清华大学天体物理优秀大学生暑期学校, 2017.07.10

原行星盘与行星形成

Protoplanetary Disks and Planet Formation

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2017.8-

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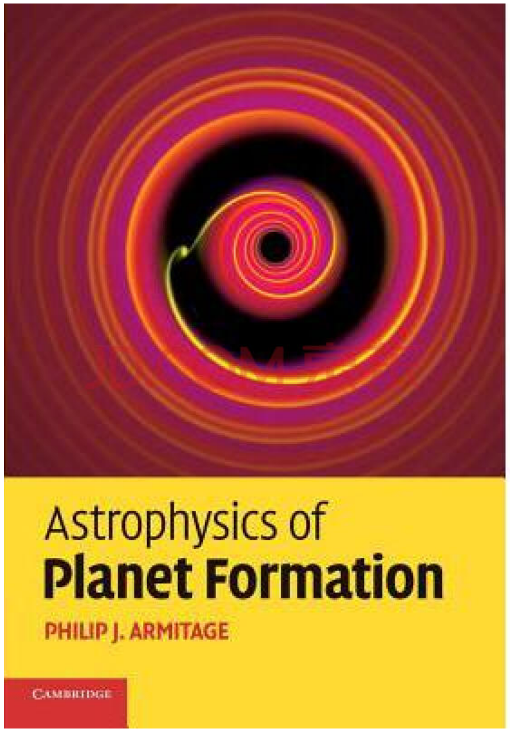
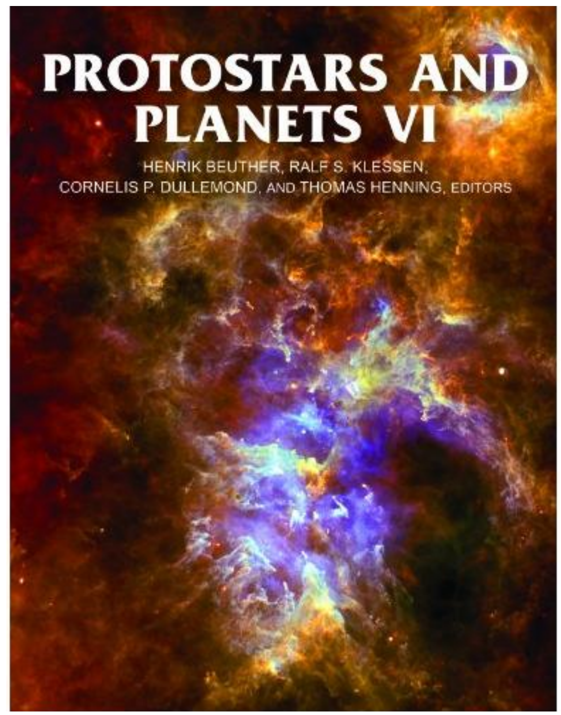
General resources:



Astrophysics of Planet Formation, Armitage, P. J., 2010, Cambridge Univ. Press.

Pedagogical introduction to the theory of planet formation. Chapters 2-5 are the most relevant to this lecture.



Protostar and Planets VI, H. Beuther, R. Klessen, C.P. Dullemond, T. Henning (eds.), 2014, Univ. of Arizona Press

Most updated in all fields of planetary studies. All chapters are available online. See also:

http://www.mpia-hd.mpg.de/homes/ppvi/

Further readings:

Reviews of disk observations:

Andrews, S., 2015, PASP, 127, 961

Williams, J. P. & Cieza, L. A., 2011, ARA&A, 49, 67

Reviews of disk dynamics and planet formation:

Armitage, P., 2007, arXiv:astro-ph/0701485, lecture notes on the formation and eary evolution of planetary systems (updated recently)

Armitage, P., 2015, arXiv: 1509.06382, more in-depth lecture notes on physical processes in protoplanetary disks.

Turner, N. J. et al. 2014, review chapter in PPVI, arXiv:1401.7306, high-level review of protoplanetary disk gas dynamics

Morbidelli, A. & Raymond, S.N., 2016, Journal of Geophysical Research (Planets), 121, 1962, review on the theoretical challenges in planet formation

Chiang, E. & Youdin, A., 2010, AREPS, 38, 493, review on planetesimal formation

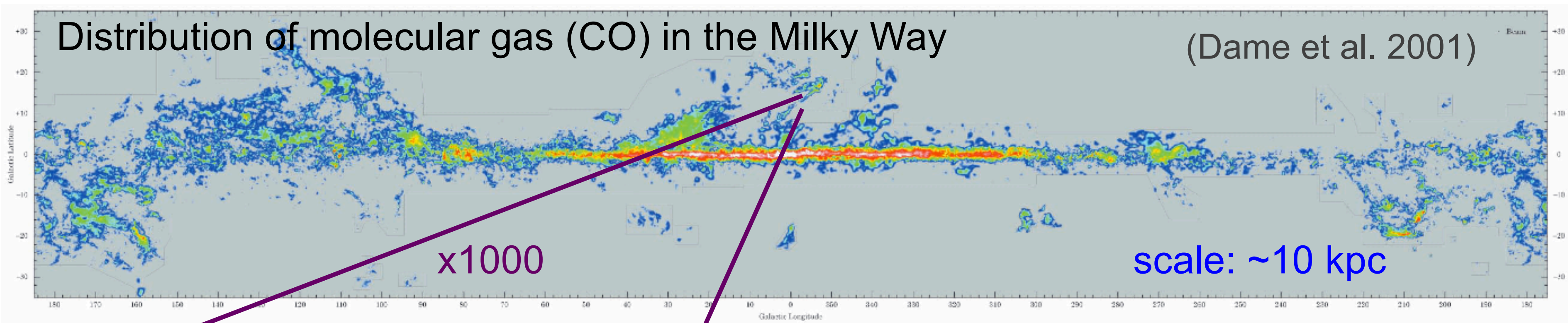
Goldreich, P., Lithwick, Y. & Sari, R., 2004, ARA&A, 42, 549, review on the gravitational accretion of solid bodies during planet formation

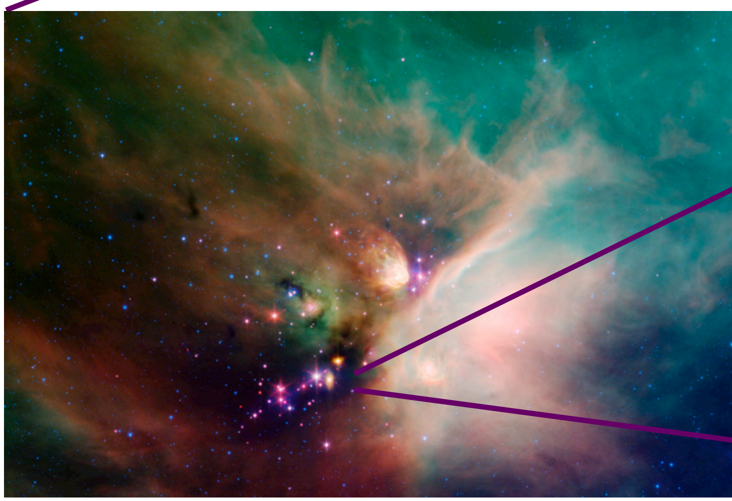
Outline

- Protoplanetary disks: observations
- Protoplanetary disks: gas dynamics
- Planet formation: basic theory
- Challenges and opportunities

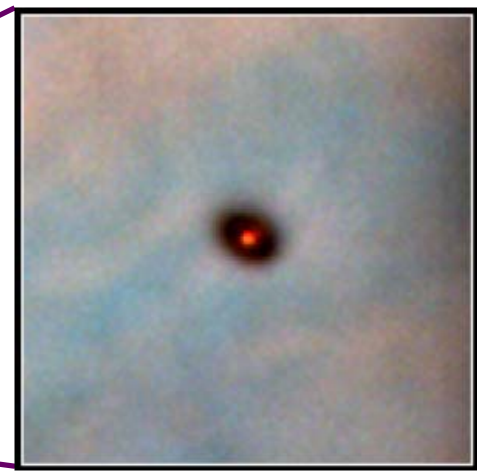
Observational Overview of Protoplanetary Disks

Protoplanetary disks in the context





x1000



Star forming region (ρ ophiuchus)

Credit: Spitzer space telescope

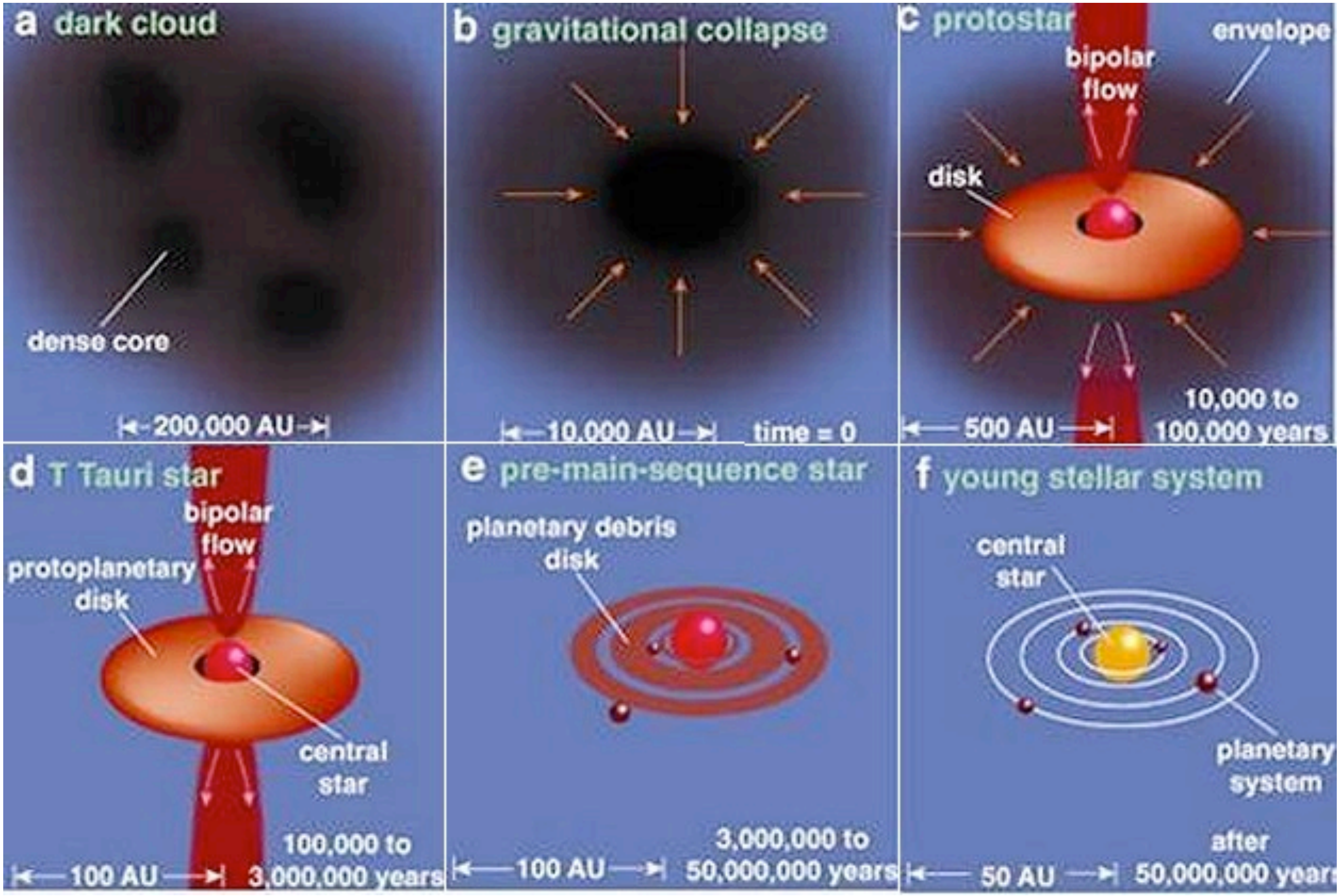
~a few x10 pc

proto-star and its disk

~a few x1000 AU

(1 pc =3.26 light years ~ $2\text{x}10^5$ AU)

(low-mass) Star and planet formation



Disk formation & size: general considerations

Size of a pre-stellar core: $R_{\rm core}$~0.05 pc

Rotational to gravitational energy: $$\beta_{\rm rot} \equiv \frac{1}{3}\frac{\omega^2 R^2}{GM/R}$$

(assuming uniform density and rotation)

~0.02 (with large scatter).

(Goodman et al. 1993)

By angular momentum conservation, infalling material settles to a disk with size $R_{\rm disk}$:

$$J \sim I\omega \sim \sqrt{GMR_{\rm disk}}$$

$$\Rightarrow \quad R_{\rm disk} \sim \frac{1}{2}\beta_{\rm rot} R_{\rm core} \sim 100{\rm AU}$$

Protoplanetary disks: fundamental properties

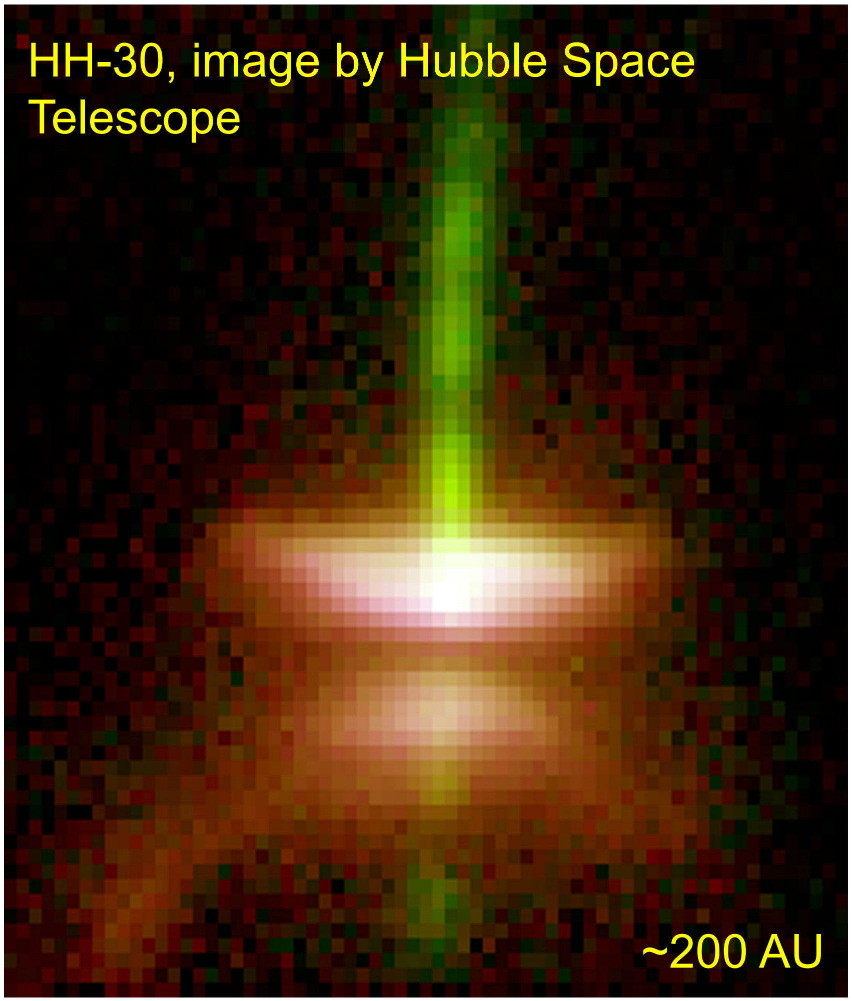


Image at optical wavelength: disk is opaque, see scattered light.

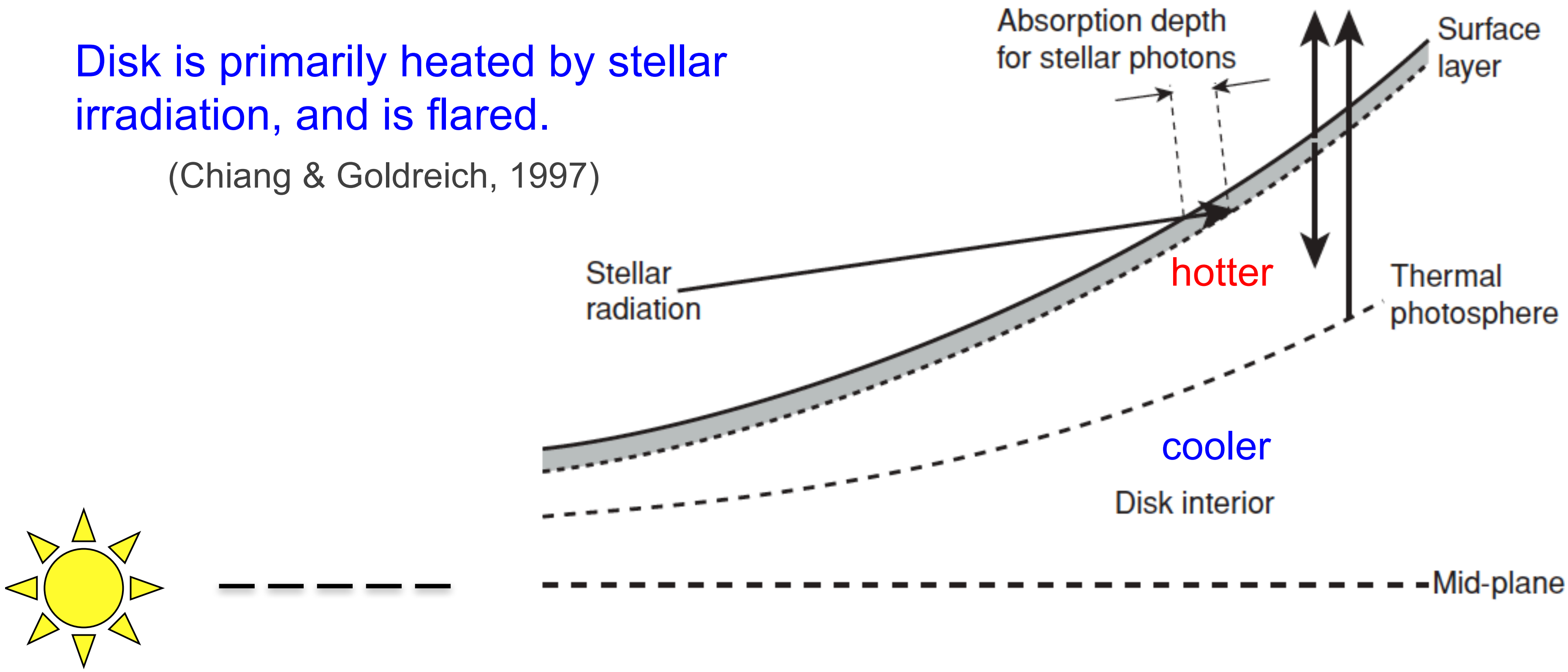
- Made of mostly gas + dust (~1% in mass)
- Disk mass: ~0.01 $M_\odot$ with large scatter
- Size: a few 10s-100s AU with large scatter.
- Lifetime ~ a few Myrs.
- Gas is being accreted to ~ 10^{-8} $M_\odot$ per yr with large scatter.

(e.g., Andrews et al. 09,10,13; Williams & Cieza, 2011, ARA&A)

Understanding disk observations: thermal structure

Disk is primarily heated by stellar irradiation, and is flared.

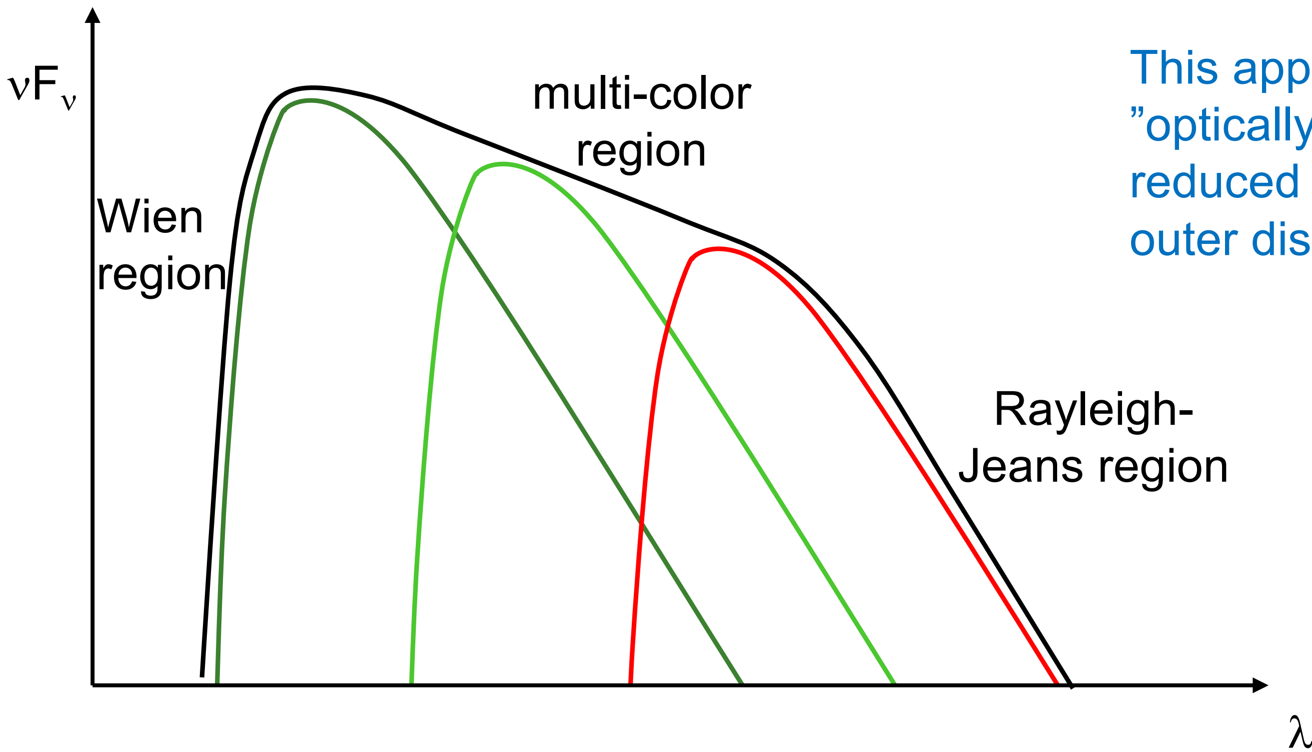
(Chiang & Goldreich, 1997)



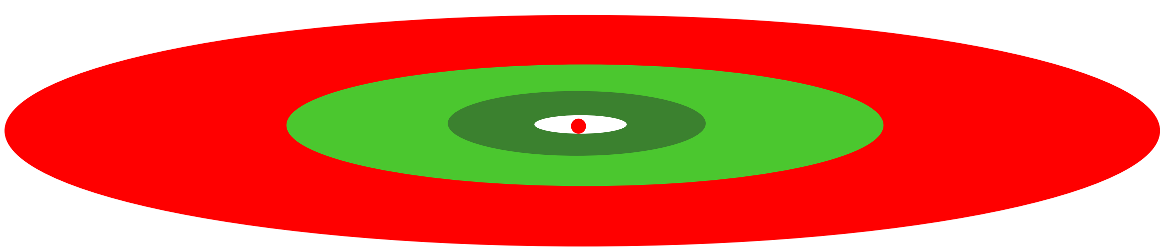
To zeroth order, approximate the disk with isothermal emitting blackbody:

$$\alpha \frac{L_*}{4\pi R^2} \approx \sigma T_{\rm disk}^4 \quad \Rightarrow \quad T_{\rm disk} \sim R^{-1/2}$$

Multi-color blackbody disk spectra



This applies when the dust is ”optically thick”. Emission is reduced in the “optically-thin” outer disk.

