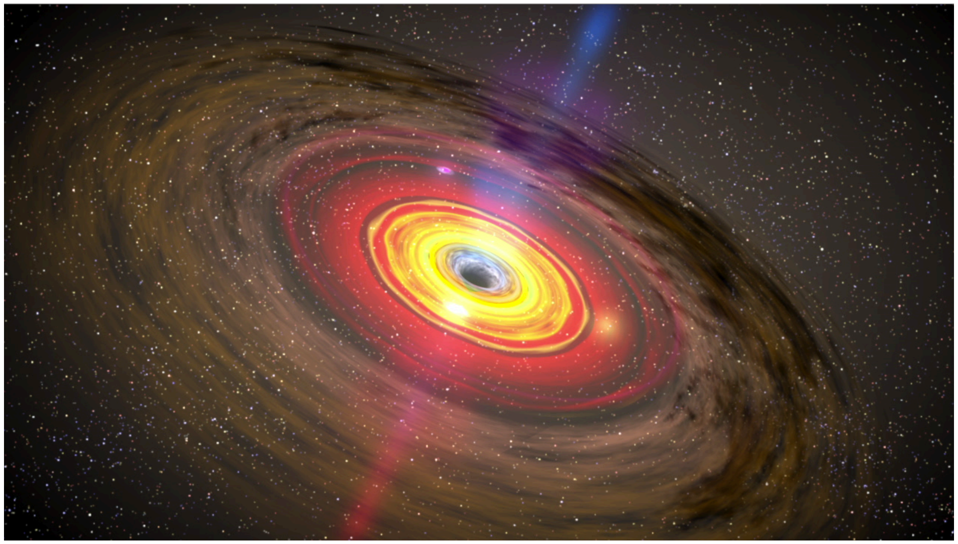
Astronomy 253, Plasma Astrophysics, Harvard University

Instabilities in Dilute Plasmas



Instructor: Xuening Bai

Mar. 28, 2016

Outline

- Dilute plasmas in astrophysical systems
- Instabilities driven by pressure anisotropy
    - Firehose instability
    - Mirror instability
    - Ion-cyclotron instability
- Collisionless accretion disks
    - Magneto-viscous instability
- Plasma physics of the intracluster medium
    - Magneto-thermal instability
    - Heat-flux-buoyancy instability

Dilute plasmas in astrophysics

- The MHD ordering generally requires

$$L \gg \lambda_{\rm mfp}, r_{L,i}, r_{L,e}$$

collisional

$$\omega^{-1} \gg \tau_{i,e}, \Omega_{L,i}^{-1}, \Omega_{L,e}^{-1}$$

low frequency, long wavelength

- Instead of having $r_{L,i} \gg r_{L,e} \gg \lambda_{\rm mfp}$, where MHD is well suited for, astrophysical systems typically satisfy

$$\lambda_{\rm mfp} \gg r_{L,i} \gg r_{L,e}$$

- Some astrophysical systems are collisionless, with

$$\lambda_{\rm mfp} \gtrsim L$$

This also applies when we are interested in small-scale physics, e.g., particle acceleration, dissipation of MHD turbulence, etc.

Properties of dilute plasmas

- Anisotropic pressure/viscosity

  Results from conservation of adiabatic invariants. Leading to velocity-space micro-instabilities. More later.

- Anisotropic heat conduction

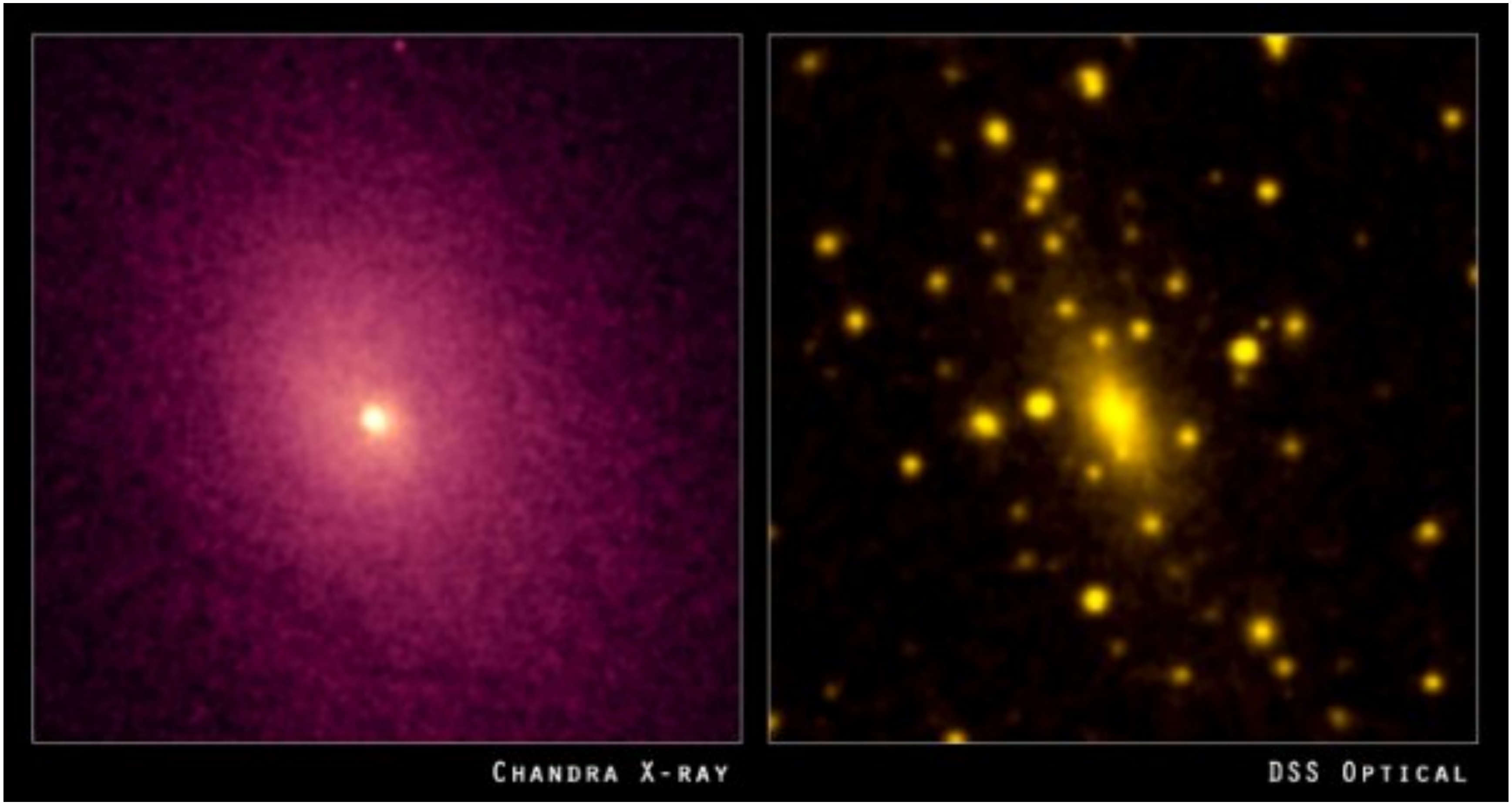
  Heat conduction is usually slow compared with dynamical timescale, but it can be very efficient in dilute, hot plasmas due to long (parallel) mean free path. More later.

