laws along poloidal magnetic field lines.

- Mass conservation: $k = 4\pi\rho v_p / B_p$
- Angular velocity of magnetic flux: $\omega = \Omega - kB_\phi/(4\pi\rho R)$
- Angular momentum conservation: $l = \Omega R^2 - RB_\phi/k$
- Energy conservation: $e = v^2/2 + h + \Phi - \omega R B_\phi / k$

Cross-field force balance: Grad-Shafranov equation (complicated…)

Wind properties



(Krasnopolsky et al. 1999)

Critical points:

v_p= slow/Alfven/fast magnetosonic speed

At the Alfvén point: $v_p^2 = B_p^2/4\pi\rho$

Below the Alfven point:

$B_p^2/8\pi \gg \rho v_p^2/2$ ⇨ Rigid rotation

Beyond the Alfven point:

Magnetic field winds up (develop B_ϕ)

Collimation by “hoop stress” due to B_ϕ.

A useful relation:

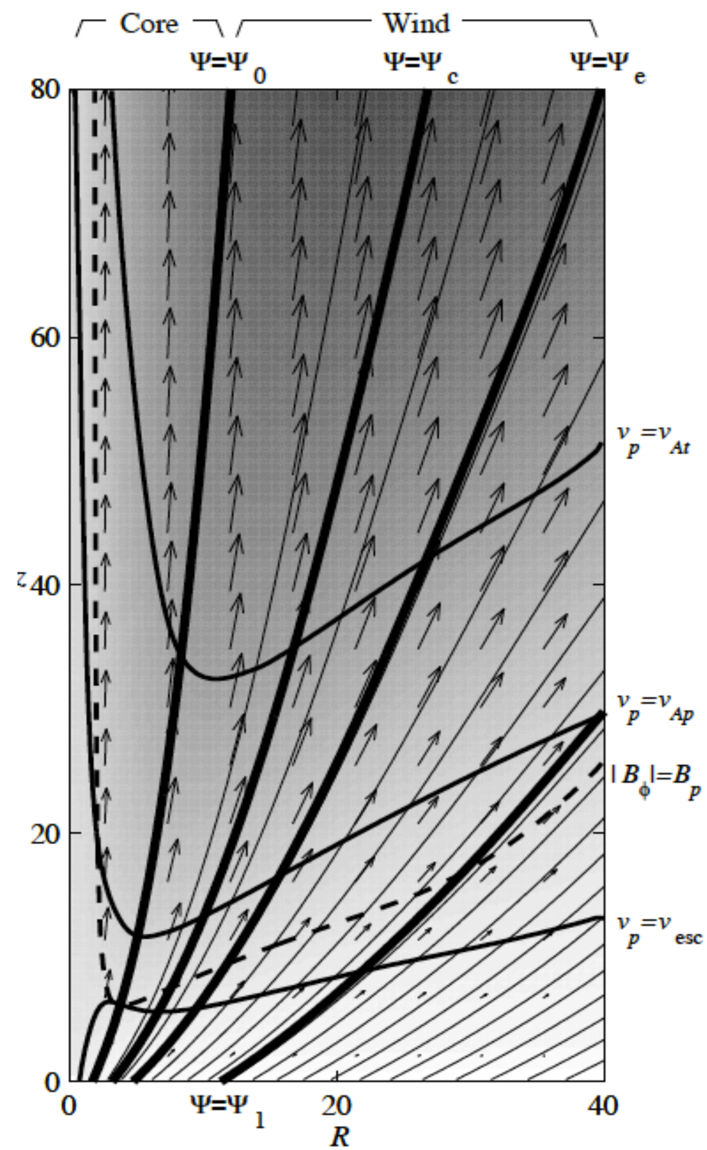
$$\frac{\dot{M}_{\rm out}}{\dot{M}_{\rm acc}} = \frac{1}{2}\left(\frac{R_0}{R_A}\right)^2 \approx 0.1$$

Open issues

- Global wind simulations so far do not resolve disk physics
  - Disk either treated as a boundary condition, or resolved with unphysical resistivity.
  - Almost always assume axisymmetry (i.e., 2D simulations, MRI does not survive).

How is mass loaded to the wind (i.e., wind launching)?

- Most simulations deal with near equi-partition poloidal field:
  - Very strong wind with excessive accretion rate and mass loss.