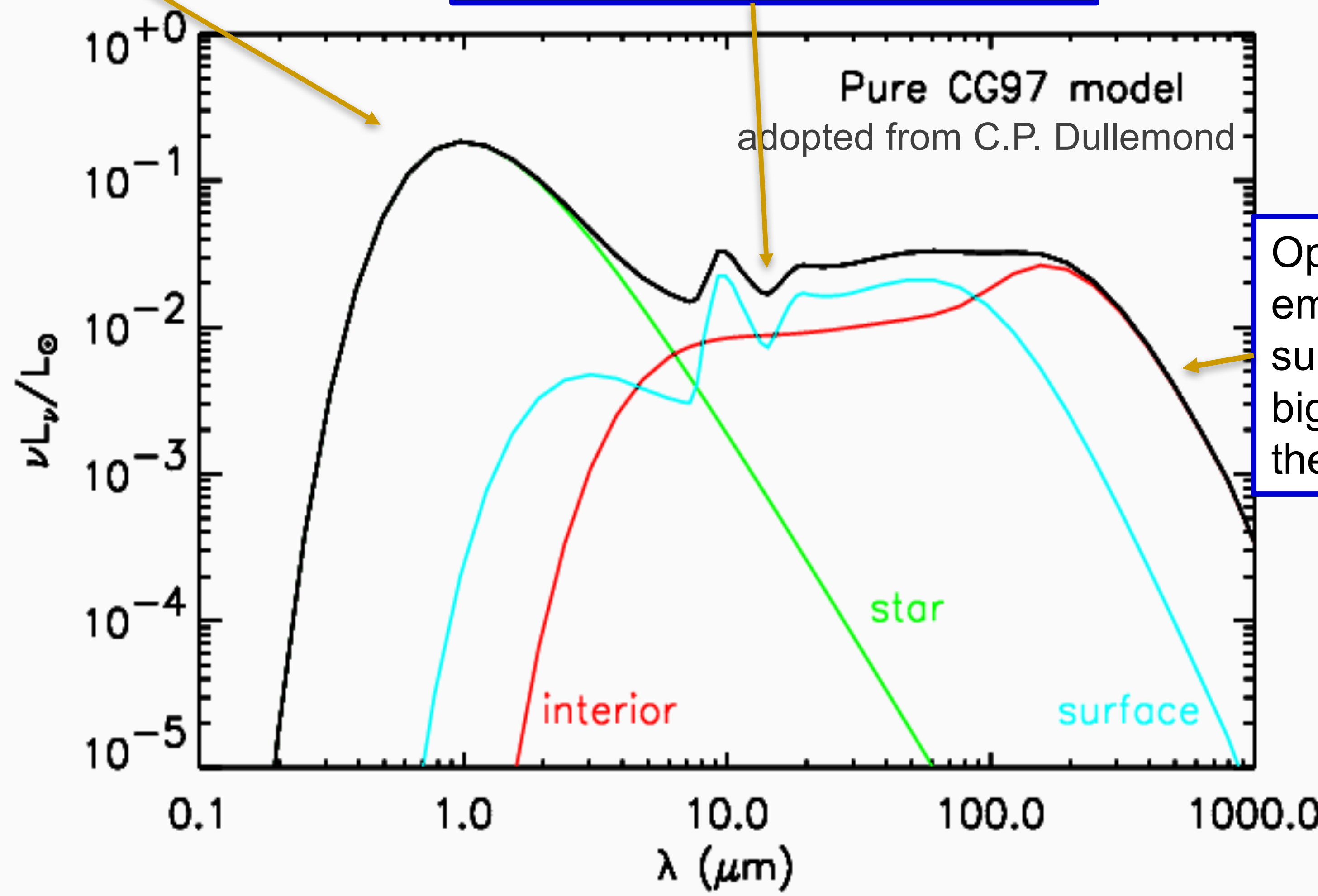


adopted from C.P. Dullemond

Spectral energy distribution (SED)

Star and scattered light

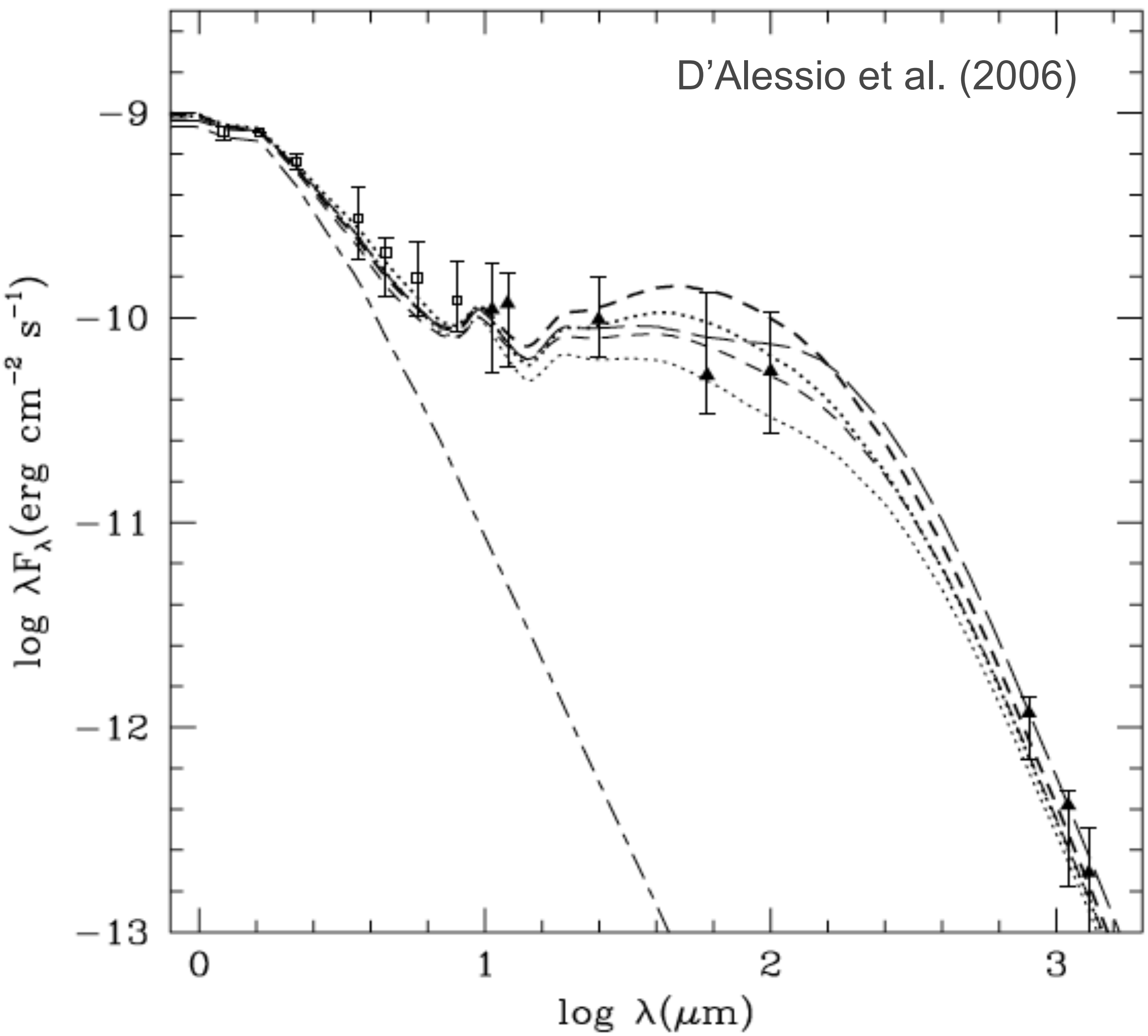
Near infrared excess: emission from super-heated surface dust



Optically thin emission at sub-mm from big grains in the outer disk

(Chiang & Goldreich, 1997)

Modeling the SEDs



To constrain:

- Dust composition, size, abundance and distribution
- Disk size, surface density and temperature
- Viewing geometry

Parameters are highly degenerate but show evidence of grain growth and settling.

The ALMA revolution

Location: Chajnantor plain, Chile (altitude ~5000m)

50 antennas of 12m each

Wavelength: 0.4-3mm

Longest baseline: ~15 km

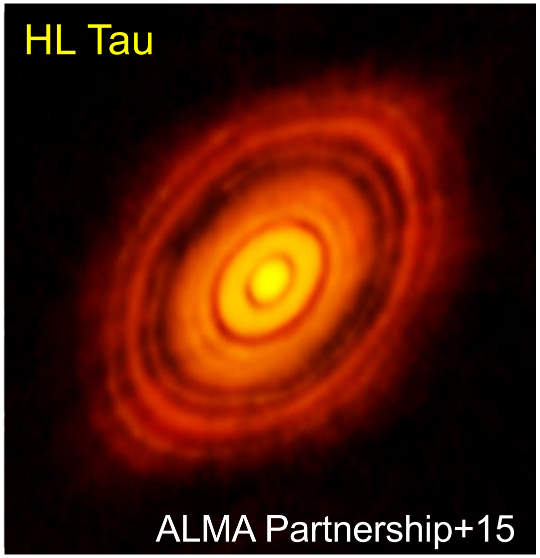
~1 order of magnitude improvement in sensitivity and resolution compared to previous instruments

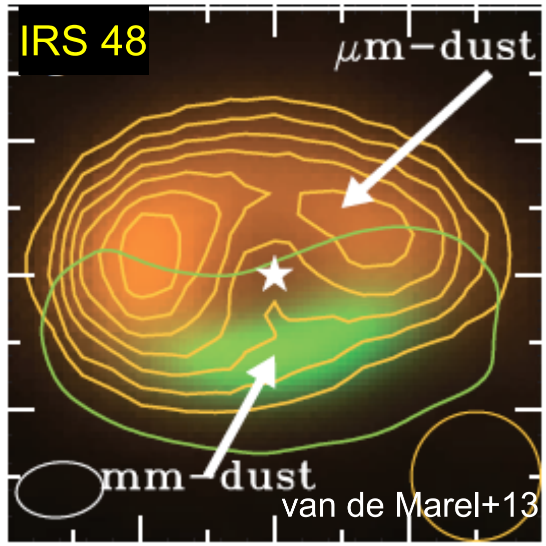
Telescope resolution in diffraction limit: $1.22\lambda/D$

Maximum resolution achieved by an interferometer: $1.22\lambda/L$

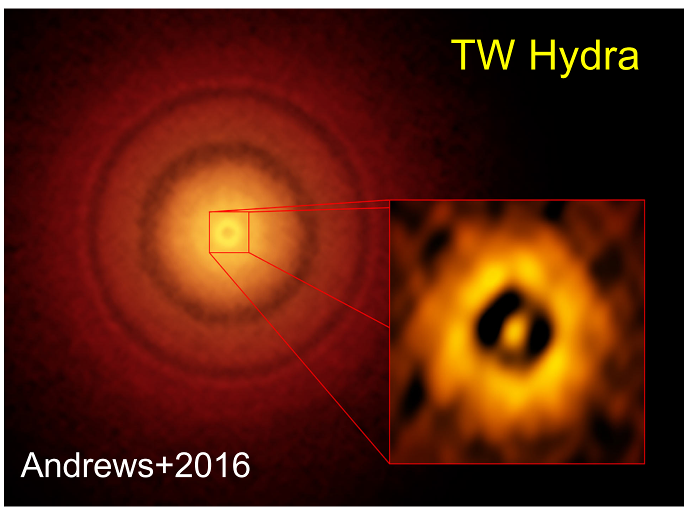
Baseline

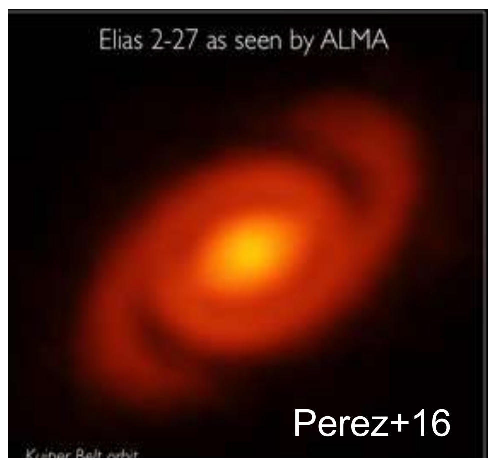
The ALMA revolution





ALMA have revealed that almost all disks have rich “sub-structures”: rings and gaps, asymmetries, spiral arms, etc.

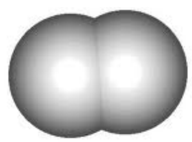


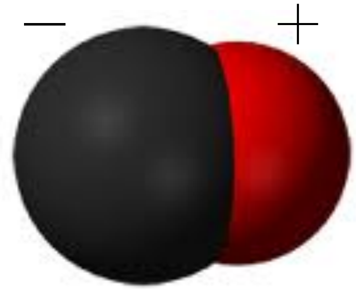


What are causing them is currently under hot debate…

Gas in protoplanetary disks

- Gas contains ~99% of disk mass, but the primary gas species, H_2, does not have a permanent electric dipole moment -> radiate very weakly.





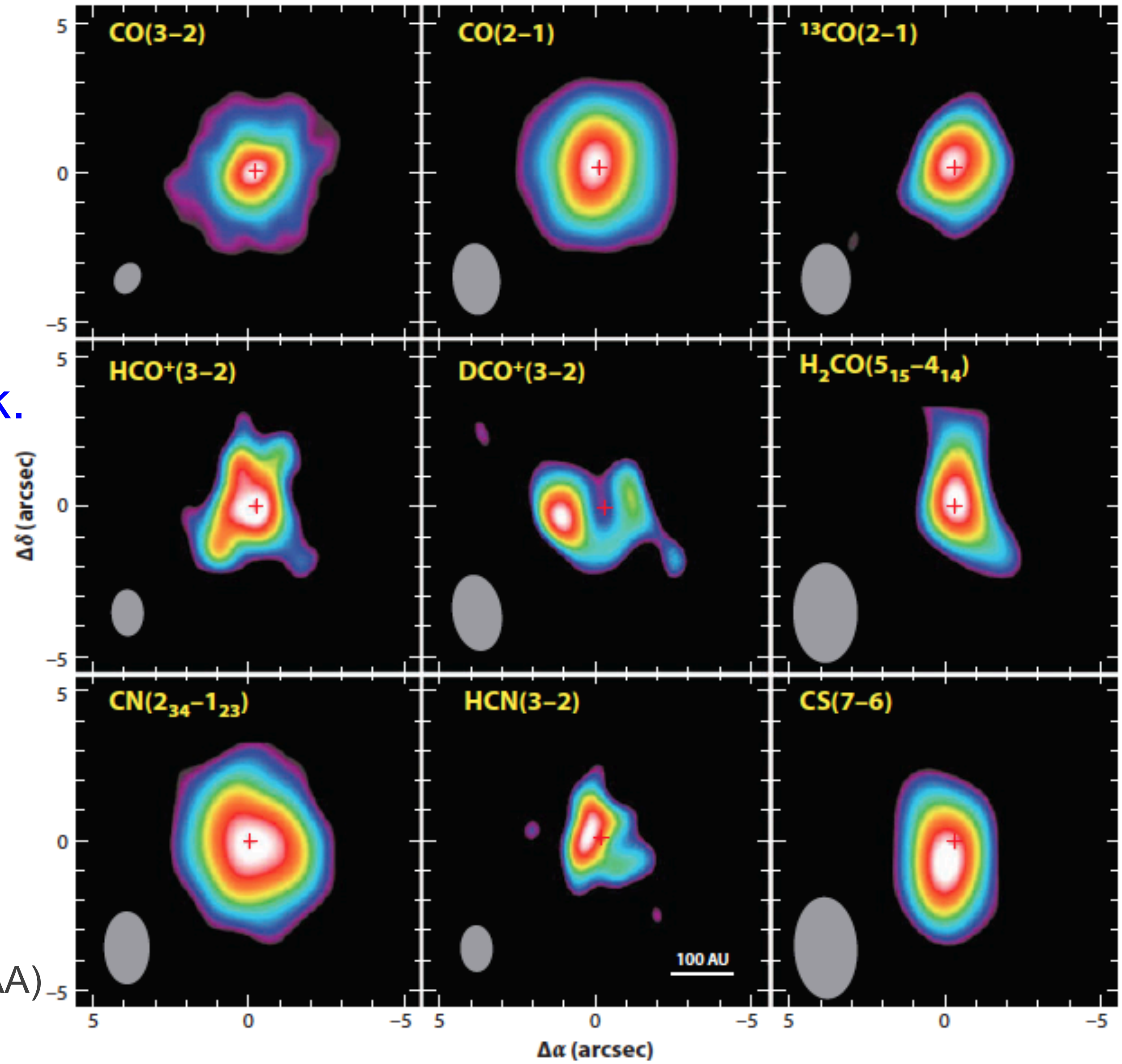
- Other major molecules, particularly CO, do radiate efficiently, although most cases they are optically thick (can not be used to directly measure disk mass).

For a uniform temperature slab in thermal equilibrium:

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T_{\rm exc})(1 - e^{-\tau_\nu})$$

Gas content in sub-mm

- Molecular rotational lines can easily be thermally excited (many are in mm): powerful gas probes of the cold outer disk.
- To probe the disk interior, to use isotopologs, or ionized species.

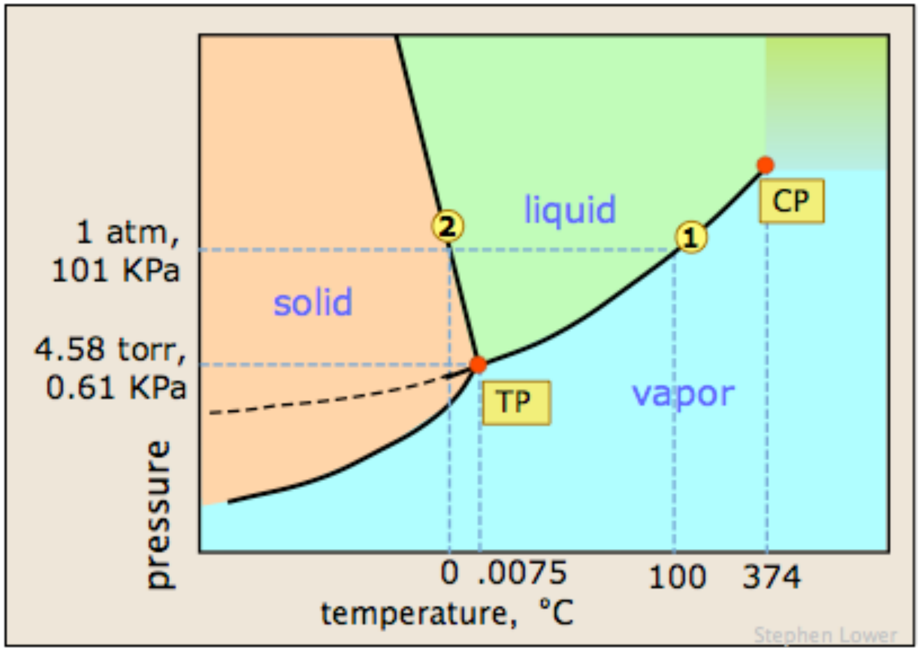


Williams & Cieza (2011, AR&AA)

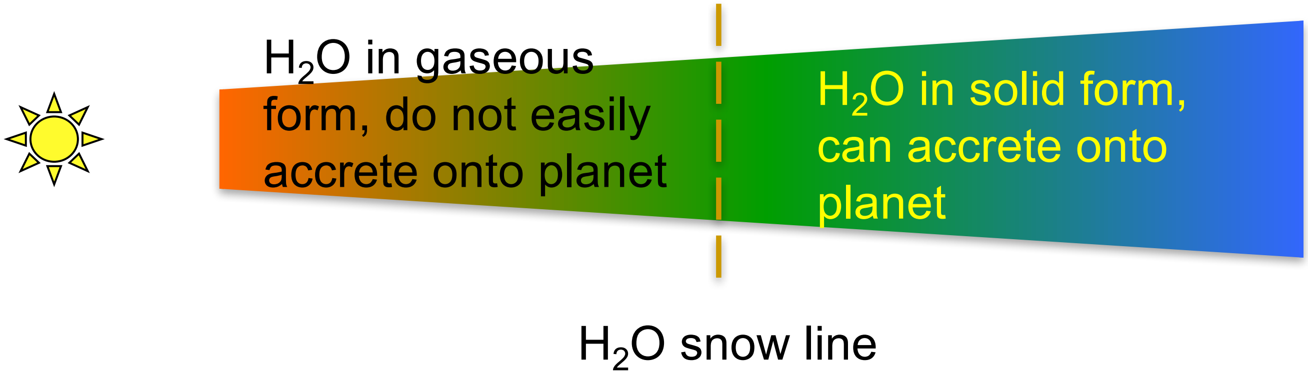
Snowline

Snowline: water freezes out to ice at ~150-170K in PPDs ($P \sim 10^{-6}$ bar).

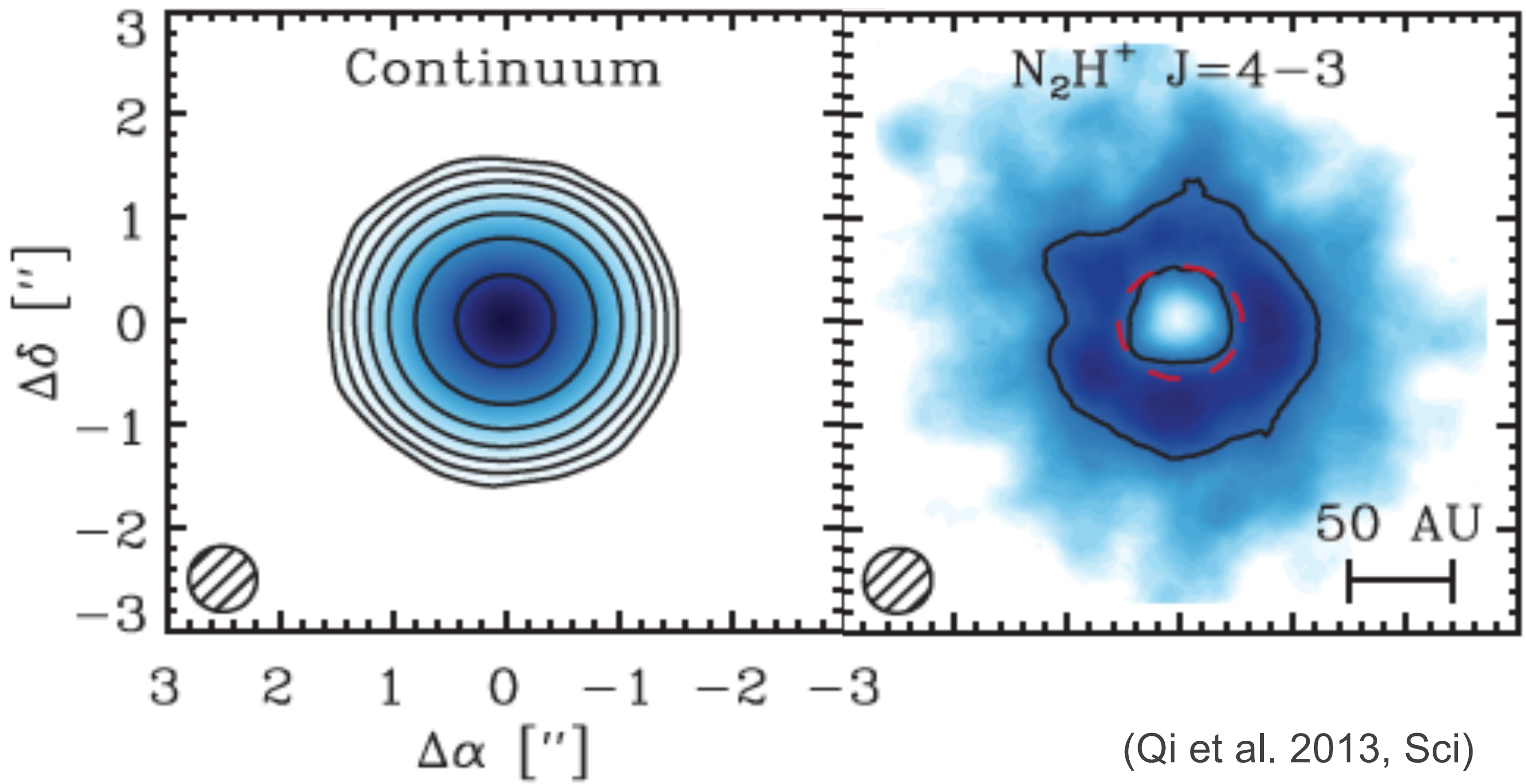
Evidence of snowline at ~2.7 AU during solar system formation.