(Krasnopolsky et al. 1999)

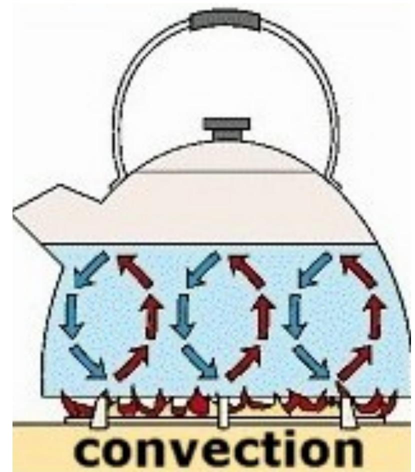
Hydrodynamic instabilities

- Convection

No heating source at midplane…

Direction of AM transport wrong…

(Stone & Balbus 1996, Cabot, 1996, Klahr & Bodenheimer, 2003, Lesur & Ogilvie 2010)



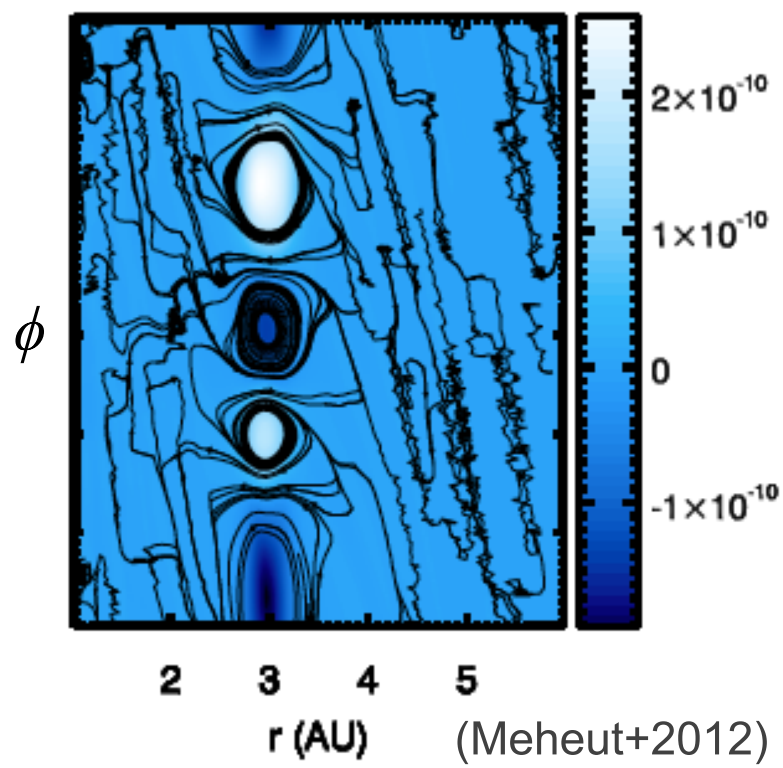
- Rossby-wave instability

Instability results from a bump in the radial pressure profile, and develops into vortices.

(Lovelace+ 1999, Li+ 2000.2001, Meheut+2010, 2012)

Need pre-existing pressure bump in the disk, which may result from the presence of massive planet in the disk.

(de Val-boro+ 2007, Lyra+2009, Zhu+ 2014)



(Meheut+2012)

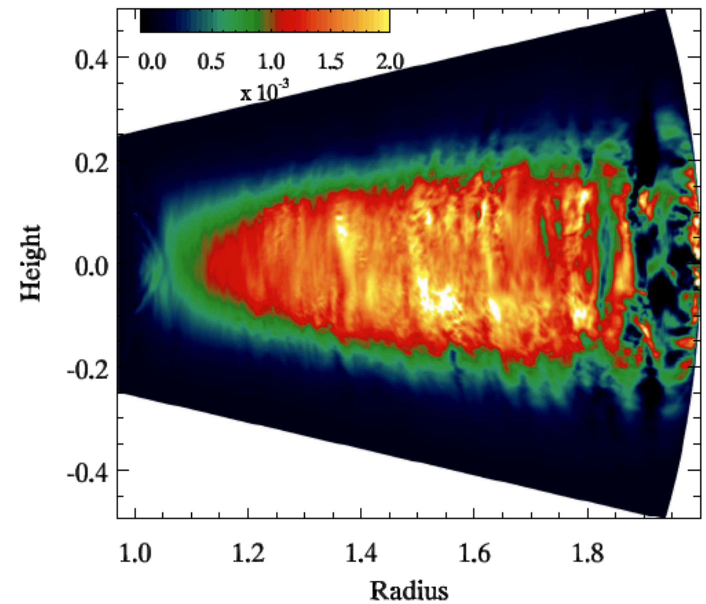
Hydrodynamic instabilities

- Goldreich-Schubert-Fricke (GSF) instability

  (Goldreich & Schubert 1967, Fricke 1968)

  In hydrostatic equilibrium, disk has vertical shear in rotation velocity, which is unstable if thermal relaxation is much faster than dynamical time => can lead to $\alpha \sim 10^{-3}$.