Question: Is the Earth abundant in water?



Snowline



(Qi et al. 2013, Sci)

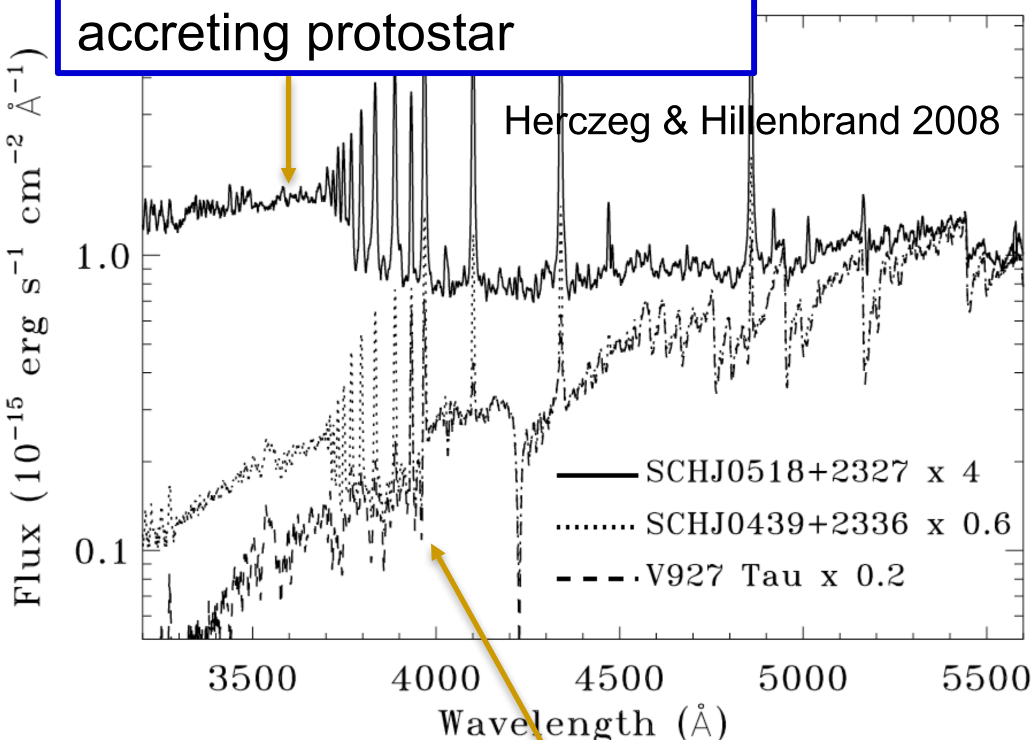
Detection of H_2O snowline in other disks has not been achieved, but detection of CO snowline (~17K) has been reported (by ALMA):

N_2H^+: can easily be destroyed by CO: $N_2H^+ + CO \Rightarrow HCO^+ + N_2$

Accretion signature

Gas falls onto the proto-star via magnetospheric accretion, which strongly heats the stellar polar cap, producing UV excess emission.

UV spectrum of an actively accreting protostar



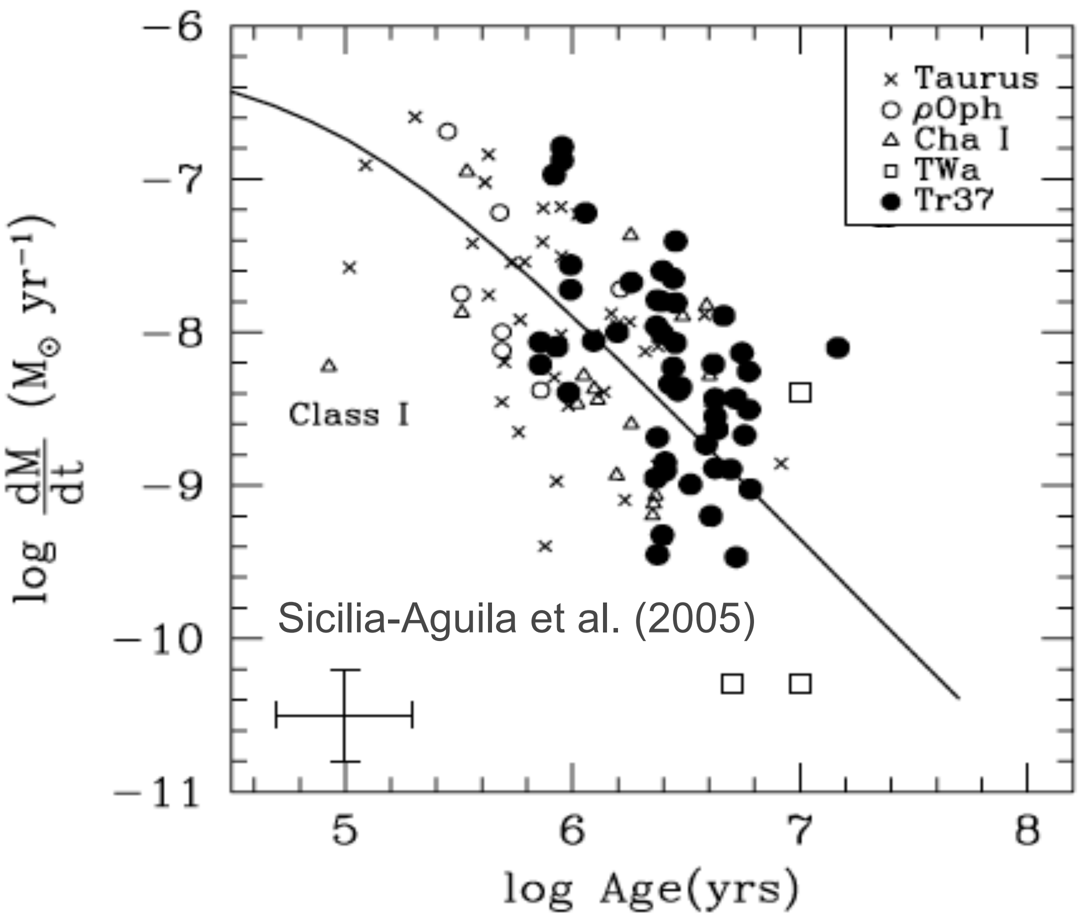
Photospheric radiation without accretion

The accretion process essentially converts gravitational energy into heat and radiation:

$$L_{\mathrm{acc}} \approx 0.8 \frac{GM\dot{M}}{R_*}$$

This is how accretion rate is measured.

Accretion rates

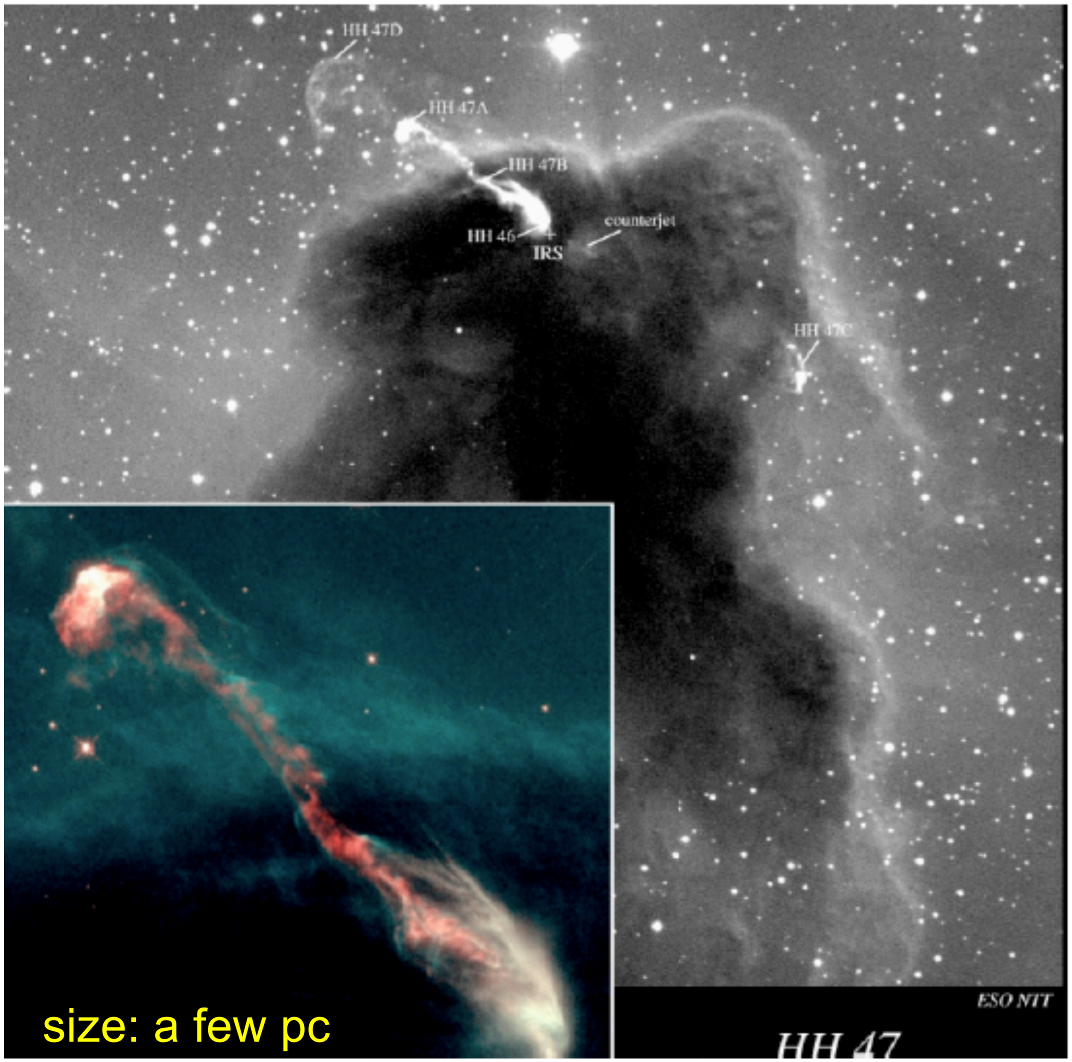


- Typical accretion rate ~ $10^{-8}\,M_\odot$ yr $^{-1}$ with large scatter.
- Acc. rate decreases with stellar age.

Note: stellar age can be approximately estimated based on stellar luminosity and photospheric temperature.

They provide crucial constraints for understanding the gas dynamics of PPDs.

Jets and outflows

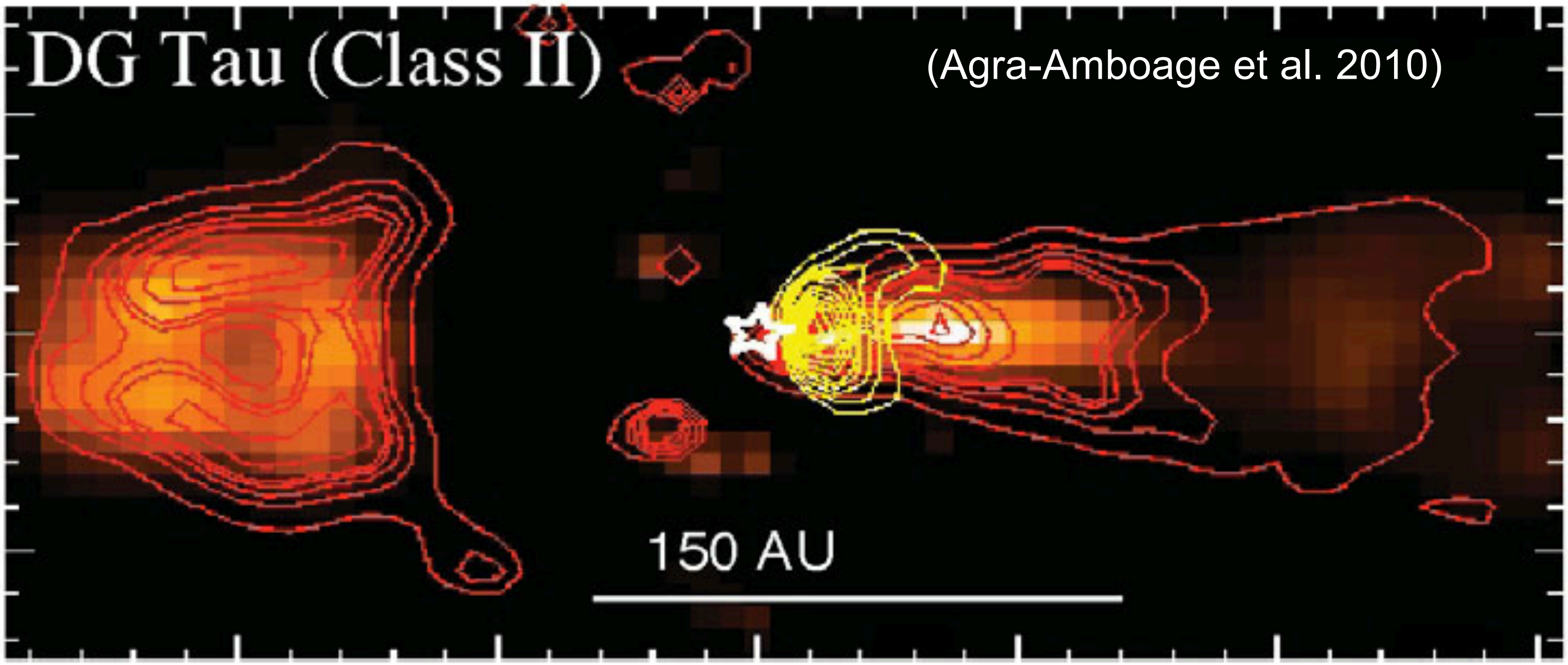


Herbig-Haro objects:

Nebular resulting from YSO jets traveling into their parent cloud/ISM.

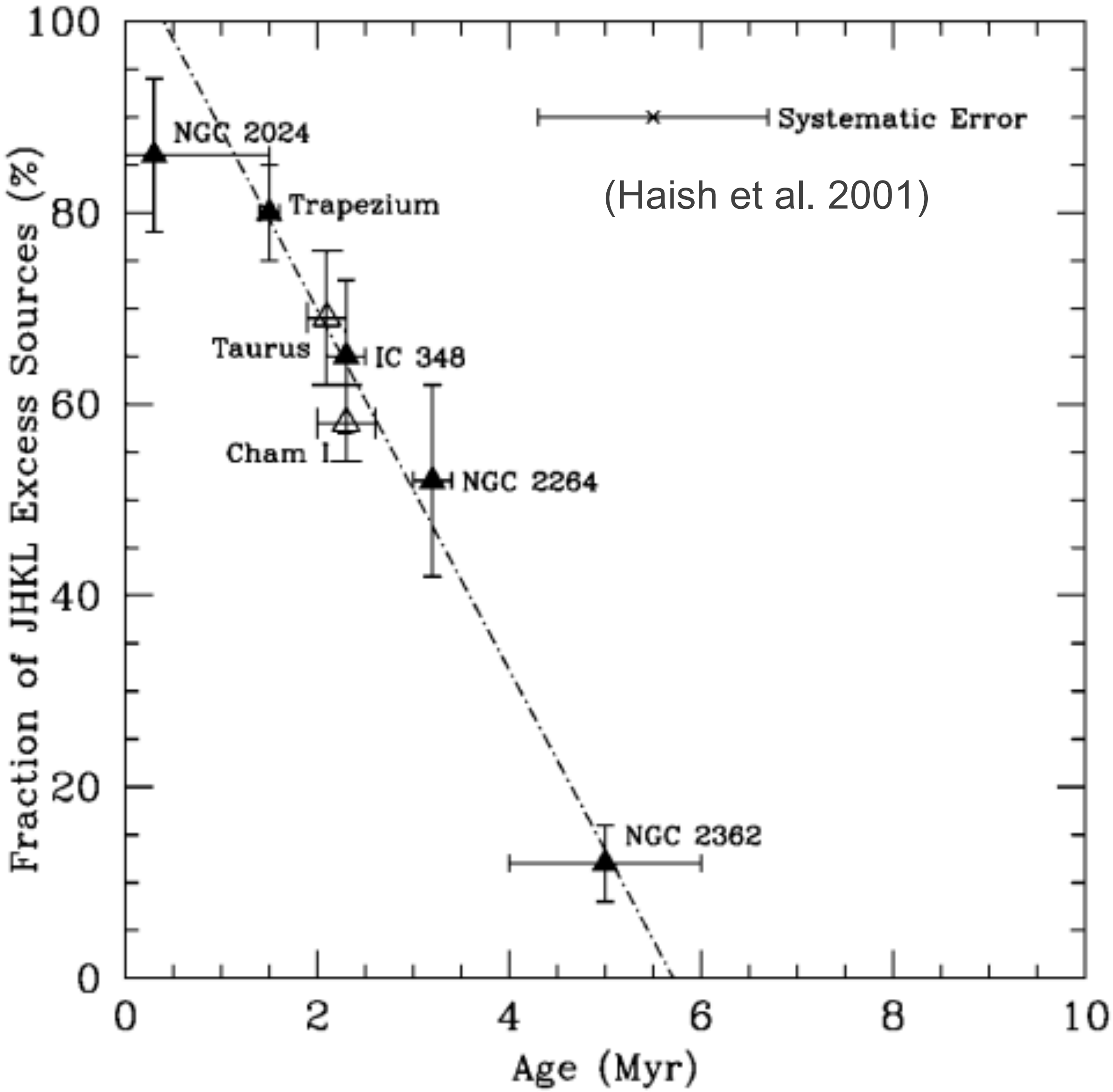
What's the central engine?

Jets and outflows



- Highly collimated high-velocity component + wide-angle low-velocity component (LVC).
- The LVC is very likely to originate from the inner disk (0.3-4AU for DG Tau, Bacciotti et al.2002, Anderson et al.2003)

Disk lifetime



Median lifetime:

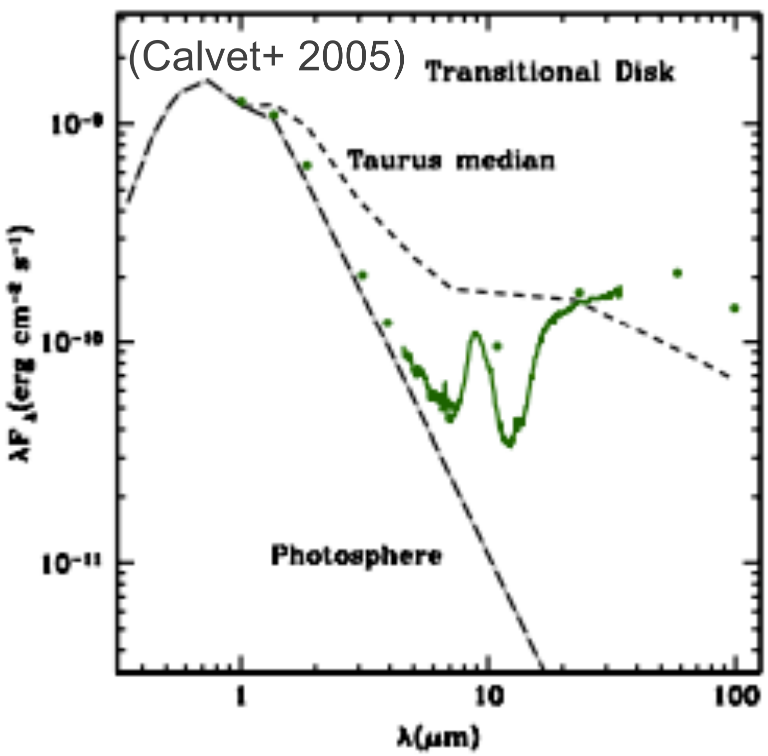
~3 Myr

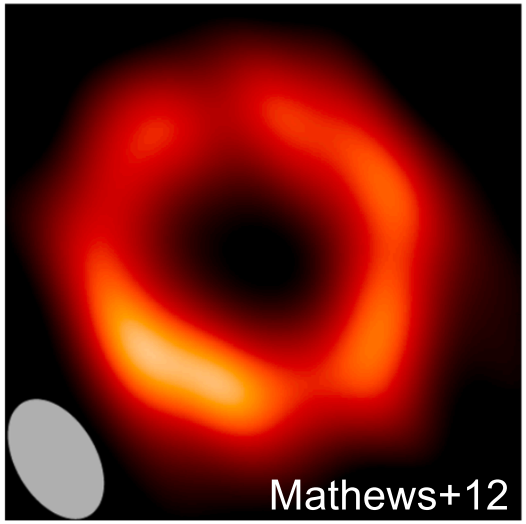
Some old systems still possess massive disks (e.g., TW Hydra).

Environmental effects (photoevaporation from nearby OB stars).

Transition disks

Deficit of NIR and/or MIR flux with respect to the median SED of CTS (Strom et al. 1989)





Interpretation: inner disk is deficient of dust (but generally has gas with less depletion). Cause still unclear!

Transition disks constitute of ~10% of the disk population.

Many transition disks are still actively accreting!

Summary

- Protoplanetary disks: dusty gaseous disk surrounding newly born stars.
- Spectral-energy-distribution implies disk structure and composition.
- ALMA reveals rich disk substructures, challenging conventional understandings.
- Astrochemistry and snow lines: volatile content relevant to planetary composition
- Accretion and outflows are ubiquitous, providing major constraints on disk dynamics.
- Do transition disks represent a transitional stage or signatures of planet formation?

Gas dynamics of Protoplanetary Disks

Hydrostatic equilibrium

The gas experiences gravitational force and a force from its own pressure gradient. It should equal to zero in equilibrium:

$$\rho \frac{d\boldsymbol{v}}{dt} = -\nabla P - \rho \nabla \Phi = 0$$

This equation is particularly simple in 1D, and can be easily solved once an equation of state (EoS) is known:

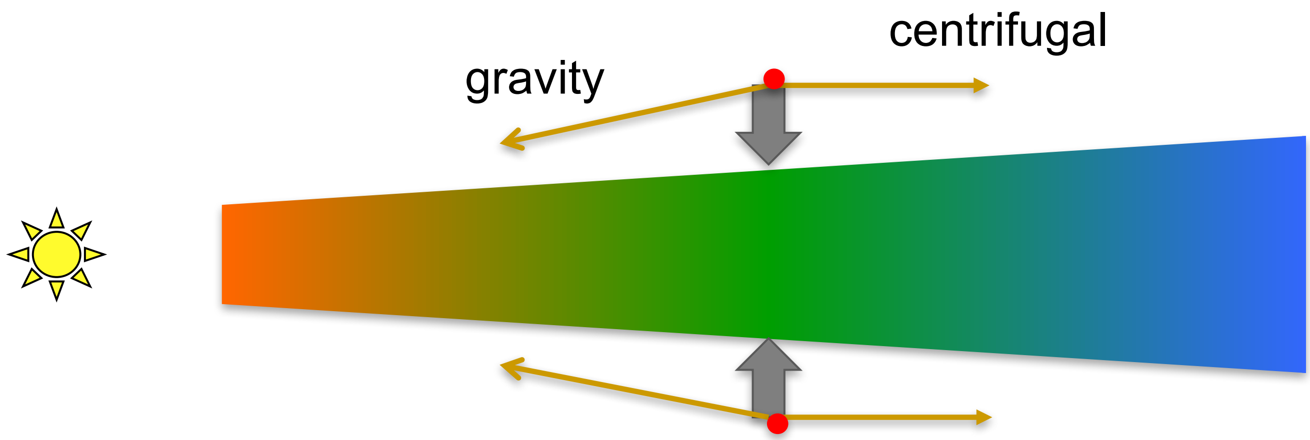
$$P = P(\rho)$$

This is called barotropic EoS. A special case is the **isothermal EoS**: $P = \rho c_s^2$

More generally, the EoS depends on temperature, which generally would require solving the energy balance.

$$P = P(\rho, T)$$ (e.g., $P = \frac{\rho}{\mu m_p} kT$)

Disk vertical structure



Vertical gravity:

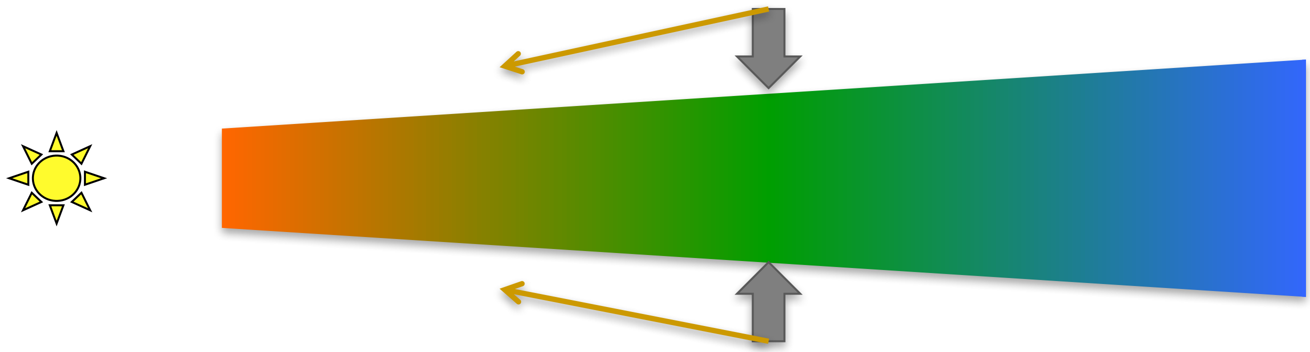
(z<<R)

$$g_z = -\frac{GM_*}{(R+z)^2}\frac{z}{(R^2+z^2)^{1/2}} \approx -\Omega^2 z$$

Pressure support (assuming isothermal EoS $P = \rho c_s^2$):

$$c_s^2 \frac{d\rho}{dz} = \rho g_z$$

Disk vertical structure



Vertical structure: $$\rho = \rho_0 e^{-z^2/2H^2}$$

Disk scale height (*H*): $$H = c_s/\Omega$$

Isothermal sound speed (c_s): $$c_s = \frac{kT}{\mu m_p}$$

$\mu \sim 2.33$
(for molecular gas)

Exercise: the Earth's atmosphere

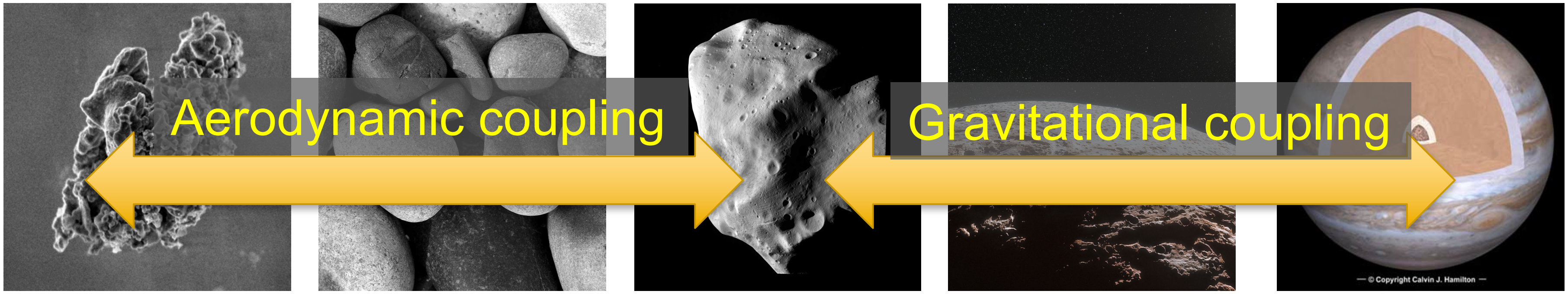
The pressure of the Earth's atmosphere is about 10^5 Pa at sea level, and the gas density is about 1 kg/m^3. Assume the Earth's atmosphere is isothermal, estimate the characteristic scale height of the Earth's atmosphere.

The answer is about 10 km, as we know. See if you can get it on your own.

From hydrostatic equilibrium: $$\frac{P}{H} \approx \rho g$$

Therefore: $$H \sim \frac{P}{\rho g} \approx 10^4 \mathrm{m}$$

Why do we care about gas dynamics?



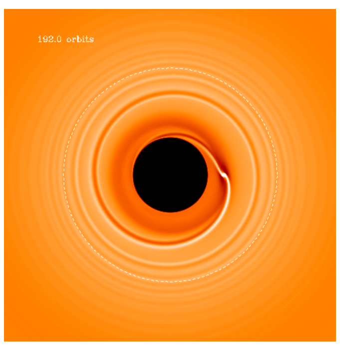
μm | cm | km | 10^3km | 10^5km

Grain growth

Planetesimal formation

Planetesimal growth to cores

growth/accretion to gas giants



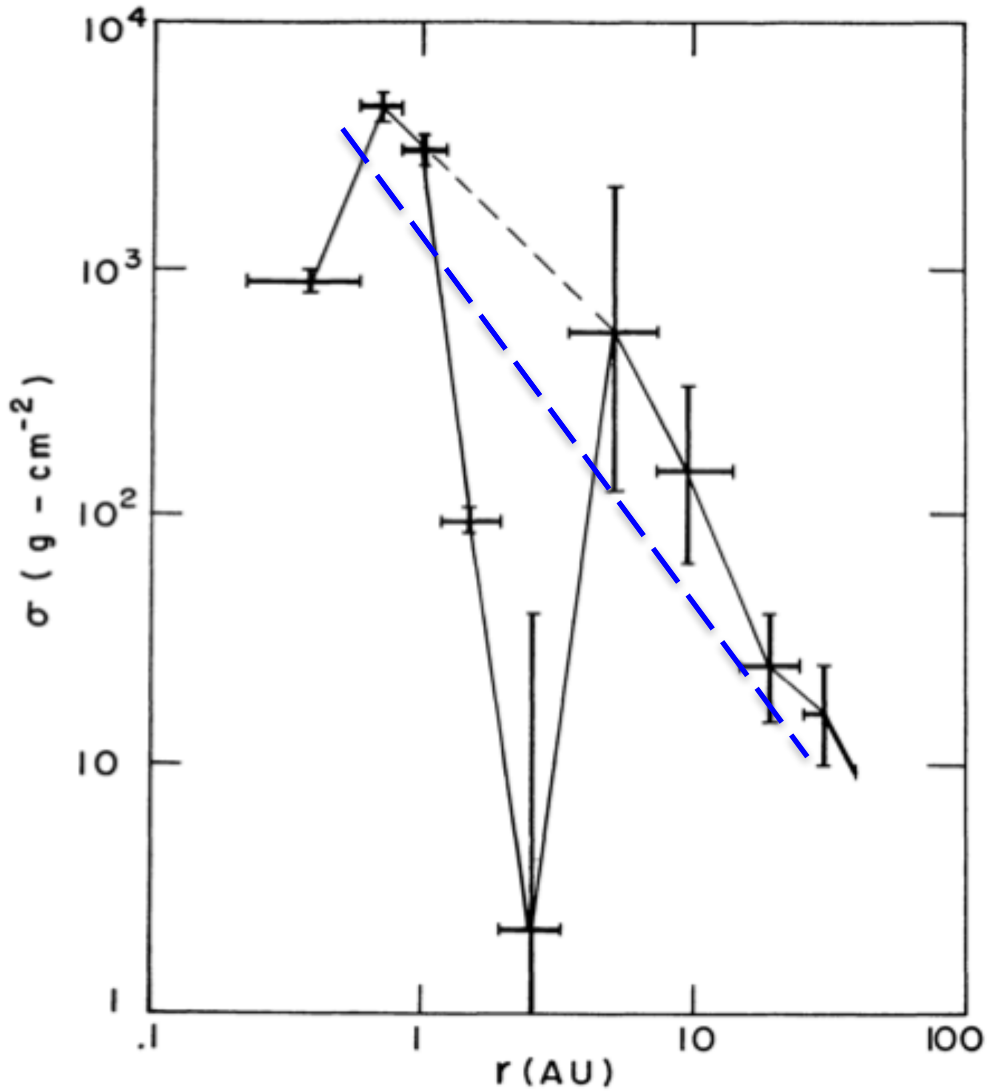
Planet migration

Essentially all processes depend on the gas dynamics of protoplanetary disks.

Most importantly:

- Global structure and disk evolution
- Level of turbulence

Minimum mass solar nebular (MMSN)



(Weidenschilling 1977; Hayashi 1981)

Assume solar-system planets are formed *in-situ*, reconstruct surface density profile by smoothly spreading required gas mass across the disk:

$$\Sigma = 1700 \text{ g cm}^{-2} \cdot r_{\mathrm{AU}}^{-3/2},$$

Total mass ~0.01 $M_{\odot}$

Lower mass limit for the pre-solar nebular.

Not too bad a guess based on PPD observations.

Minimum mass solar nebular (MMSN)

$$\Sigma = 1700 \text{ g cm}^{-2} \cdot r_{\rm AU}^{-3/2}, \quad T = 280 \text{ K} \cdot r_{\rm AU}^{-1/2}$$

$$v_K = 30 \text{ km s}^{-1} \cdot r_{\rm AU}^{-1/2}, \quad c_s = 1.0 \text{ km s}^{-1} \cdot r_{\rm AU}^{-1/4}$$

Vertically isothermal:

Disk is flared

$$H = c_s/\Omega, \qquad H/r = 0.03 r_{\rm AU}^{1/4}$$

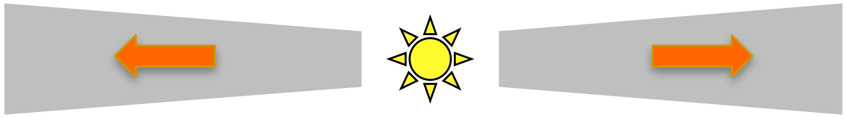
$$\rho(r,z) = \rho_0(r) \exp(-z^2/2H^2)$$

$$\rho_0(r) = 1.4 \times 10^{-9} \text{ g cm}^{-3} \cdot r_{\rm AU}^{-11/4}$$

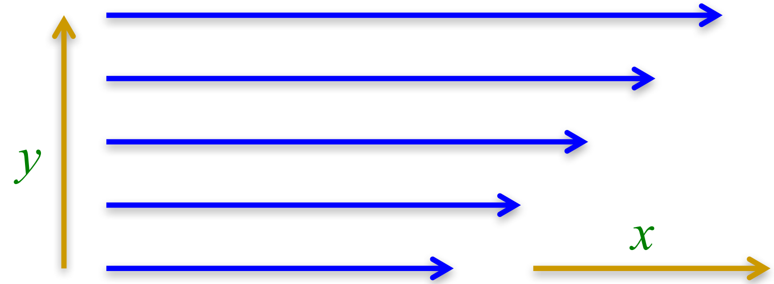
What drives accretion and disk evolution?

Angular momentum transport: the most fundamental question in accretion disk theory.

Radial transport of angular momentum



Viscosity tends to reduce shear:



What happens if the disk has viscosity?

Inner disk falls in,
outer disk expands

In this case, viscosity produces a momentum flux:

$$\sigma_{xy} = -\rho\nu \frac{\partial v_x}{\partial y}$$

viscosity

Microphysics of viscosity

At microscopic level, viscosity is due to momentum exchange in the fluid from thermal motion and molecular collisions.

$$\nu \approx \frac{1}{3} v_{\rm th} \lambda_{\rm mfp}$$

For the disk problem, it is more convenient to parameterize

$$\nu \equiv \alpha c_s H$$ (Shakura & Sunyaev, 1973)

Required value of α to explain PPD accretion rate: ~10^{-3}-10^{-2}

However, one can easily find that $\lambda_{\rm mfp} \ll 10^{-3} H$

(Exercise: show this result based on the MMSN disk model)

Need anomalous viscosity (e.g., turbulence) to boost α.

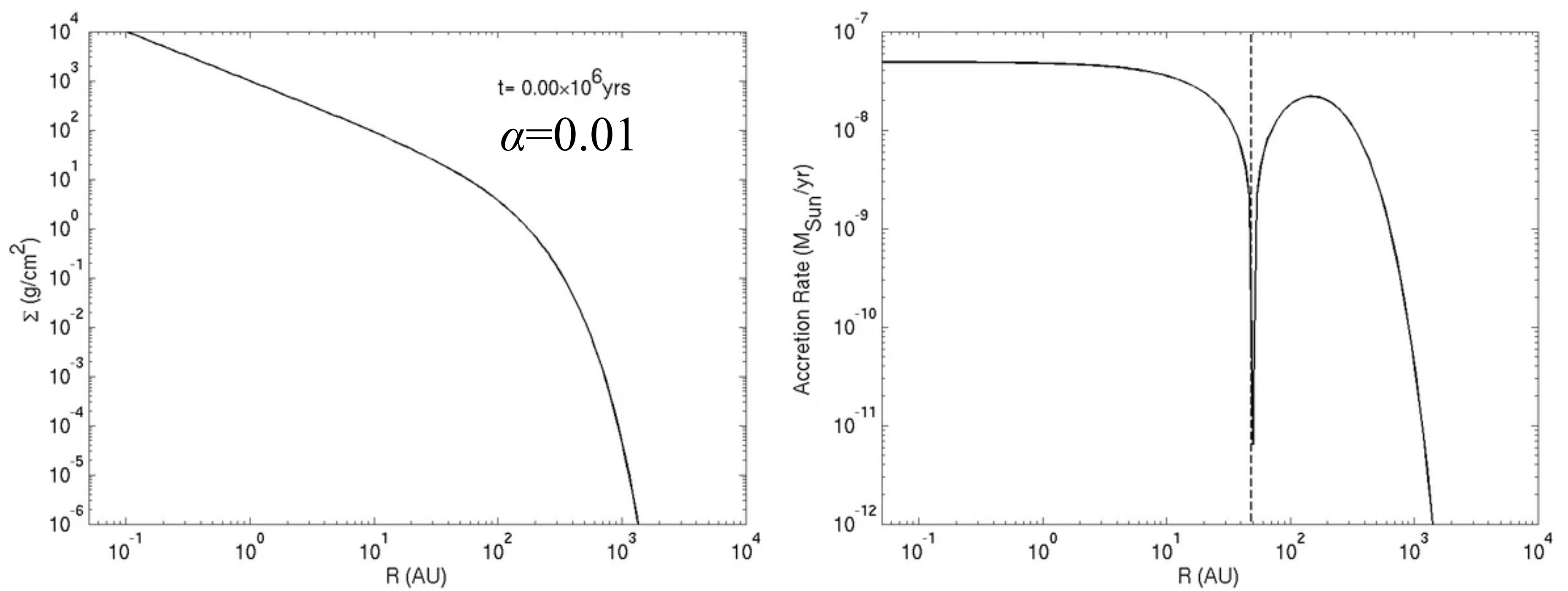
Viscous evolution based on the α-disk model

Viscous evolution equation:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[\frac{dR}{dj_z}\frac{\partial}{\partial R}\left(\alpha c_s^2 \Sigma R^2\right)\right]$$

Note: $j_z(R) = \Omega R^2$

Gas is assumed to be locally isothermal.



Where is "viscosity" coming from?

- Turbulence: large-scale mixing of the gas leads to momentum exchange over scales ~H.

  Turbulence is typically excited via a number of hydrodynamic and/or magneto-hydrodynamic instabilities.

- Large-scale magnetic stress: leading to magnetic braking.

  Requires the presence of large-scale B field.

Hydrodynamic instabilities

- Gravitational instability

Toomre's Q parameter:

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma}$$

Disk is gravitationally unstable if $Q<1$.

It may be achieved in very early stages of PPDs when it is massive, but not over most of their lifetime.

- Rossby-wave instability
- Baroclinic instability
- Vertical shear instability
- Zombie vortex instability
-

Many of these instabilities require special thermodynamic conditions to operate, and are not sufficiently powerful to drive rapid accretion.

Magnetorotational Instability (MRI)

- Rayleigh criterion for unmagnetized rotating disks:

Unstable if: $$\frac{d(\Omega R^2)}{dR} < 0$$ (Rayleigh, 1916)

Confirmed experimentally (Ji et al. 2006).

All astrophysical disks should be stable against this criterion.

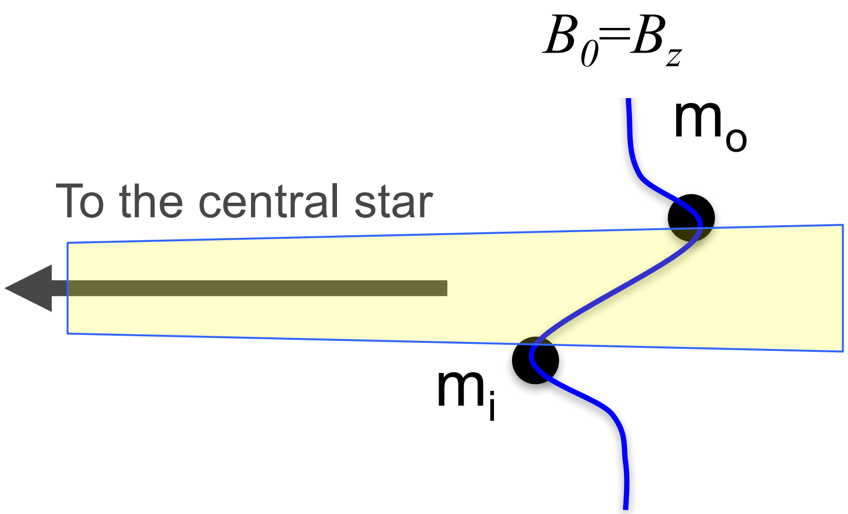
- Including (a vertical, well-coupled) magnetic field qualitatively changes the criterion (even as B->0):

Unstable if: $$\frac{d\Omega}{dR} < 0$$ Velikhov (1959), Chandrasekhar (1960), Balbus & Hawley (1991)

All astrophysical disks should be unstable!

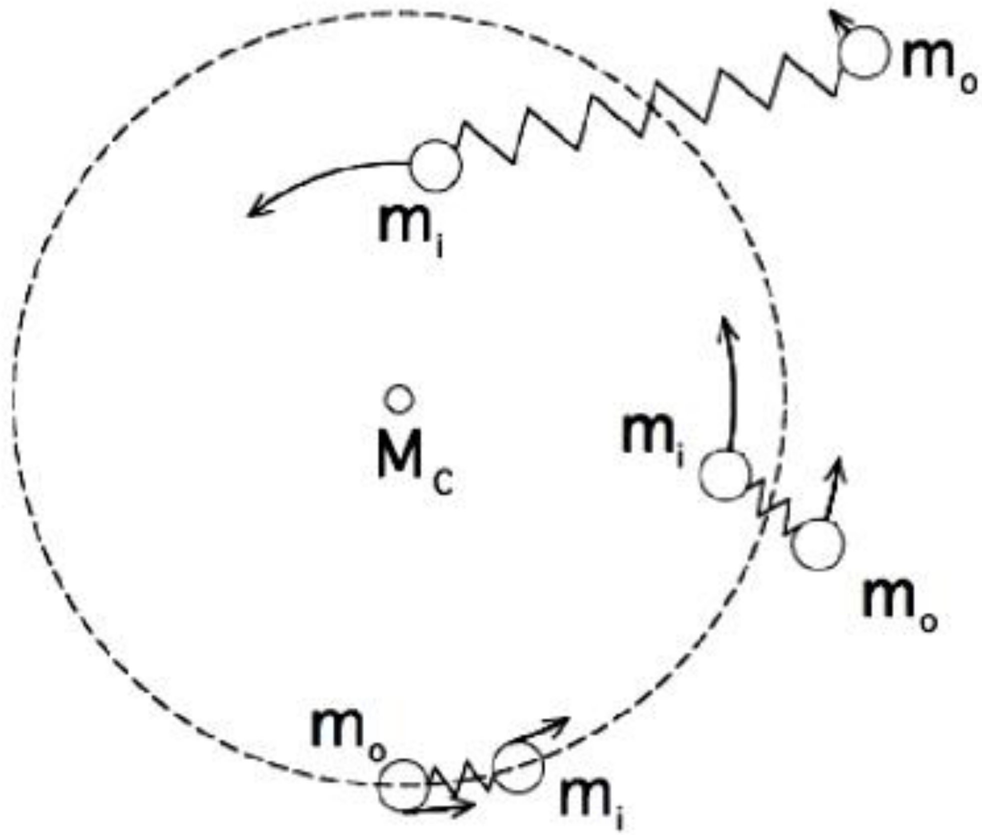
Magnetorotational Instability (MRI)

Edge on view:

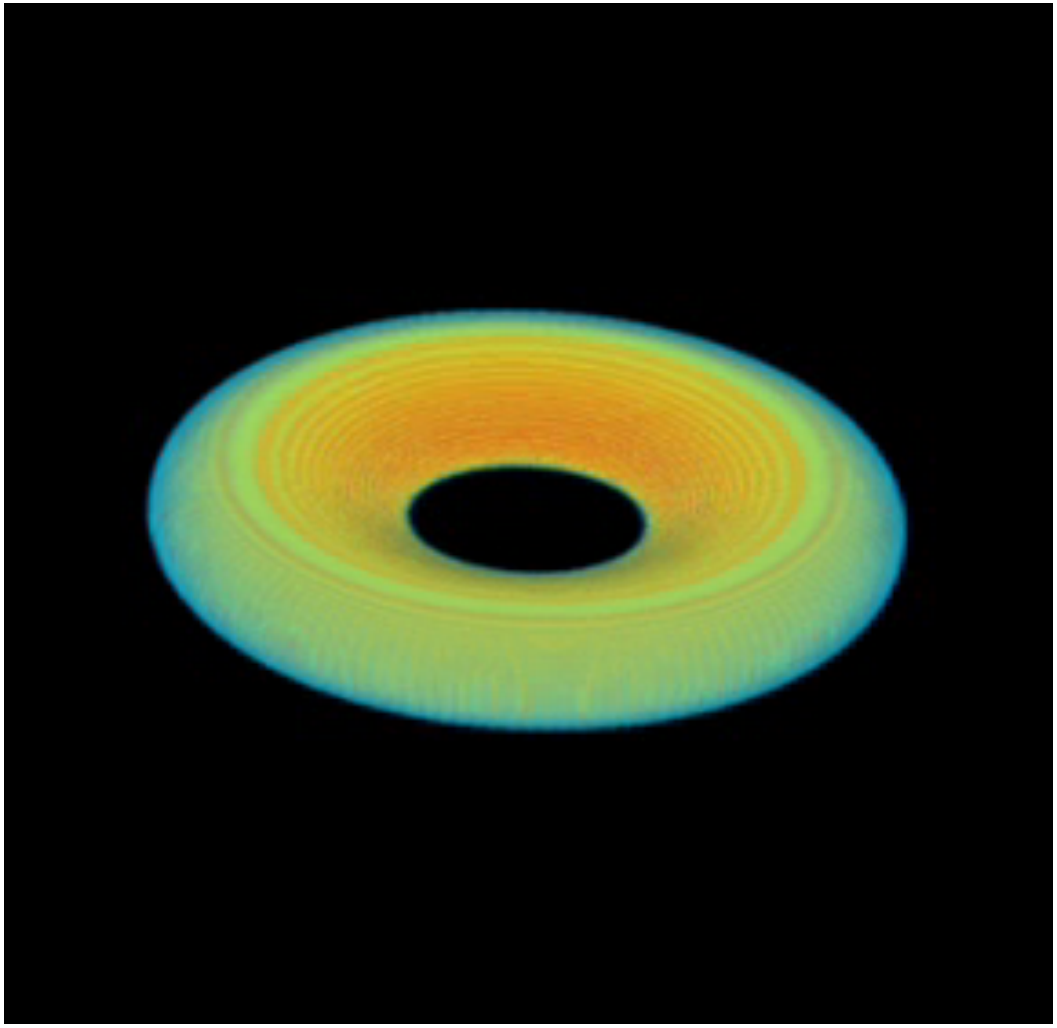


Magnetic tension force behaves like a spring.