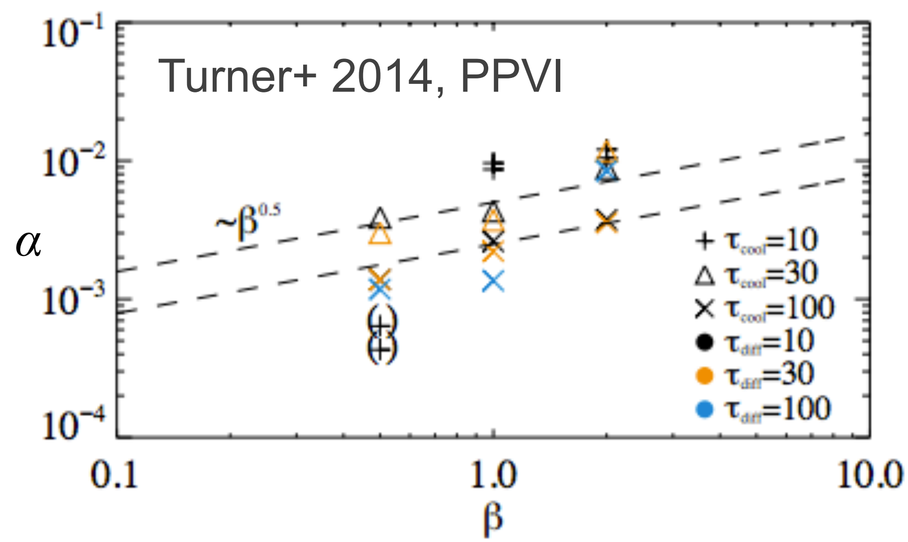
Nelson+2013



- Baroclinic vortex amplification

  (Klahr & Bodenheimer 2003, Peterson+ 2007, Lesur & Papaloizou 2010, Raettig+2013)

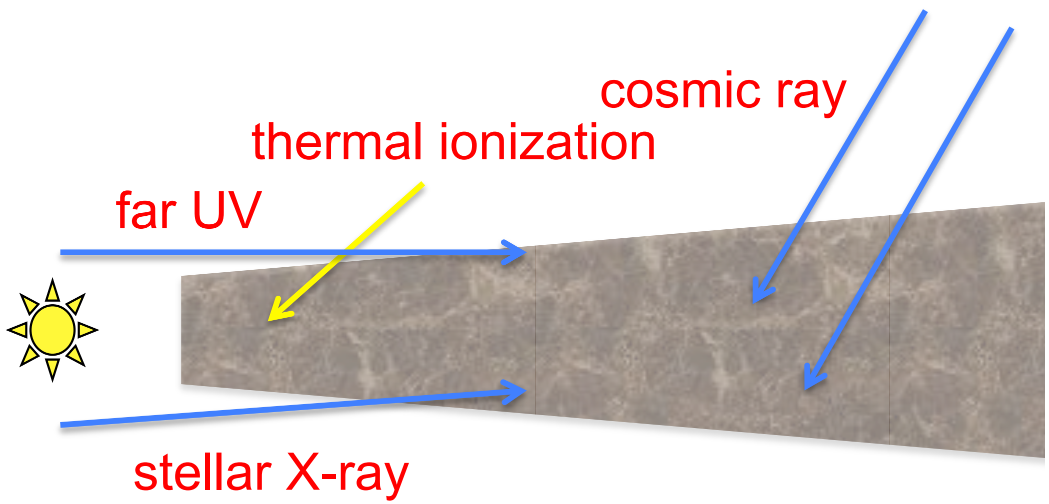
  Disks generally possess negative radial entropy gradient, which results in amplification of pre-existing vortices if thermal relaxation is not too slow => can lead to $\alpha \sim 10^{-4}$-10^{-2}.