Face on view:



Magnetorotational Instability (MRI)



(J. Hawley)

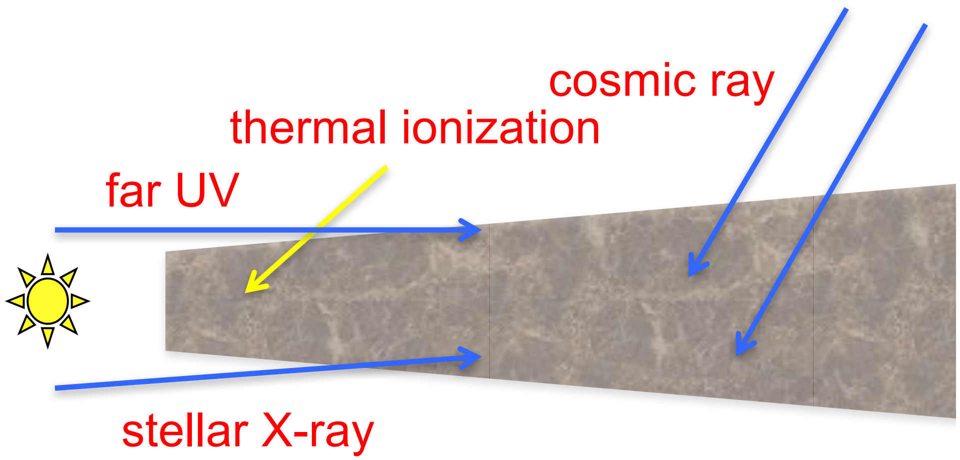
This is for a black hole accretion disk, but the physics is similar.

$$\alpha \sim (\delta v / c_s)^2$$

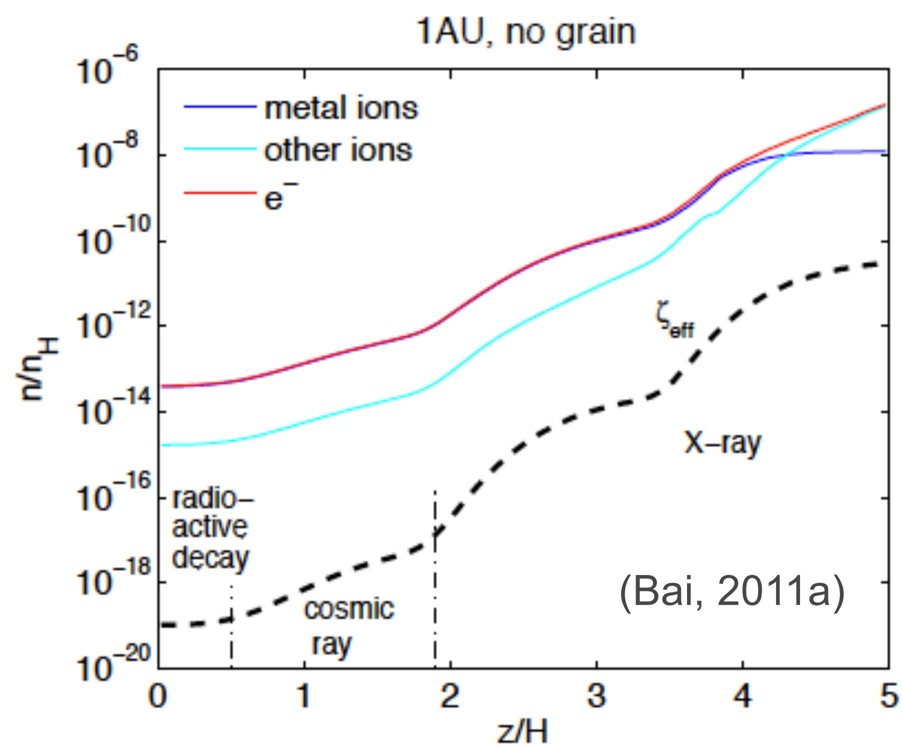
Typically, α ~0.01-0.1

This is sufficient to explain the accretion rate, **BUT**…

PPDs are extremely weakly ionized



Umibayashi & Nakano (1981)
Igea & Glassgold (1999)
Perez-Becker & Chiang (2011b)

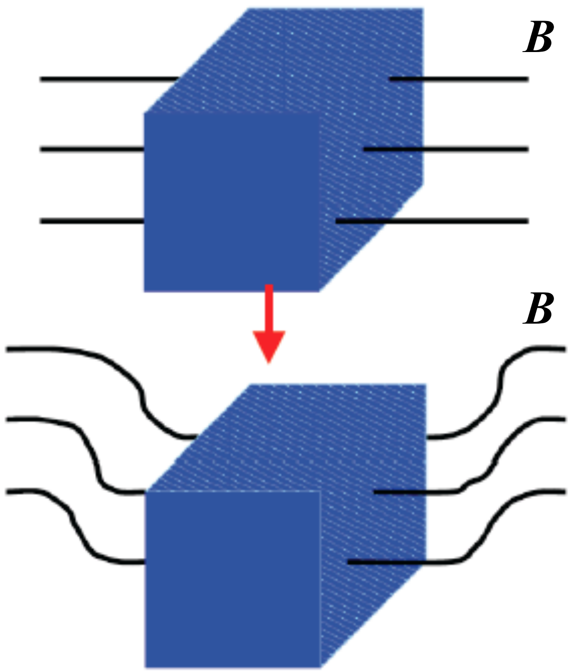


Very weak ionization in the gas substantially reduces the coupling between gas and magnetic fields, which can suppress the MRI!

Gas-B field coupling in weakly ionized gas

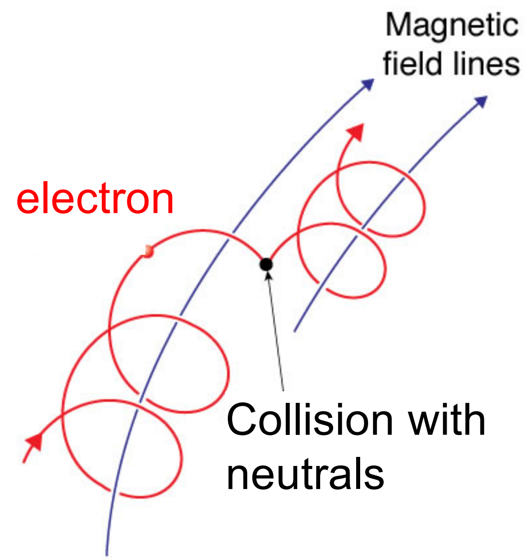
Ionized gas: well coupled

B field is frozen in the gas
(flux freezing)



Ideal Magnetohydrodynamics (MHD)

Weakly ionized gas: poorly coupled



B field is frozen in the electrons
(most mobile charged species)

+

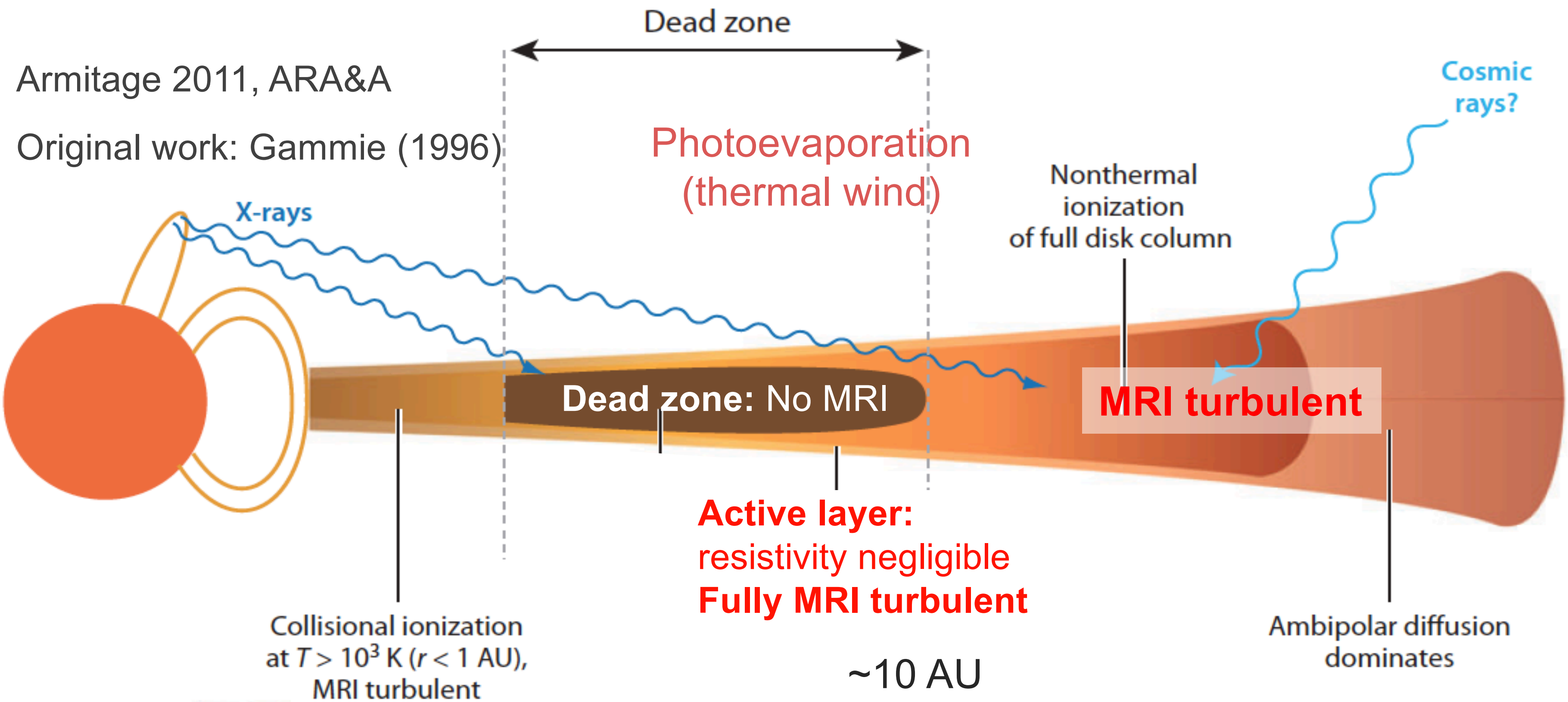
Resistivity: B field slips through the electrons due to collisions.

Non-ideal MHD

Conventional understanding: resistivity only

Armitage 2011, ARA&A

Original work: Gammie (1996)

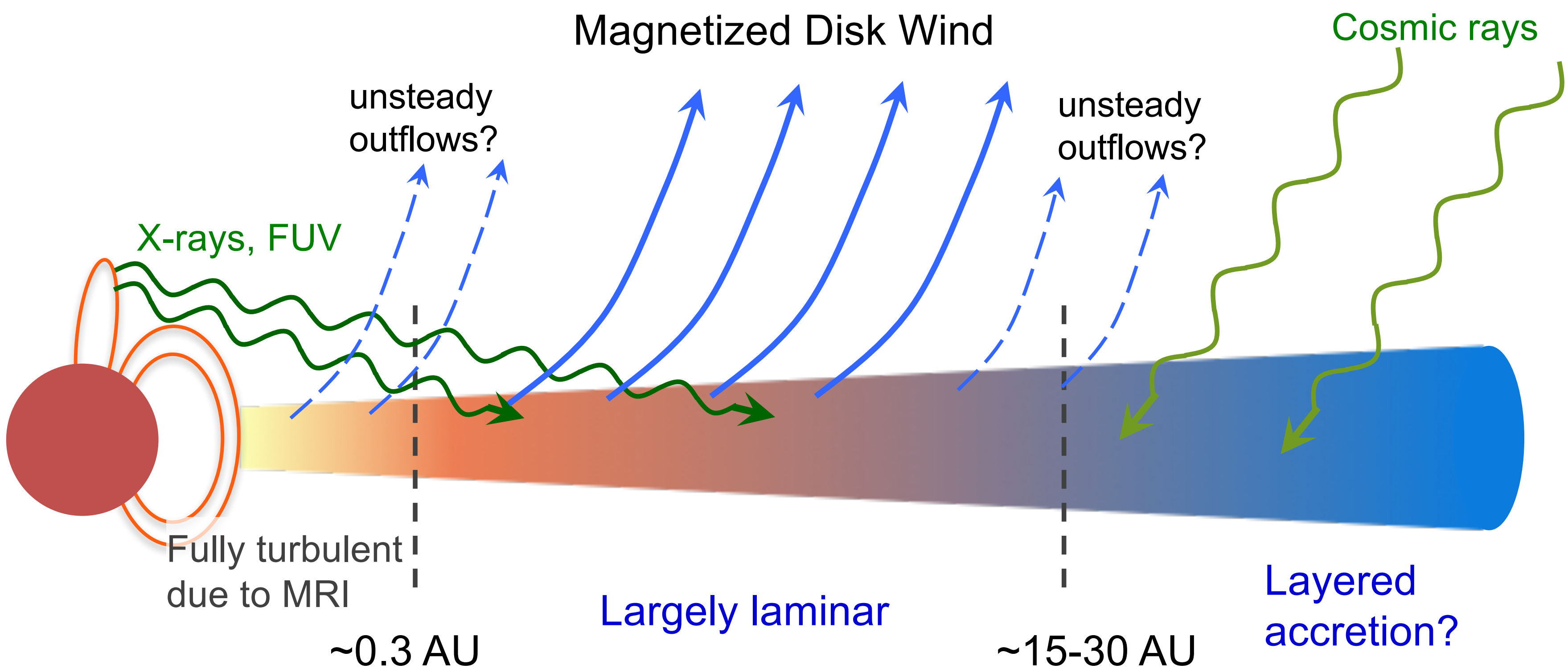


But…There are additional effects from weak ionization, called the Hall effect and ambipolar diffusion.

$$\boldsymbol{v}_e = \boldsymbol{v} + (\boldsymbol{v}_e - \boldsymbol{v}_i) + (\boldsymbol{v}_i - \boldsymbol{v})$$

More recent understanding

(Bai, 2013)

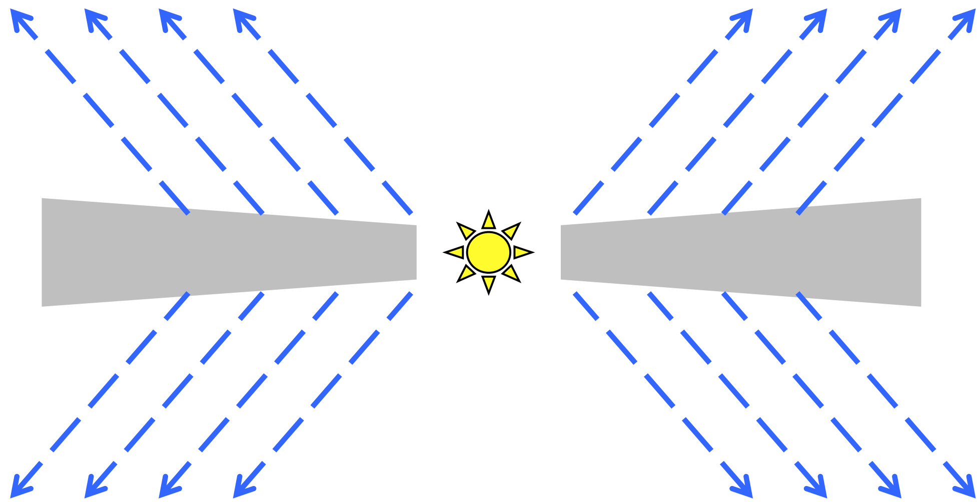


External magnetic flux is essential to drive disk evolution.

The inner disk is largely laminar. Accretion is driven by disk wind.

Layered accretion picture applies at the outer disk (>~30 AU).

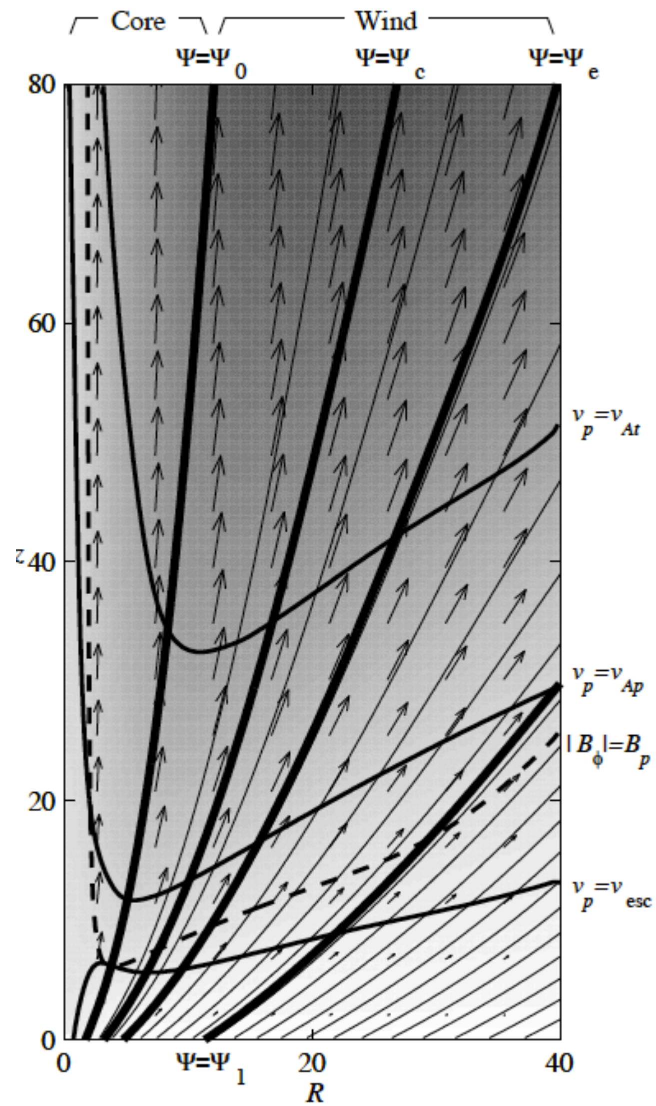
Angular momentum transport



Vertical extraction by:

Magnetized disk wind

The entire disk falls in.

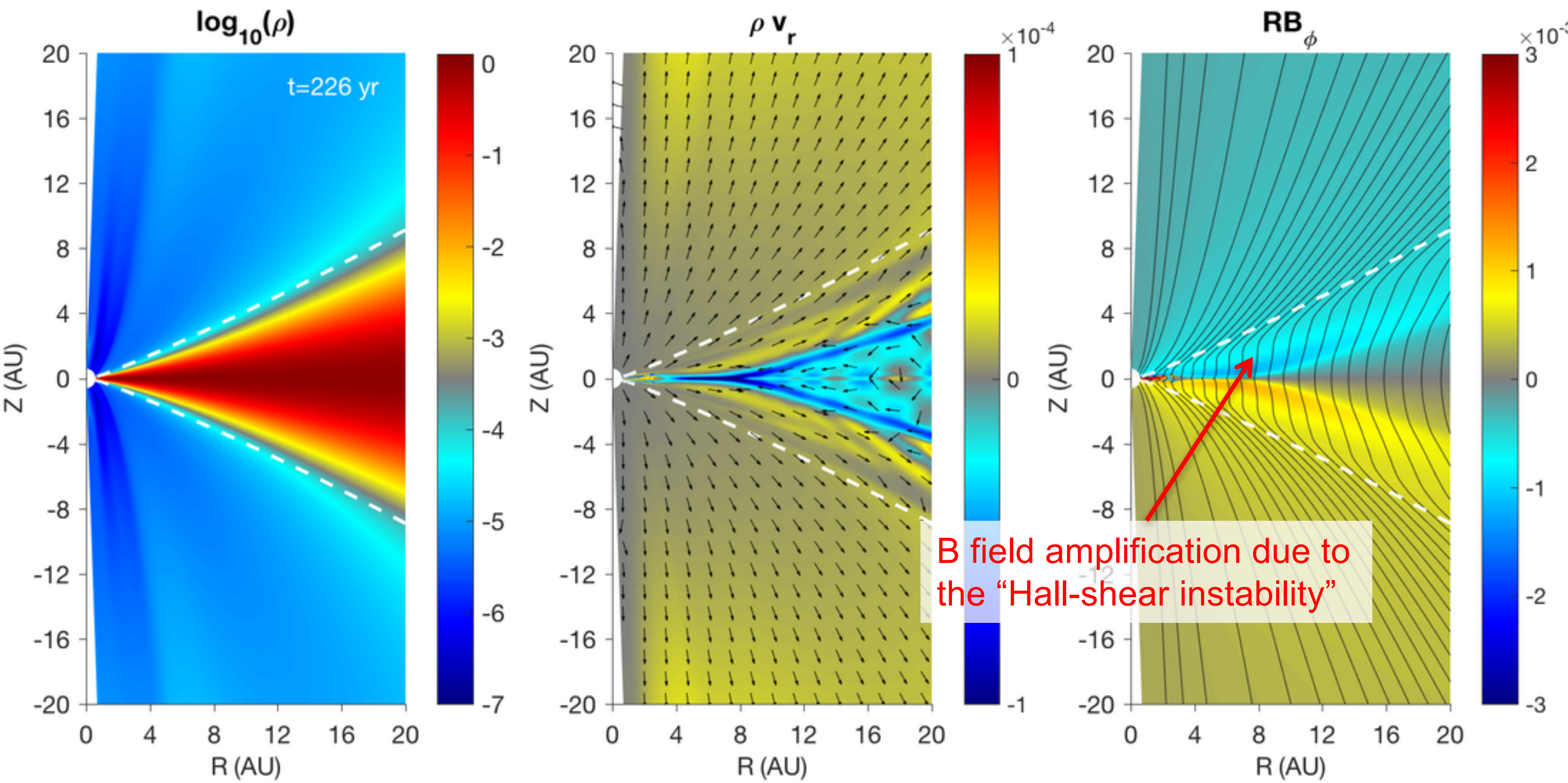


(Krasnopolsky+99)

Towards realistic simulation of PPDs

2D axisymmetric, all 3 non-ideal MHD effects included.

Bai, 2017



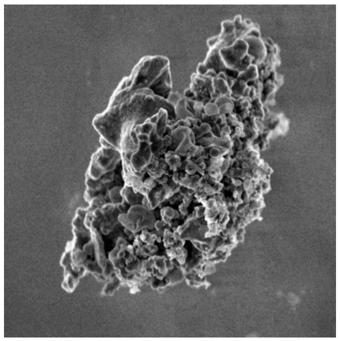
Complex flow structures with major implications for planet formation.

Summary

- To zeroth order, the disk can be understood as being in Keplerian rotation with vertical hydrostatic equilibrium.
- Angular momentum can be transported either radially (by "viscosity") or vertically by a magnetized disk wind. It is the main driver of disk evolution.
- A number of hydrodynamic / magneto-hydrodynamic instabilities can potentially drive turbulence, but are unlikely to be universal.
- The bulk part of the disk is unlikely to possess strong turbulence, with accretion driven by a disk wind.

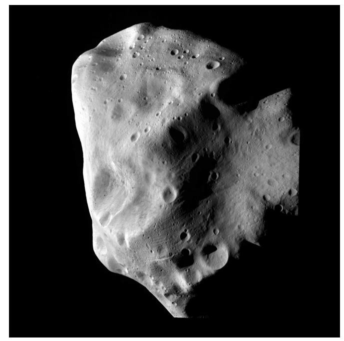
Early Stages of Planet Formation

Stages of planet formation

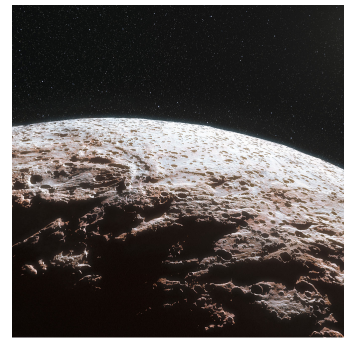


μm

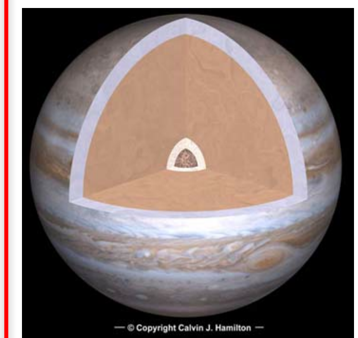
cm



km



10^3km



10^5km

Grain growth

Planetesimal formation

Planetesimal growth to cores

growth/accretion to gas giants

Sticking by surface forces

Miracle?

Gravitational accretion

Dust particles: aerodynamics

Aerodynamic drag: $$\boldsymbol{F}_{\rm drag} = -M\frac{\Delta\boldsymbol{v}}{t_{\rm stop}}$$ (Weidenschilling 1977)

For subsonic motion, $t_{\rm stop}$ only depends on particle size (a):

Epstein regime: $$t_{\rm stop} = \frac{\rho_s a}{\rho_g c_s} \qquad a < 9\lambda_{\rm mfp}/4$$

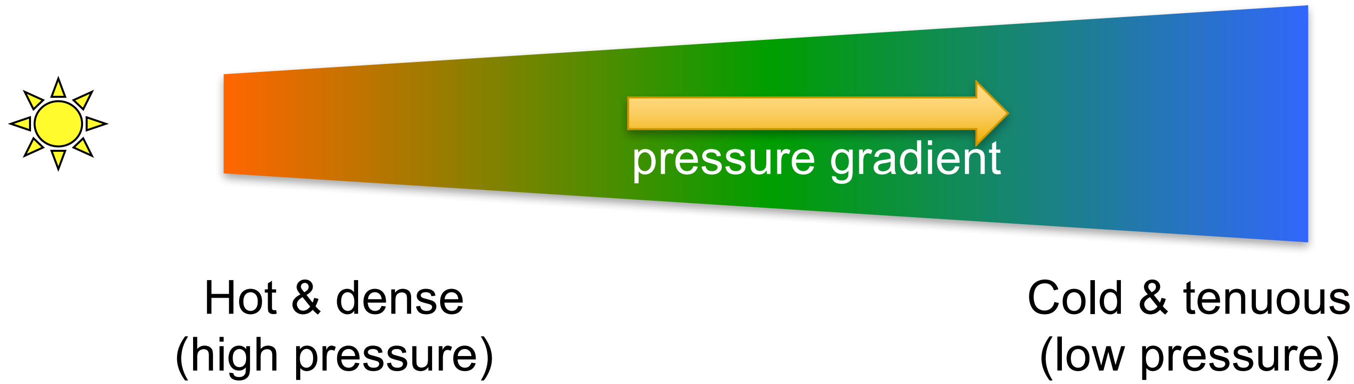
Stokes regime: $$t_{\rm stop} = \frac{4a}{9\lambda_{\rm mfp}}\frac{\rho_s a}{\rho_g c_s} \qquad a \geq 9\lambda_{\rm mfp}/4$$

Define dimensionless stopping time, and for MMSN disk at midplane:

$$\boxed{\tau_s \equiv \Omega t_{\rm stop}} = \max\Big[\underbrace{4.4\times10^{-3}a_{\rm cm}r_{\rm AU}^{3/2}}_{\text{Epstein}},\ \underbrace{1.4\times10^{-3}a_{\rm cm}^2 r_{\rm AU}^{-5/4}}_{\text{Stokes}}\Big]$$

For mm size particles: $\tau_s \sim 10^{-3}$ at 1AU; $\tau_s \sim 0.1$ at 30 AU.

Radial drift of particles



Gas rotates at sub-Keplerian velocity due to pressure support.

$\Delta u_\phi \equiv -\eta v_K$ where for MMSN: $\dfrac{\eta v_K}{c_s} = -\dfrac{1}{2}\dfrac{d\ln P}{d\ln R}\dfrac{c_s}{v_K} \approx 0.05\ r_{\rm AU}^{1/4}$

Solid particles feel headwind => lose angular momentum and drift inward.

In general, particles drift toward regions with higher pressure.

Radial drift velocity

Consequence: particles spiral inwards, gas slowly drifts outward.

$$u_r = -\eta v_K \frac{2\tau_s}{(1+\epsilon)^2 + \tau_s^2}$$

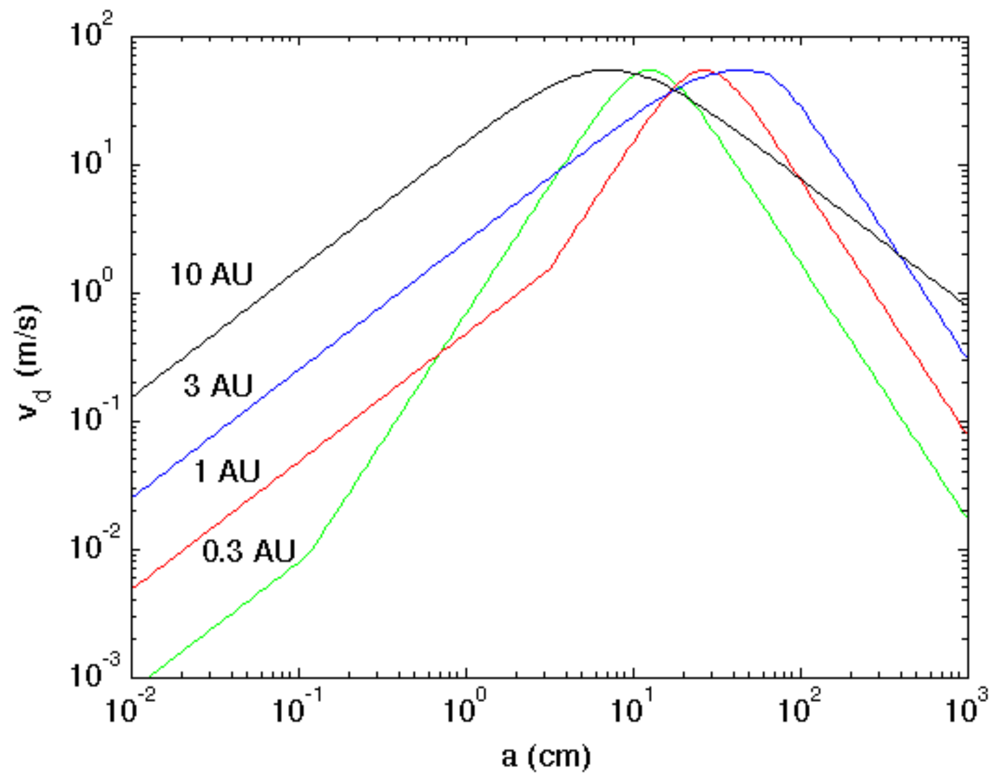
(Nakagawa, Sekiya & Hayashi 1986)

ε: solid to gas density ratio

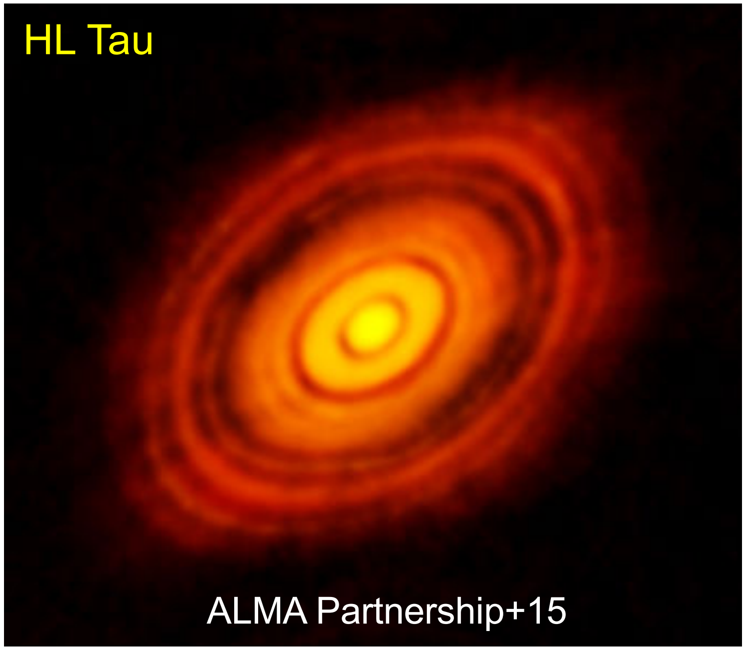
=> dust feedback to gas

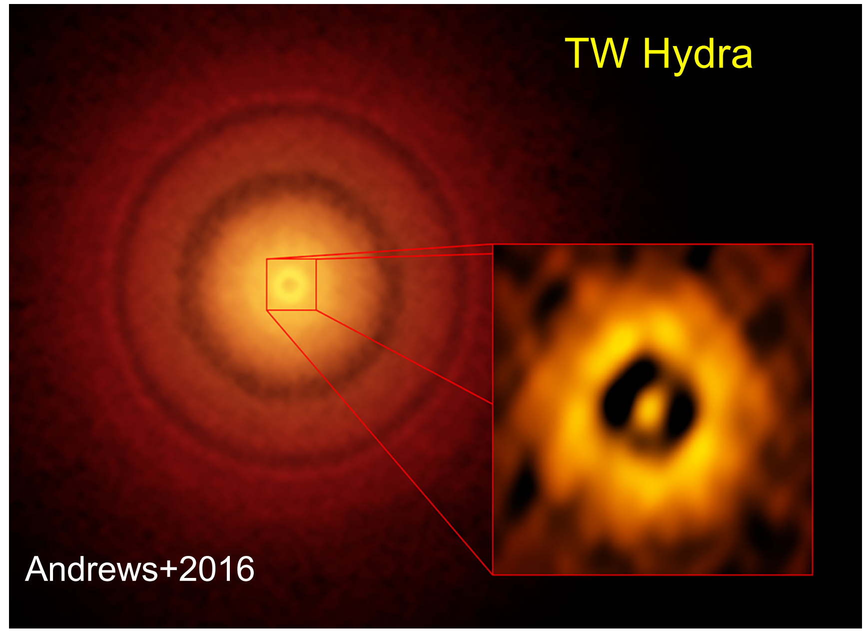
Characteristic lifetime for meter sized bodies is only ~100 years at 1 AU!

Severe constraint on the timescale for planetesimal formation!



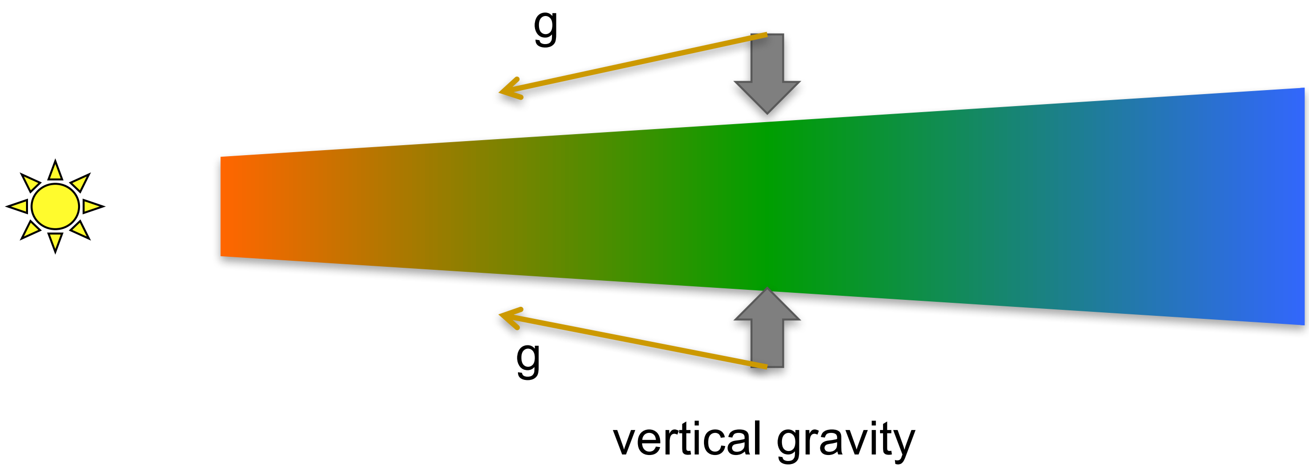
Are we seeing dust traps?





Very likely, though it is less clear whether the disk forms the traps on its own, or whether they are the outcome of planet formation.

Vertical settling



Dust feels vertical stellar gravity + gas drag (but no gas pressure).

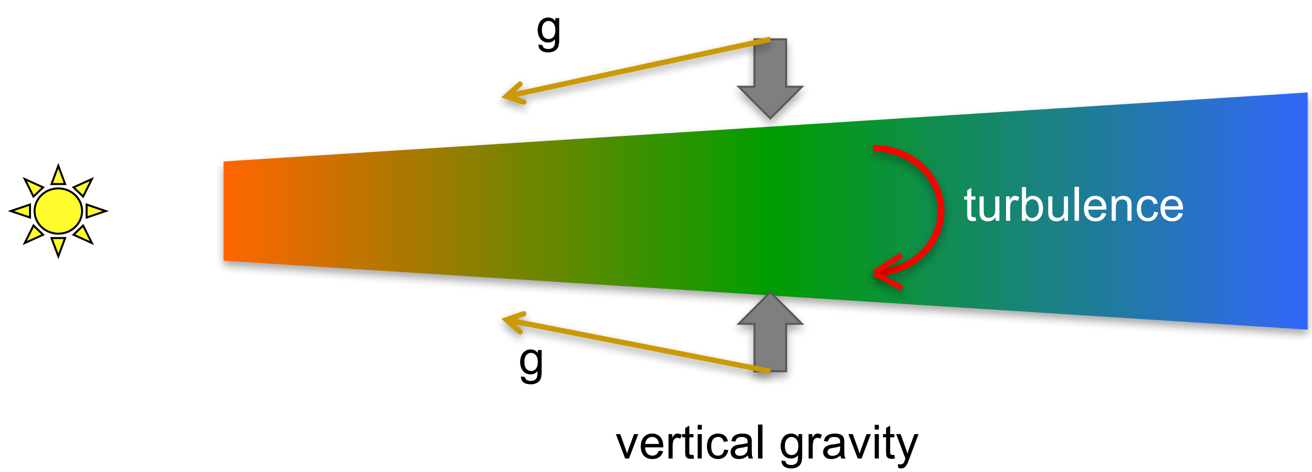
$$\frac{dv_{d,z}}{dt} = -\Omega^2 z - \frac{v_{d,z} - v_{g,z}}{t_{\rm stop}}$$

Large grain => vertical damped oscillation — settling timescale ~τ_s

Small grain => terminal velocity: $v_{d,z} = -(\Omega z)\tau_s$ — settling timescale ~$1/\tau_s$

Overall settling timescale: $\Omega t_{\rm sett} \sim \tau_s + 1/\tau_s$

Turbulent diffusion



Turbulent diffusion prevents particles from settling indefinitely, which occurs on timescale of

$$t_{\rm diff} \sim H_p^2/D_p$$ where D_p is the particle diffusion coefficient.

$D_p \sim D_g$ for small particles (τ_s<<1), but $D_p < D_g$ for big particles.

Particle vertical scale height: $$H_p \sim \sqrt{\frac{D_g}{\Omega \tau_s}}$$

Note:

$$D_g \sim \nu \sim \alpha c_s H$$

Grain growth

Coagulation equation:

(Smoluchowski 1916)

$$\frac{\partial n(m)}{\partial t} = \frac{1}{2}\int_0^m A(m', m-m')n(m')n(m-m')dm' - n(m)\int_0^\infty A(m', m)n(m')dm'$$

$n(m)dm$: number density of solids in mass range between m and $m+dm$.

$A(m_1, m_2)$: coagulation kernel

$$A(m_1, m_2) = P(m_1, m_2, \Delta v)\Delta v(m_1, m_2)\sigma(m_1, m_2)$$

Further generalizations/complications to account for:

Fragmentation/bouncing

Different kinds of grains (composition/porosity, etc.)

Distribution of collision velocity

Radial and vertical transport/diffusion of grains

Coupled with global disk evolution.

Relative velocities

(e.g., Birnstiel+ 2010)

Differential radial drift: $$\Delta u_{RD} = \left| \eta v_K \left(\frac{2\tau_{s,1}}{1+\tau_{s,1}^2} - \frac{2\tau_{s,2}}{1+\tau_{s,2}^2} \right) \right|$$

Differential azimuthal motion: $$\Delta u_{\phi} = \left| \eta v_K \left(\frac{1}{1+\tau_{s,1}^2} - \frac{1}{1+\tau_{s,2}^2} \right) \right|$$

Effective for collisions between big and small grains.

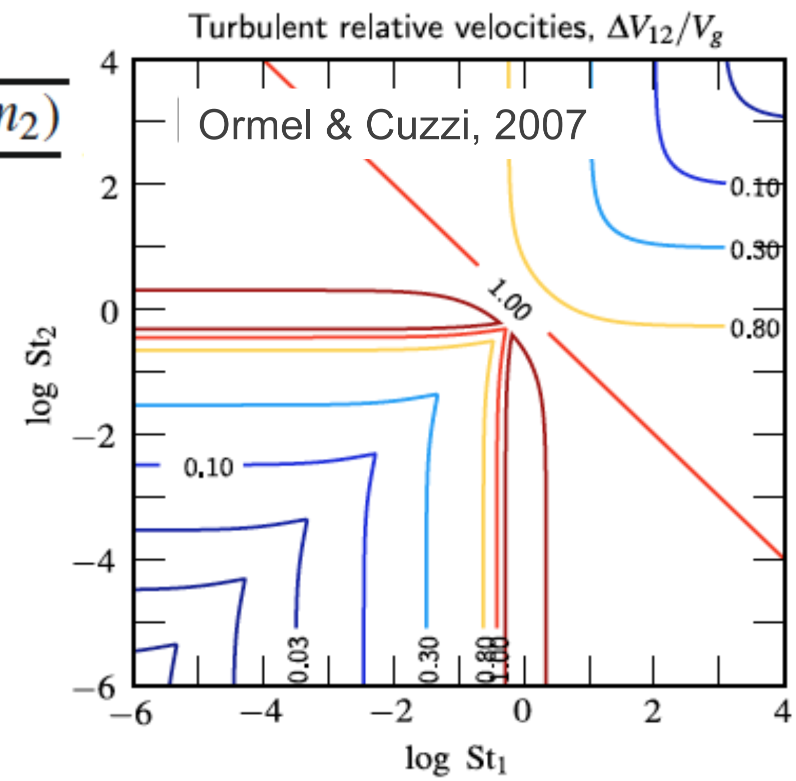
Brownian motion: $$\Delta u_{\mathrm{BM}} = \sqrt{\frac{8 k_{\mathrm{B}} T (m_1 + m_2)}{\pi m_1 m_2}}$$

Effective for small grains.

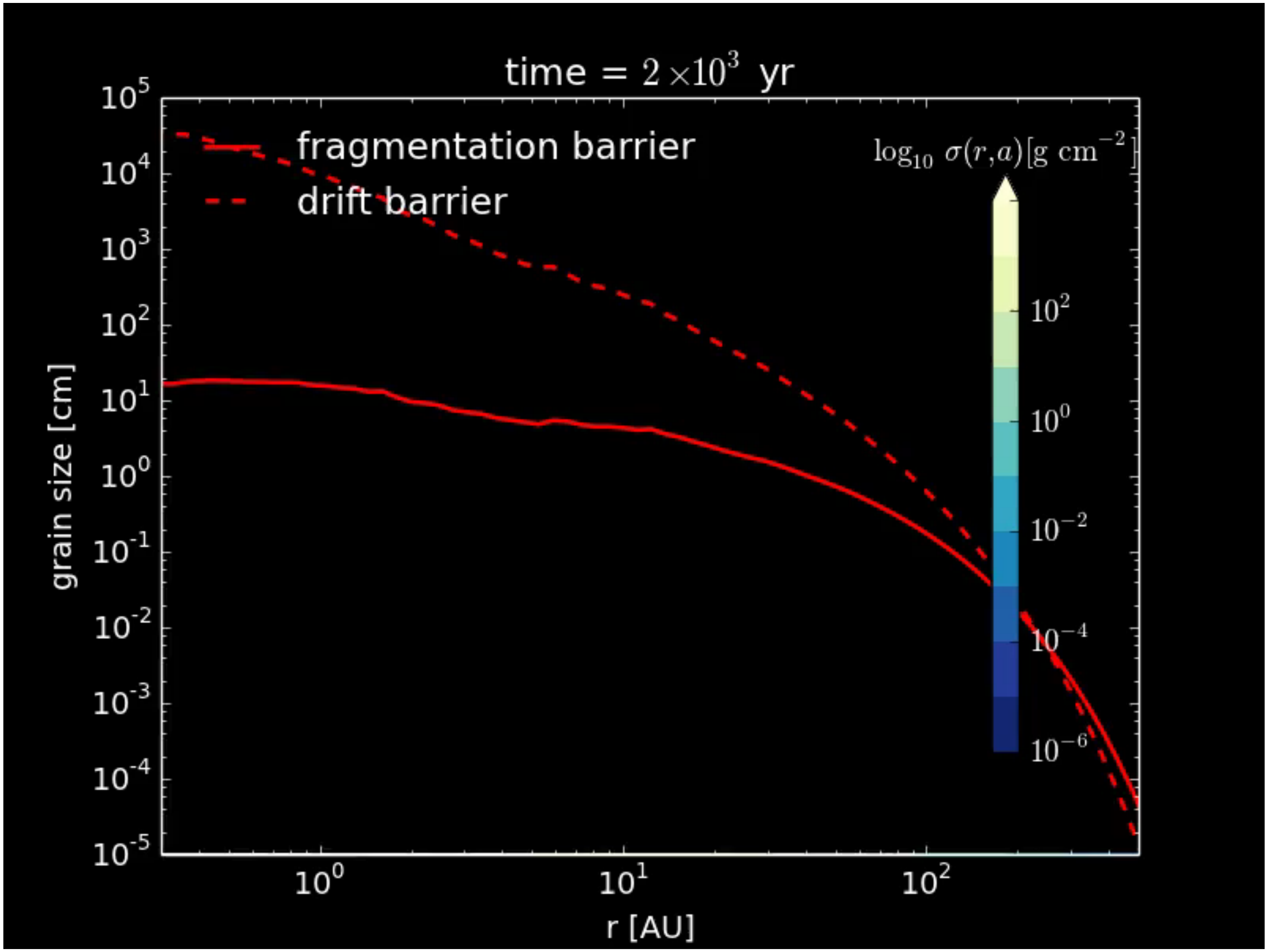
Turbulent motion

More complex physics, and depends on the nature of turbulence.

Effective when at least one body is big.



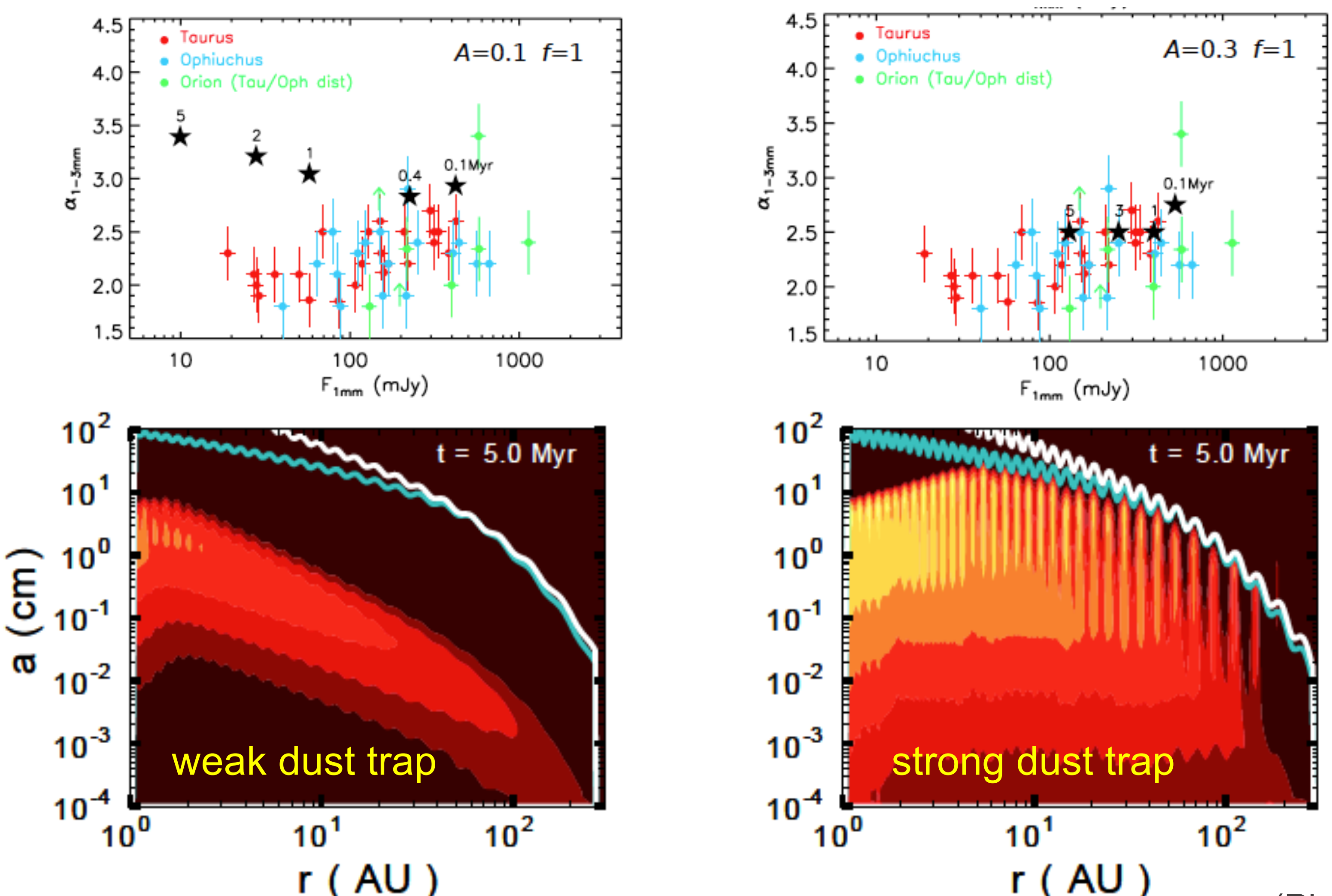
Coagulation simulations



Credit: Til Birnstiel

Radial drift revisited and need for dust traps

Disk observations indicate that disks remain dust-rich for several Myrs.