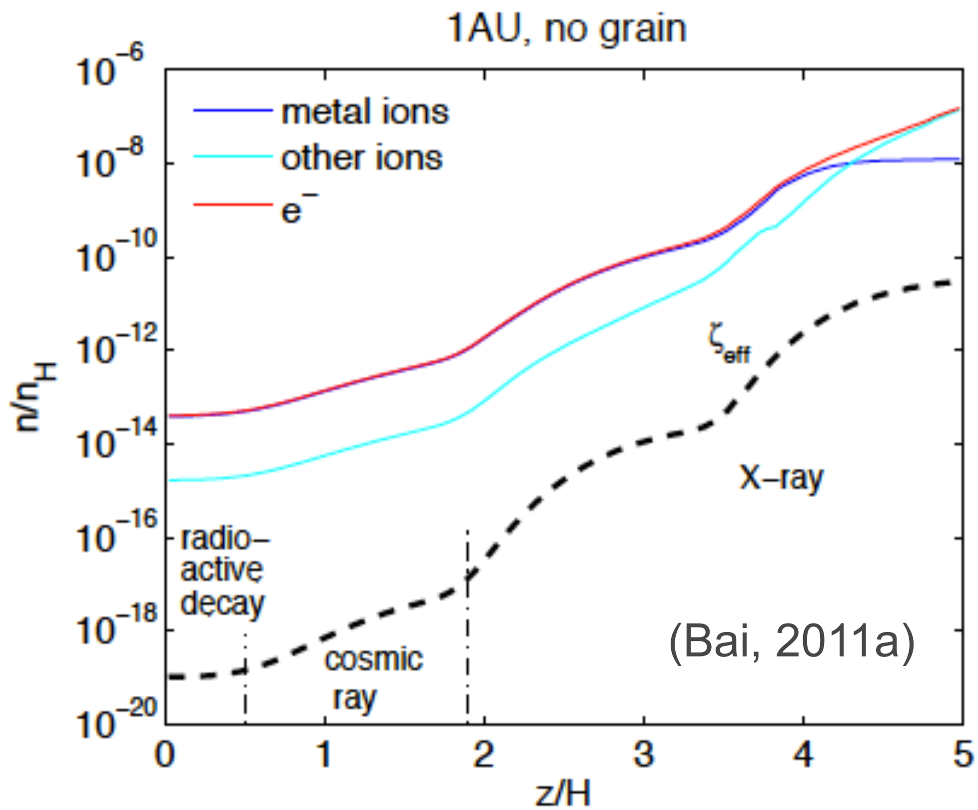
Topics to be covered

- Hydrodynamics and magnetohydrodynamics (MHD)
- Angular momentum transport in accretion disks
- Gravitational instability
- Magnetorotational instability
- Magnetocentrifugal wind
- Hydrodynamic instabilities
- Non-ideal MHD physics
- Summary: current understandings

PPDs are extremely weakly ionized



Umibayashi & Nakano (1981)
Igea & Glassgold (1999)
Perez-Becker & Chiang (2011b)



Ionization fraction rapidly decreases from surface to midplane.

Including small grains further reduce disk ionization.

Conductivity in weakly ionized gas

In the absence of magnetic field: $\boldsymbol{J} = \sigma \boldsymbol{E}$

In the presence of magnetic field: $\boldsymbol{J} = \boldsymbol{\sigma}(\boldsymbol{B}, \boldsymbol{E})\boldsymbol{E}$

Motion of charged particles is set by the Hall parameter: $\beta_j \equiv \frac{Z_j e B}{m_j c} \frac{1}{\gamma_j \rho}$ (Wardle, 1999)

Dense
Weak B

$\beta_i \ll |\beta_e| \ll 1$: **Ohmic regime**, both e$^-$ & ions coupled to the neutrals

$\beta_i \ll 1 \ll |\beta_e|$: **Hall regime**, e$^-$ coupled to B, ions coupled to neutrals

Sparse
Strong B

$1 \ll \beta_i \ll |\beta_e|$: **ambipolar diffusion regime**, both e$^-$ & ions coupled to B.

Conductivity in weakly ionized gas

Induction equation (no grain):

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times \left[\frac{4\pi\eta}{c} \boldsymbol{J} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{e n_e} - \frac{(\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B}}{c\gamma\rho\rho_i} \right]$$

inductive **Ohmic** **Hall** **AD**

$$\sim \frac{n}{n_e} \qquad \sim \frac{n}{n_e}\frac{B}{\rho} \qquad \sim \frac{n}{n_e}\frac{B^2}{\rho^2}$$

Dense Weak B

$\beta_i \ll |\beta_e| \ll 1$: **Ohmic regime**, both e⁻ & ions coupled to the neutrals

$\beta_i \ll 1 \ll |\beta_e|$: **Hall regime**, e⁻ coupled to B, ions coupled to neutrals

Sparse Strong B

$1 \ll \beta_i \ll |\beta_e|$: **ambipolar diffusion regime**, both e⁻ & ions coupled to B.

Conductivity in weakly ionized gas

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inductive **Ohmic** **Hall** **AD**

$$\sim \frac{n}{n_e} \qquad \sim \frac{n}{n_e}\frac{B}{\rho} \qquad \sim \frac{n}{n_e}\frac{B^2}{\rho^2}$$