Dust trap: artificially add radial pressure variations.

(Pinila+ 2012)

Planetesimal formation

- Gravitational instability

Goldreich & Ward (1973)

Without external turbulence, particles settle to an infinitely thin layer, which is unstable to GI, and collapse to planetesimals.

Instability requires: $$Q \equiv \frac{c\kappa}{\pi G \Sigma} < 1$$

c: velocity dispersion of dust.

κ: epicyclic frequency = Ω

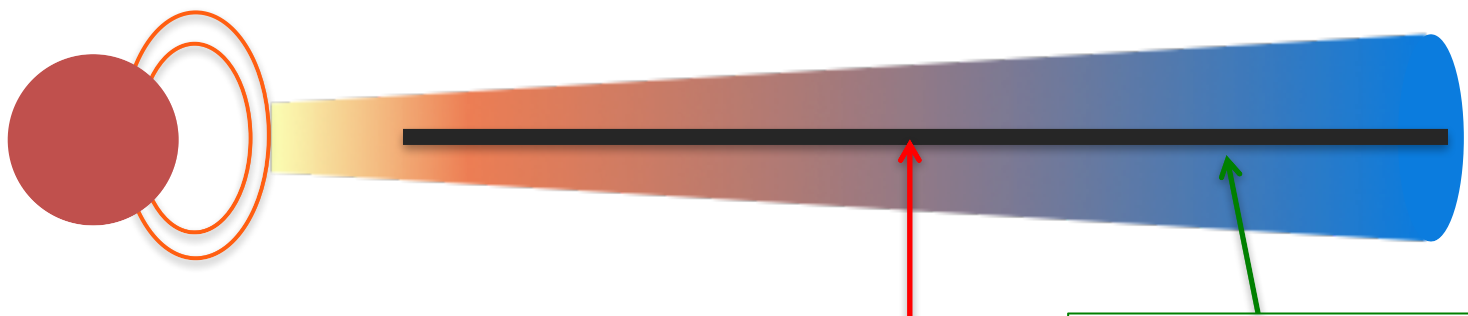
Σ: surface den of the dust layer

Requires excessively low level of turbulence to meet Q<1.

Issue with GI: Kelvin-Helmholtz instability

Before the onset of GI, the dust layer is unstable to the KHI:

(Weidenschilling 1980)



Gas beyond dust layer: sub-Keplerian rotation

Gas in the dust layer: close to Keplerian rotation due to dust feedback

Vertical shear in azimuthal velocity

GI requires enrichment of solids by a factor of at least 4 ($Z>0.06$) (Lee+ 2010a,b)

Streaming instability (SI)

Momentum feedback from particles to the gas leads to a linear instability.

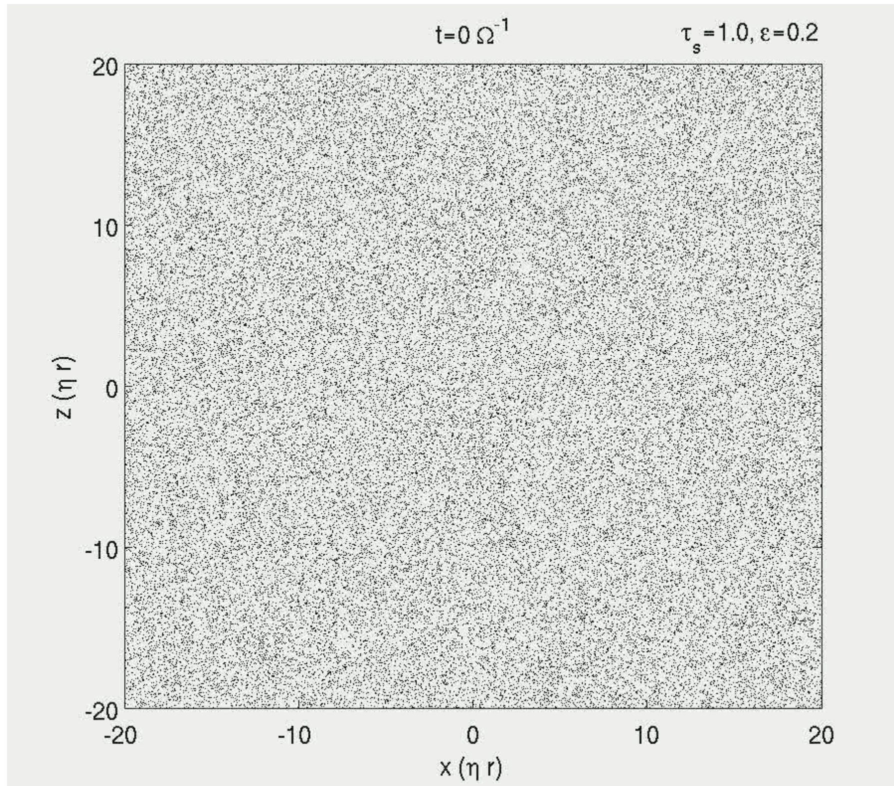
(Goodman & Pindor 2001, Youdin & Goodman 2005, Jacquet et al. 2011)

Most efficient for marginally coupled particles

$$\tau_s \sim 0.1 - 1$$

Energy budget:
radial pressure gradient

Non-linear evolution can efficiently concentrate particles!



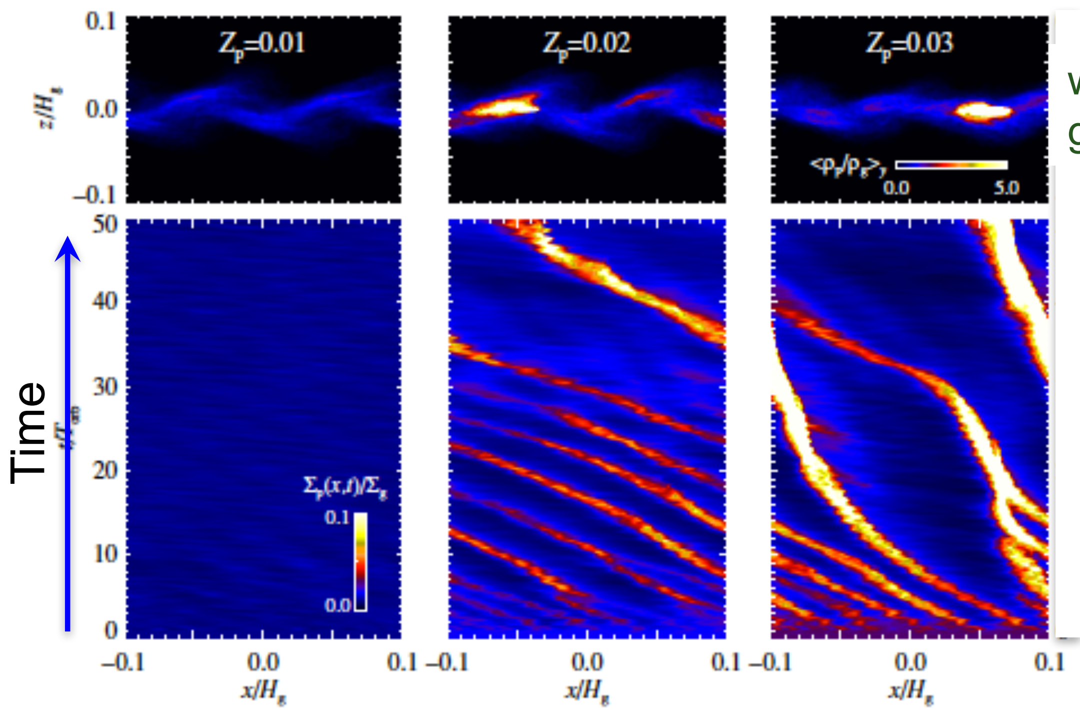
(Johansen &Youdin 2007, Bai & Stone 2010a)

Planetesimal formation from the SI

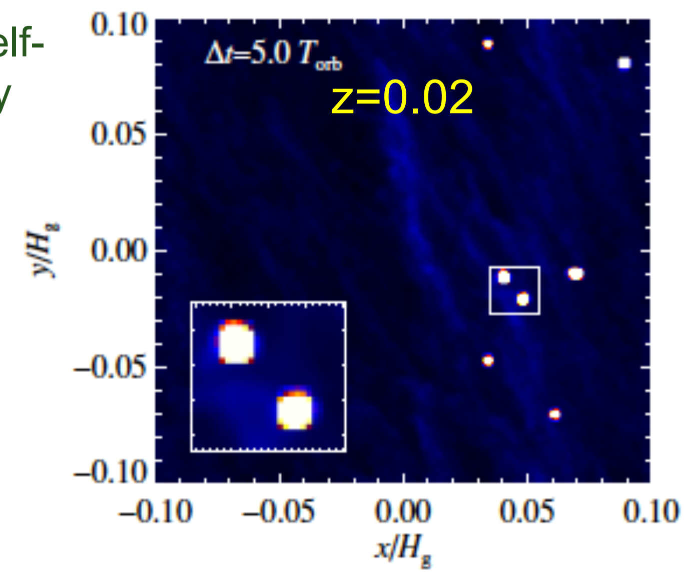
Local shearing box simulations with vertical stratification

Particle stopping time $\tau_s = 0.1 - 0.4$ (Johansen+ 2009,2011,2012)

Height-integrated solid to gas mass ratio (dust abundance): Z=0.01-0.03



w/o self-gravity



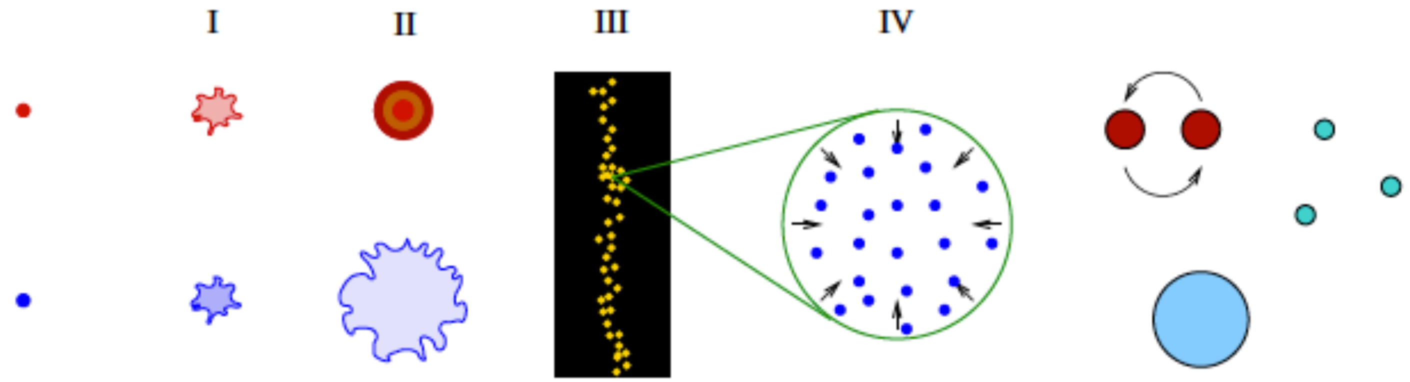
self-gravity turned on after saturation

Strong metallicity dependence!

Planetesimal formed in the computer!

Current understanding

Planetesimal formation by coagulation and self-gravity!



I initial growth of fluffy aggregates

II compactification and mass transfer or continued fluffy growth

III concentration in pressure bumps, in vortices, or by streaming instabilities

IV gravitational collapse with collisions to yield planetesimal Initial Mass Function

from Johansen+ 2014, PPVI talk

Very active field of research: involves complex physics of gas dynamics, dust dynamics, and their mutual interaction.

Planetesimals to cores: naive timescales

Consider a disk of planetesimals with size R, surface density Σ (MMSN scaling), velocity dispersion u. Assume collisions always lead to growth.

If collision cross section is geometric:

Collision rate: $n\pi R^2 u$

Planetesimal disk scale height: $h \approx u/\Omega \Rightarrow n \approx \Sigma/Mh \approx \Sigma\Omega/Mu$

Growth rate: $$\frac{dR}{dt} = \frac{R}{3M}\frac{dM}{dt} \approx \frac{\Sigma\Omega}{4\rho_p} \approx \frac{30\mathrm{cm}}{\mathrm{yr}}\left(\frac{a}{\mathrm{AU}}\right)^{-3}$$

To build Earth:	~ 10^7 yr
Jupiter's core:	~ 10^9 yr
Neptune:	~ 10^{12} yr

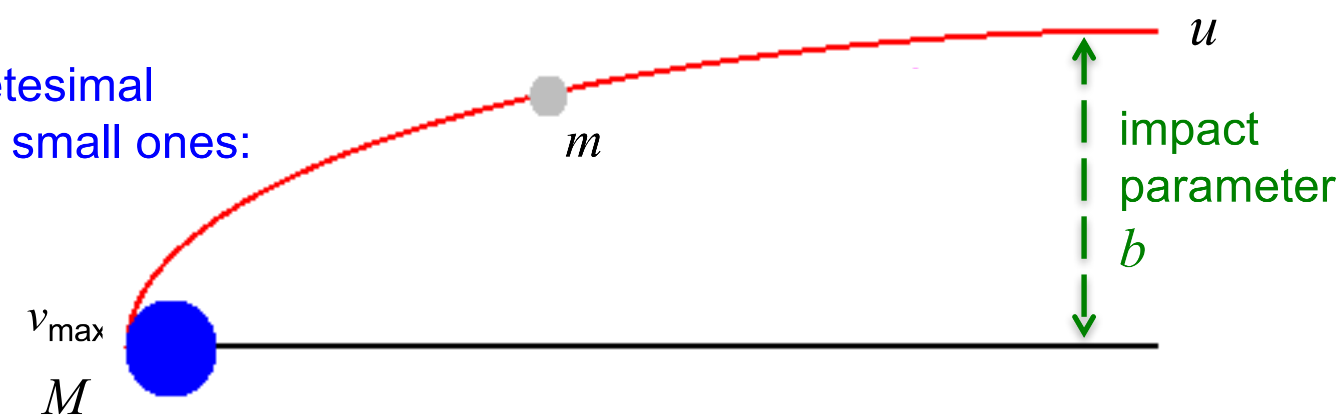
Way too long!

Need acceleration!

Gravitational focusing

Consider two-body encounter, ignoring disk rotation for the moment.

Big planetesimal
accreting small ones:



For closest approach, angular momentum conservation gives:

$$v_{\max} R = ub$$

Energy conservation gives:

$$\frac{u^2}{2} = \frac{v_{\max}^2}{2} - \frac{GM}{R}$$

$$b^2 = R^2 + \frac{2GMR}{u^2} = R^2\left(1 + \frac{v_{\rm esc}^2}{u^2}\right) \quad \text{where} \quad v_{\rm esc}^2 \equiv \frac{2GM}{R}$$

Gravitational focusing

Consider two-body encounter, ignoring disk rotation for the moment.

Enhancement of collisional cross section:

$$b^2 = R^2\left(1+\frac{v_{\rm esc}^2}{u^2}\right) \xrightarrow[\text{section}]{\text{cross}} \sigma = \pi R^2\left(1+\frac{v_{\rm esc}^2}{u^2}\right)$$

where $v_{\rm esc}^2 \equiv \dfrac{2GM}{R}$

Collision rate: $$f_{\rm coll} = n\sigma u \approx \frac{\Sigma}{mh}\sigma u = \frac{\Sigma\Omega}{m}\sigma$$

Growth rate: $$\frac{1}{R}\frac{dR}{dt} \approx \frac{1}{M}\frac{dM}{dt} \approx \frac{m}{M}f_{\rm coll}$$

$$\Rightarrow \frac{1}{R}\frac{dR}{dt} \approx \frac{\Sigma\Omega}{\rho R}\begin{cases} 1 & \propto R^{-1} \quad (u \gtrsim v_{\rm esc}) \\ (v_{\rm esc}/u)^2 & \propto R \quad (u < v_{\rm esc}) \end{cases}$$

Ordered growth

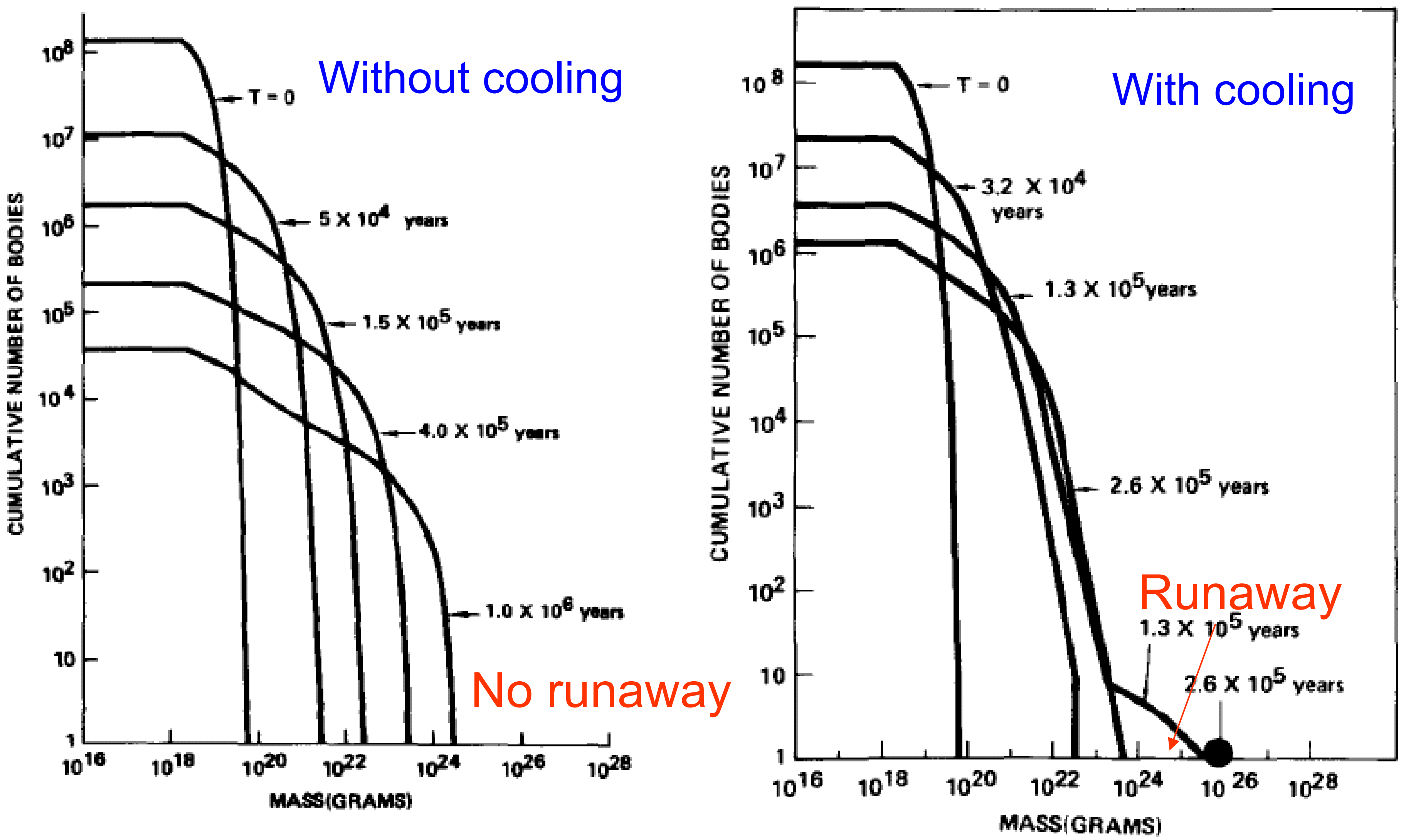
Runaway growth

(for constant u)

Need a "cold" population of planetesimals for efficient growth.

Runaway growth

(Wetherill & Stewart 1989)



Oligarchic growth

(Kokubo & Ida 1998, 2000)

Once big bodies grow sufficiently massive, they can strongly stir up the eccentricities of neighboring small bodies (u increases with M).

$$\frac{1}{R}\frac{dR}{dt} \approx \frac{\sigma\Omega}{\rho_p R}\left(\frac{v_{\rm esc}}{u}\right)^2 \propto R/u^2$$

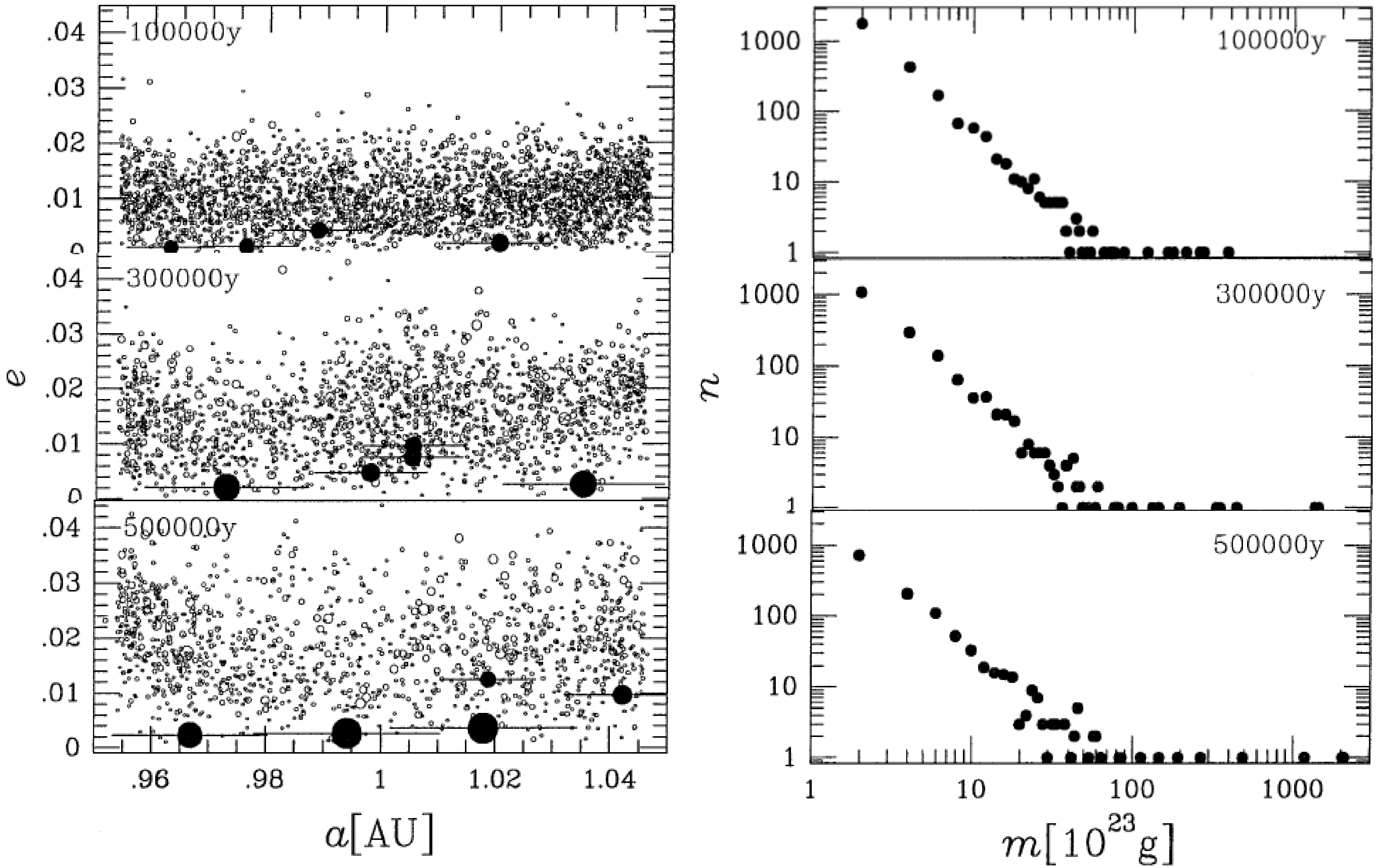
If u~R (empirical), then $$\frac{1}{R}\frac{dR}{dt} \propto R^{-1}$$ Bigger bodies now grow slower.