midplane region of the inner disk

inner disk surface and outer disk

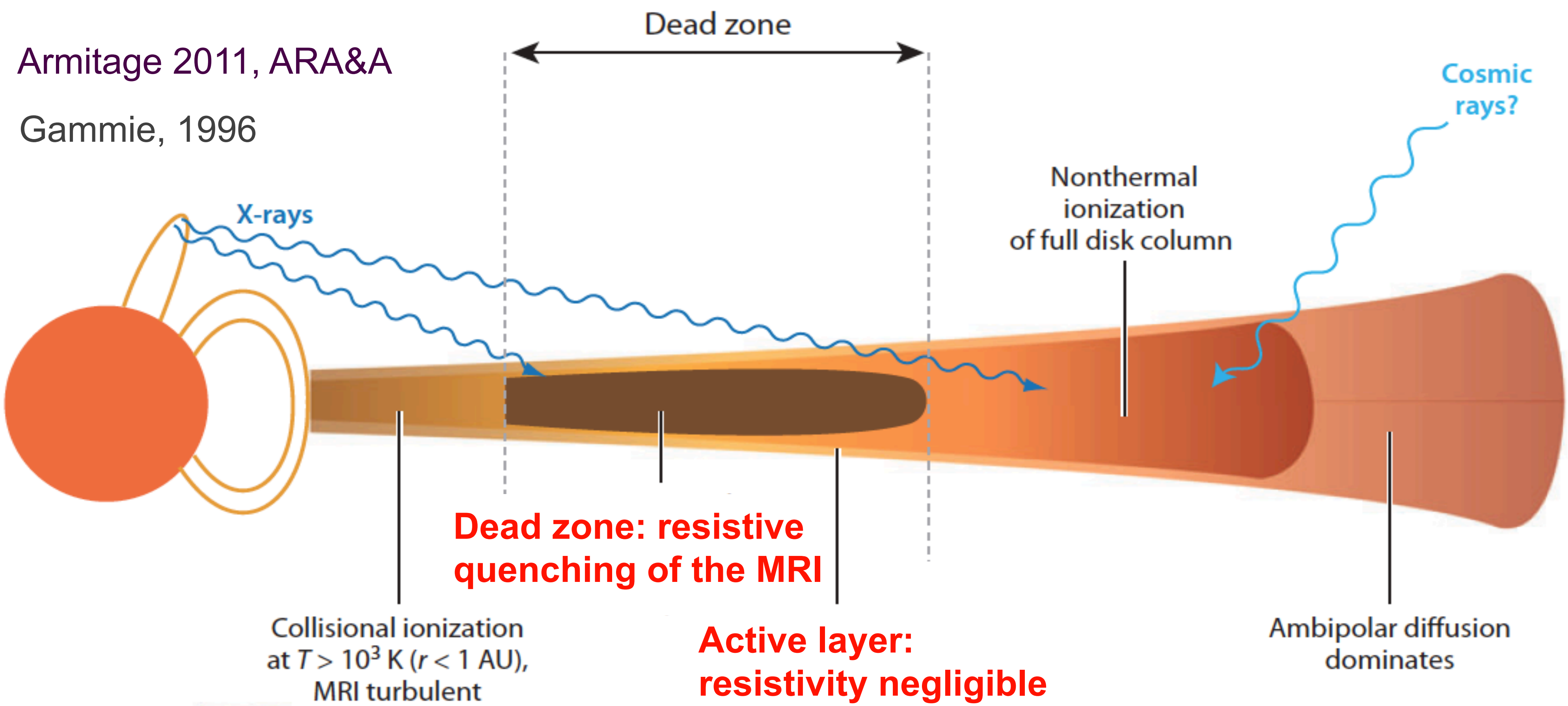
Intermediate heights in the inner disk

Midplane in the outer disk (up to ~60 AU)

Conventional picture of layered accretion

Armitage 2011, ARA&A

Gammie, 1996



- Semi-analytical studies already indicated that MRI is insufficient to drive rapid accretion when including the effect of ambipolar diffusion (Bai & Stone, 2011, Bai, 2011a,b, Perez-Becker & Chiang, 2011a,b).

What happens if we flip the magnetic field?

Lorentz force: $\sim \boldsymbol{J} \times \boldsymbol{B}$ is unaffected.

Note $\boldsymbol{J} = \frac{c}{4\pi} \nabla \times \boldsymbol{B}$

Induction equation (no grain):

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times \left[\frac{4\pi\eta}{c} \boldsymbol{J} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{e n_e} - \frac{(\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B}}{c \gamma \rho \rho_i} \right]$$

inductive **Ohmic** **Hall** **AD**

– – – $(-)^2 = +$ $(-)^3 = -$

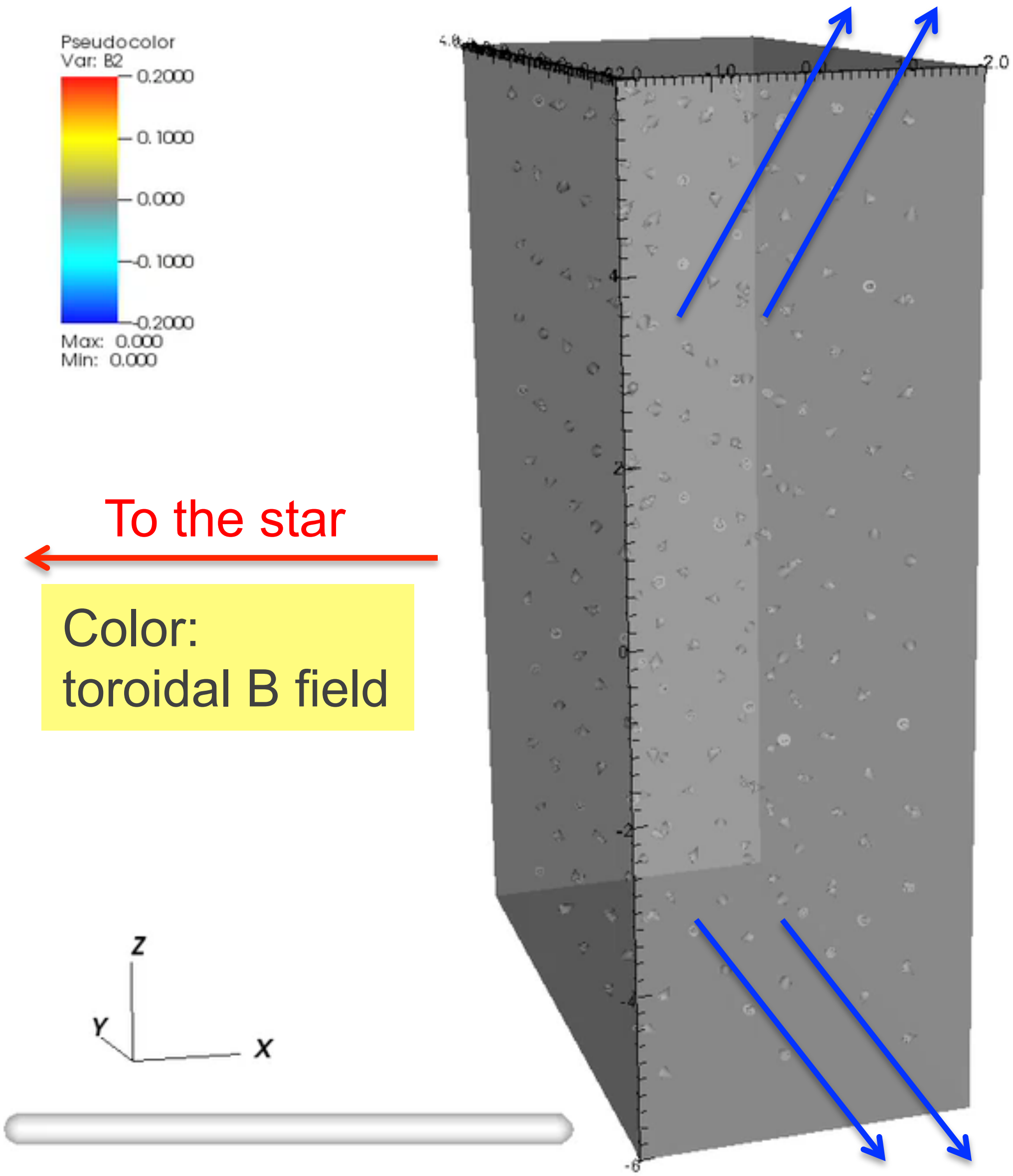
The Hall term is Polarity Dependent!

What do we expect from non-ideal MHD effects?

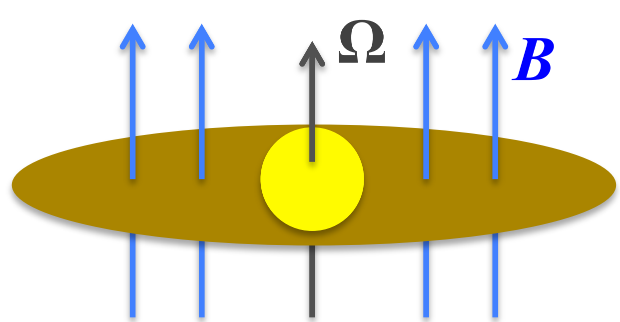
	Ohmic resistivity	Hall effect	Ambipolar diffusion
Form	$\boldsymbol{E}_O = \eta_O \boldsymbol{J}$	$\boldsymbol{E}_H \sim \frac{\boldsymbol{J} \times \boldsymbol{B}}{e n_e}$	$\boldsymbol{E}_A \sim \frac{-(\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B}}{\gamma \rho \rho_i}$
Nature	Dissipative	Non-Dissipative	Dissipative
Outcome	Suppresses the MRI when strong, especially in weak fields	A bit tricky to summarize…	Suppresses the MRI when strong, especially in strong fields.
References	Jin 96; Sano et al. 98, 00, Fleming et al. 00, Turner et al. 07, + dozens	Wardle, 99, Balbus & Terquem, 01; Wardle & Salmeron 11… Sano & Stone, 02a,b, Kunz & Lesur, 13	Blaes & Balbus 94, Kunz & Balbus, 04; Desch, 04; … Hawley & Stone, 98, Bai & Stone, 11

Representative results at 5 AU

β_{z0}~10^5



(Bai & Stone, 2014, in prep)