Self-regulated growth, a few oligarchs have comparable sizes and grow at similar rates.

The overall growth rate of oligarchs is still larger than planetesimals.

Oligarchic growth

(Kokubo & Ida 2000)



Timescale revisited

(Kokubo & Ida 2000)

In equilibrium, eccentricities of neighboring planetesimals excited by a protoplanet under gas drag is approximately

$$< e^2 >^{1/2} \sim 6R_{H,\mathrm{Max}}/a \qquad \text{where} \quad R_H = \left(\frac{GM}{3\Omega_K}\right)^{1/3}$$

Recall: $$\frac{1}{R}\frac{dR}{dt} = \frac{1}{3M}\frac{dM}{dt} \approx \frac{\Sigma_p \Omega}{\rho_p R}\left(\frac{v_{\mathrm{esc}}}{u}\right)^2$$ and $u \sim e v_K$

The growth timescale in the oligarchic regime can be estimated to be

$$T_{\mathrm{grow}} \equiv \frac{M}{dM/dt} \approx 7 \times 10^5 \left(\frac{e}{6R_H/a}\right)^2 \left(\frac{M}{10^{26}\mathrm{g}}\right)^{1/3} \left(\frac{\Sigma_p}{10\ \mathrm{g\ cm^{-2}}}\right)^{-1} \left(\frac{a}{1\ \mathrm{AU}}\right)^{1/2} \mathrm{yrs}$$

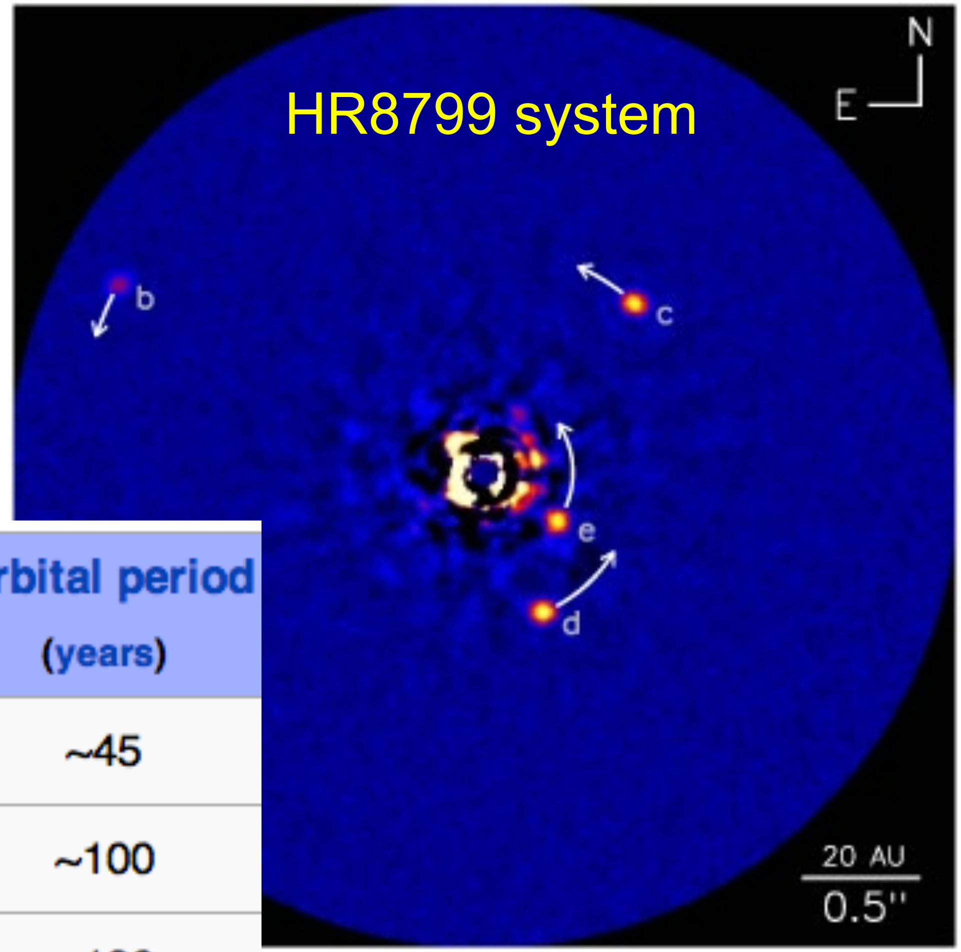
OK for terrestrial planets, but problematic toward outer region:

Timescale to build giant planet cores is excessively long at large separations.

(takes >10 Myr beyond 5 AU)

Challenges to core accretion theory

How to form giant planets at large separations?



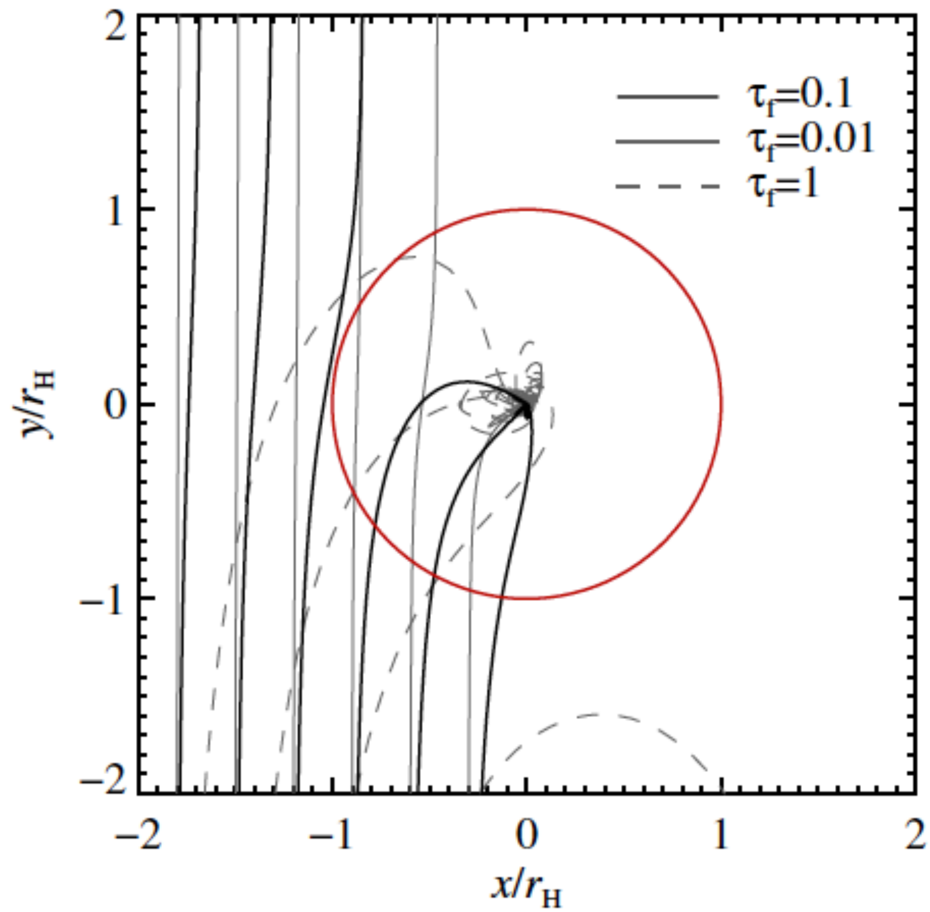
Companion (in order from star)	Mass	Semimajor axis (AU)	Orbital period (years)
e	$7^{+3}_{-2}\ M_J$	14.5 ± 0.5	~45
d	$7^{+3}_{-2}\ M_J$	24±0	~100
c	$7^{+3}_{-2}\ M_J$	38±0	~190
b	$5^{+2}_{-1}\ M_J$	68±0	~460

(Morois+ 2009, Nature)

New regime of pebble accretion

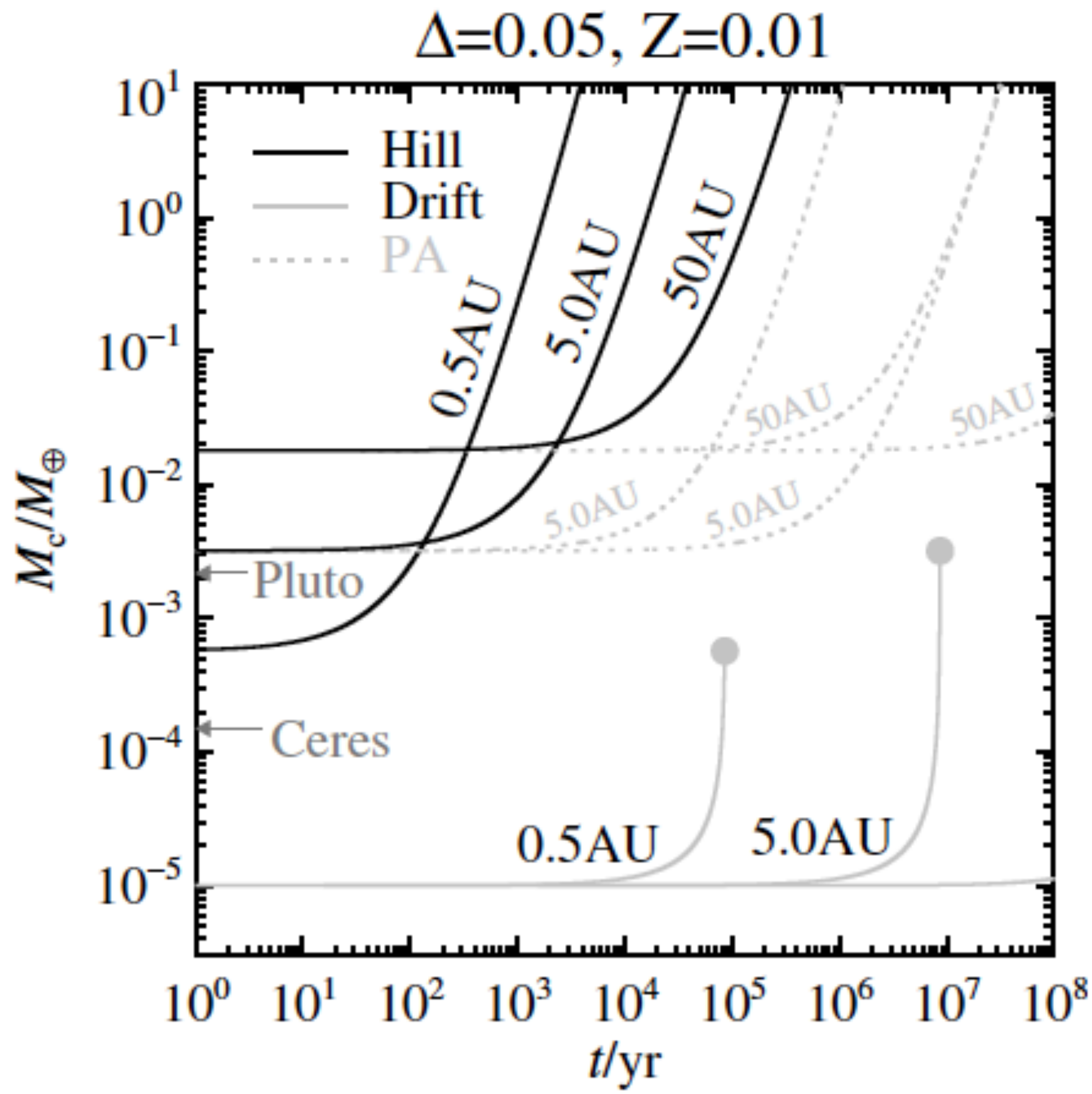
- Pebbles of mm-cm sizes are abundant in PPDs inferred from observations.
- These particles experience strong gas drag hence do not suffer from eccentricity excitations.
- At proper regime, all pebbles entering the Hillsphere spiral in, significantly enhancing the cross section.

$$\frac{dM}{dt} \sim 2R_H \Sigma_p v_H$$



(Lambrechts & Johansen, 2012)

New regime of pebble accretion



Timescale to reach critical core mass:

$$\Delta t \approx 4 \times 10^4 \left(\frac{M_{\mathrm{crit}}}{10M_\oplus}\right)^{\frac{1}{3}} \left(\frac{a}{5\mathrm{AU}}\right) \mathrm{yr}$$

weak dependence on a!

Most efficient for particles with τ_s=0.1-1

independent of core size!

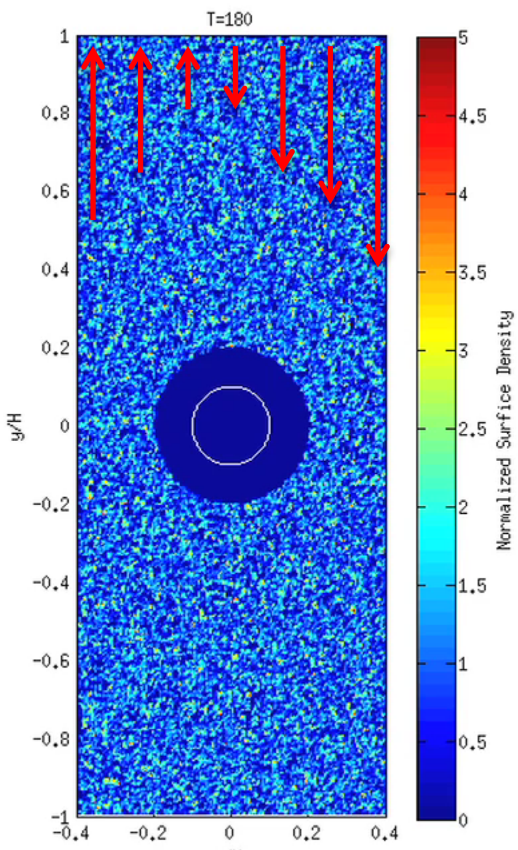
(Lambrechts & Johansen, 2012)

Pebble accretion depends on gas dynamics

- Giant planet formation at large separation becomes easier in low-turbulence environment.

Non-ideal MHD, $\Omega t_{stop}=1$

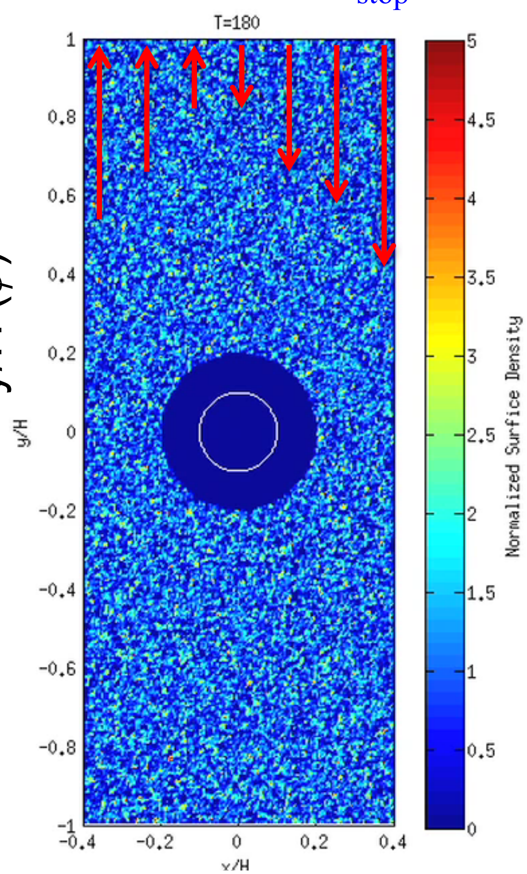
y/H (ϕ)



x/H (R)

Ideal MHD, $\Omega t_{stop}=1$

y/H (ϕ)



x/H (R)

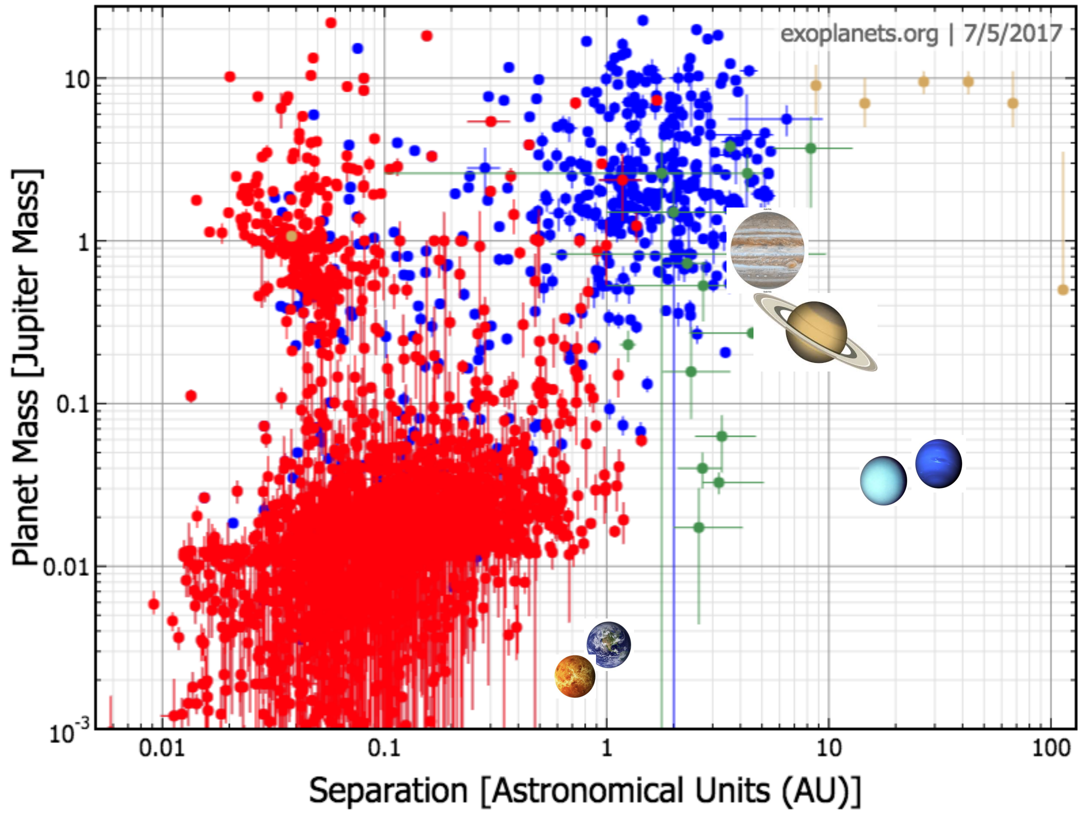
Rate of pebble accretion decreases in stronger turbulence.

Xu, Bai & Murray-Clay, submitted

Summary

- Grain growth: sensitive to dust properties, disk structure (traps?), and gas dynamics (flow structure/turbulence).
- Planetesimal formation: concentration+self-gravity, but the condition for the former is not well understood.
- Growth towards planetary embryos: gravitational focusing leads to runaway growth; oligarchic growth is inefficient; pebble accretion is the most promising.
- Not mentioned — planet migration: planets do not necessarily form in-situ.
- Almost all stages are sensitive to the disk structure / evolution / level of turbulence in PPDs.

Challenges and Opportunities

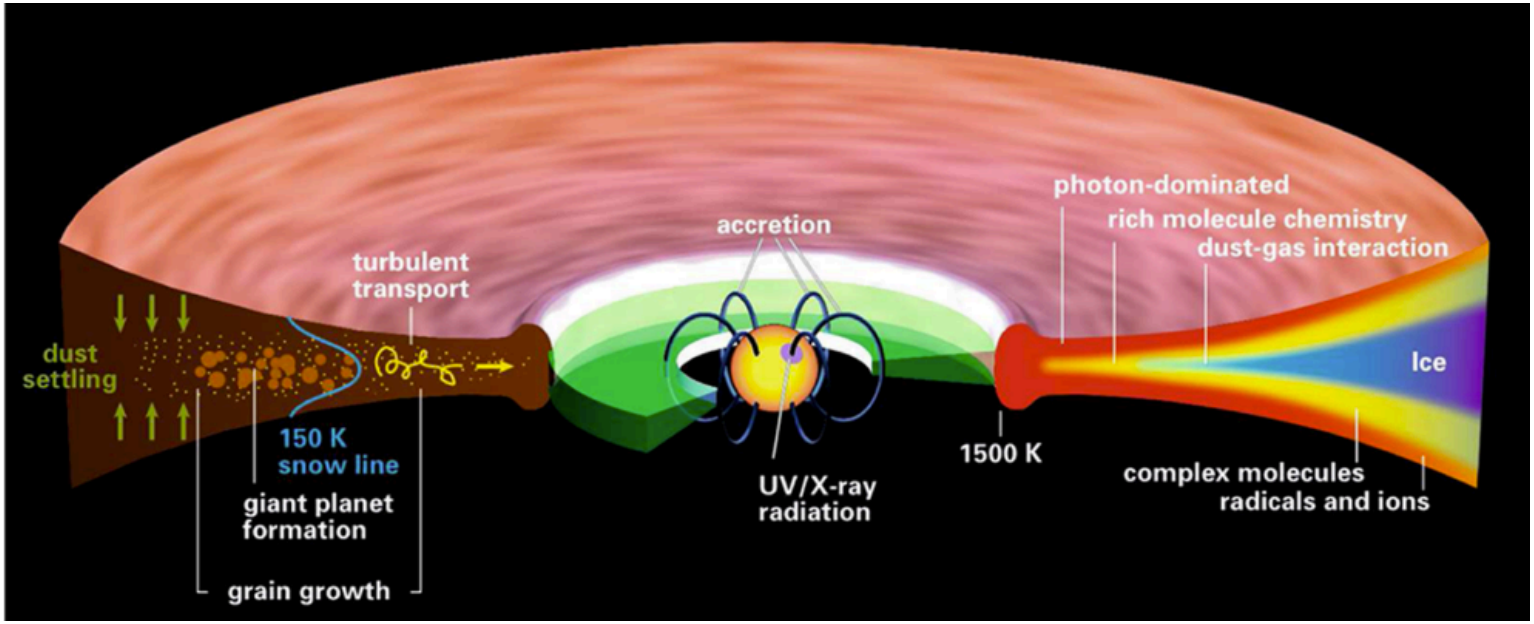


Observationally, not yet sensitive to solar system analogs. Discovery space remains plenty (e.g., TESS, JWST, WFIRST + ground).

Characterize exoplanet properties: interior structure, atmospheric composition, etc. (JWST, TMT, etc.)

Theoretically, we do not understand why certain patterns appear in the exoplanet demographics at all, let alone more detailed properties.

Challenges and Opportunities



Very rich and complex physical and chemical processes take place in PPDs: we have only touched the very basics.

ALMA has been revolutionizing. SKA for the future starting from early 2020s.

Astrophysicists are developing comprehensive tools towards better understanding PPDs and planet formation.

Homework

- Read Chpt. 2-5 of Armitage's book.
- Pick 1-2 references to read in more detail.
- Derive the equations in the slides.

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