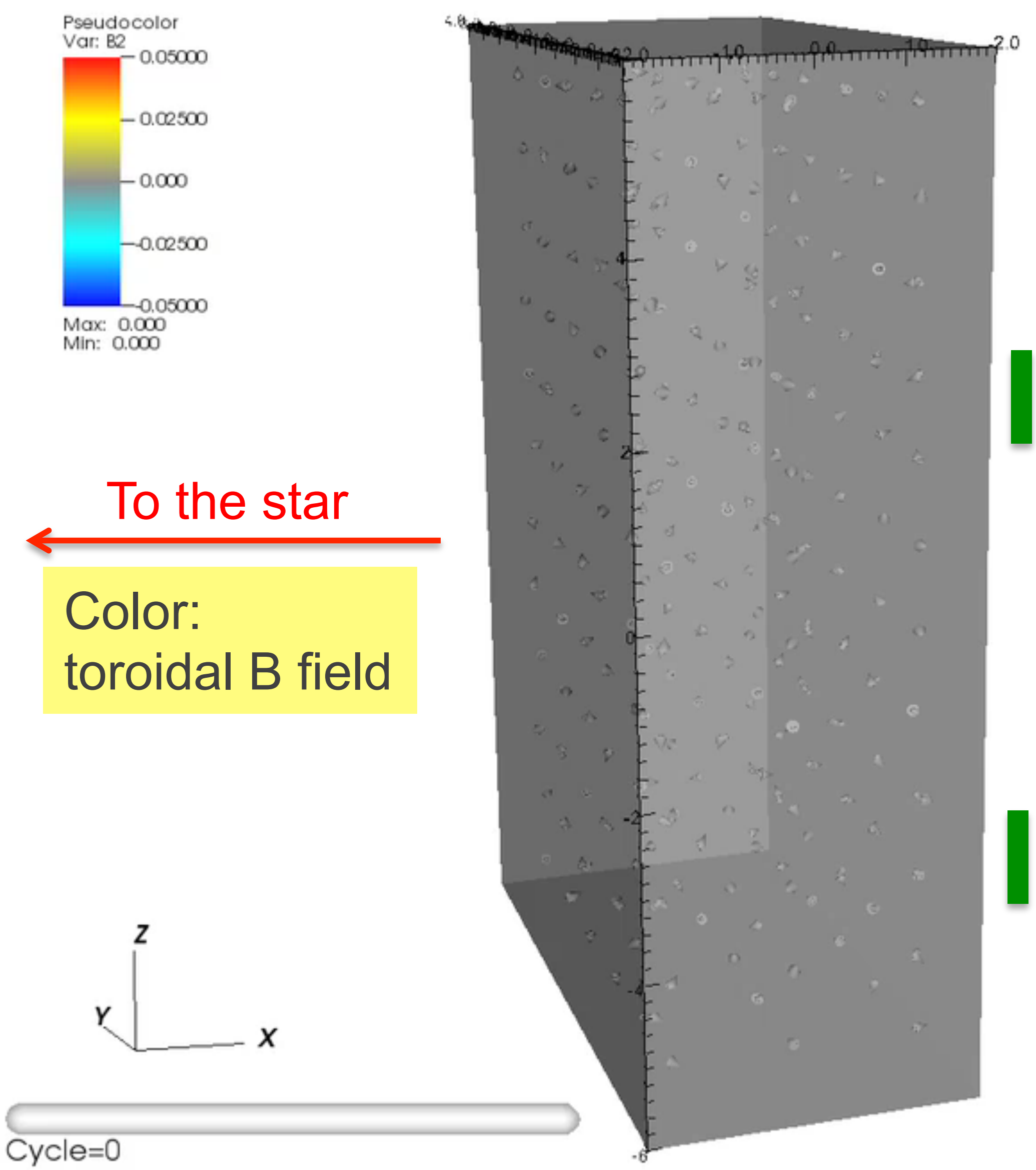
Midplane strongly magnetized, with B_ϕ reversing sign.

System is stable to MRI, and midplane is weakly turbulent (resulting from reconnection).

Launching of magneto-centrifugal wind.

Representative results at 5 AU

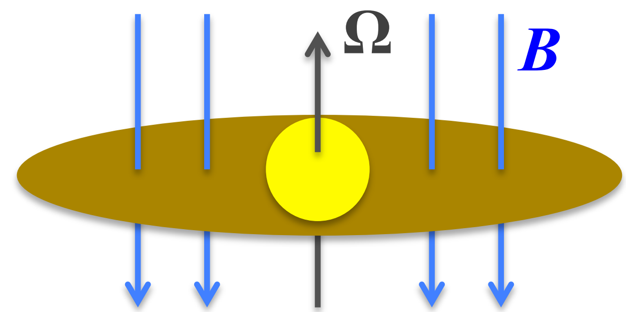
β_{z0}~10^5



To the star

Color:
toroidal B field

(Bai & Stone, 2014, in prep)



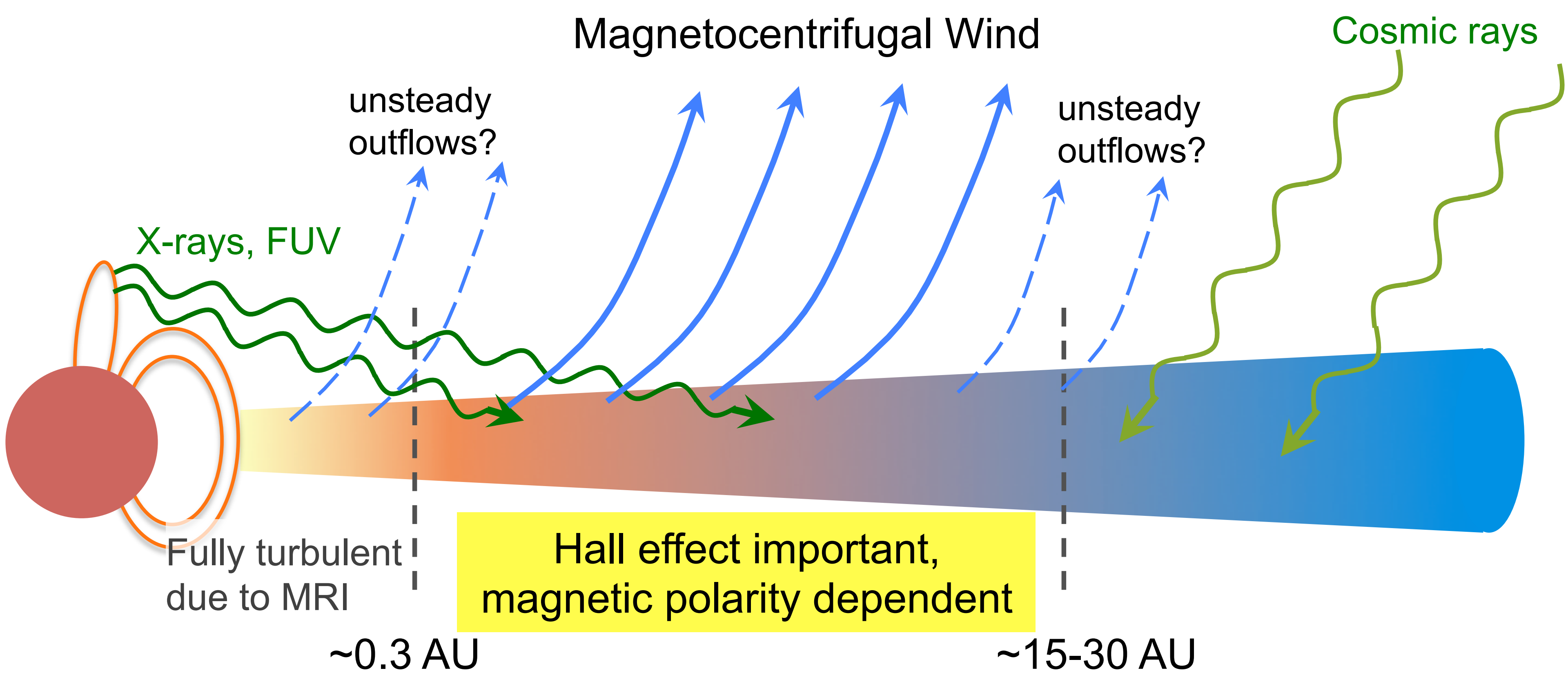
Midplane region is weakly magnetized and weakly turbulent (from surface MRI turbulence)

The system is unstable to the MRI ~2-3H off the midplane.

B_ϕ and outflow alternating directions due to MRI

A paradigm shift

(Bai, 2013)



External magnetic flux is essential to drive disk evolution.

The inner disk is largely laminar. Accretion is driven by disk wind.

Layered accretion picture applies at the outer disk (>~30 AU).

Summary

- Transport of angular momentum in accretion disks: radial transport (by "viscosity") and vertical transport (by wind).
- Gravitational instability requires massive disks, and can be important in the early phase of PPD.
- The MRI is a powerful instability to drive viscous accretion, but requires the B field to be coupled to the gas.
- Magnetocentrifugal wind, if launched, transports angular momentum more efficiently than MRI (by R/H).
- PPDs are extremely weakly ionized, resulting in non-ideal MHD effects => crucial for PPD gas dynamics.
- Hydrodynamic mechanisms can also be viable, which are under active investigation.

Open questions

- Need to better understand the ionization/recombination chemistry and role of grains.
- Need to explore thermodynamics/equation of state.
- Need to explore global disk evolution can with net vertical magnetic flux, with large domain size and resolved microphysics (numerically challenging).
- There are a suite of hydrodynamical instabilities confirmed in very recent times. Studies on these effects remain in early stages.