- Two-temperature plasma

  In collisionless systems, electrons and ions can develop into different temperatures because their energy equilibration time >> dynamical time.

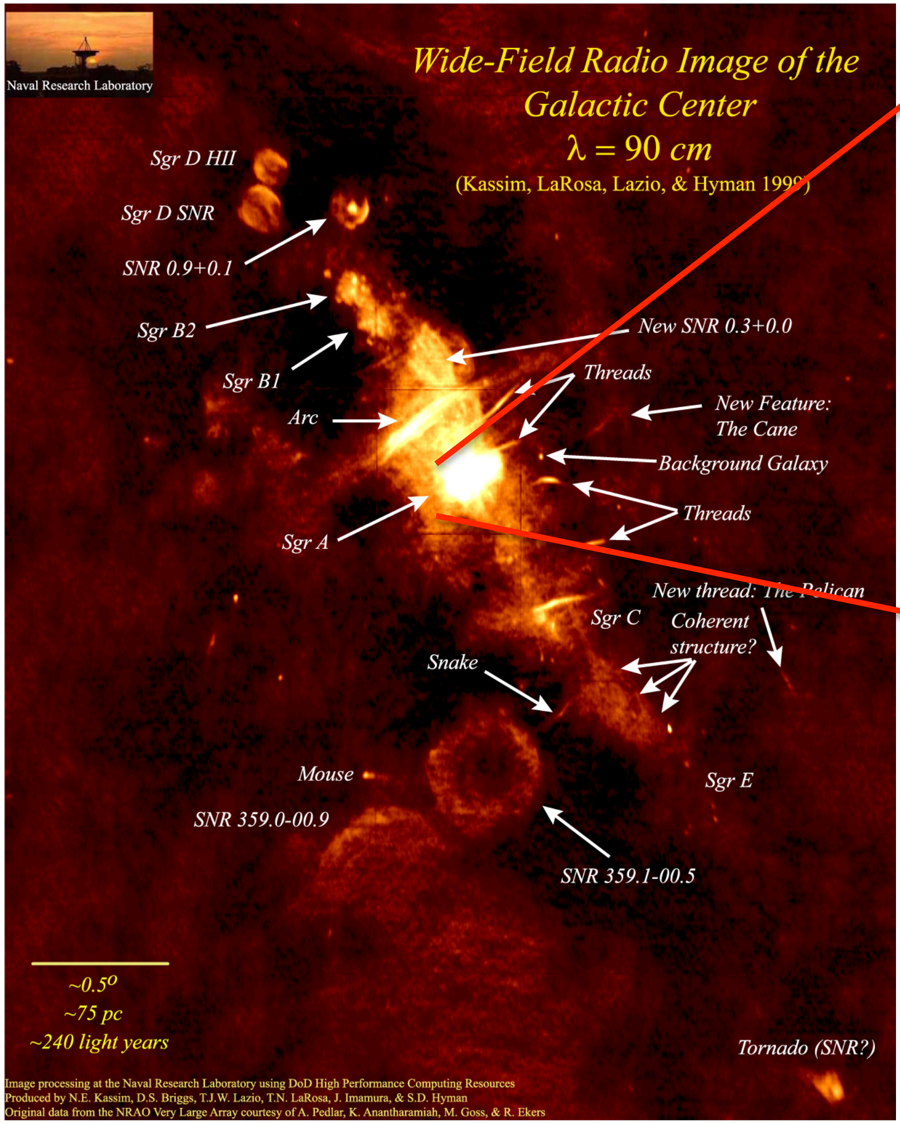
  Example: radiatively inefficient accretion flow (see Yuan & Narayan, 2014 for a review)

Galaxy clusters

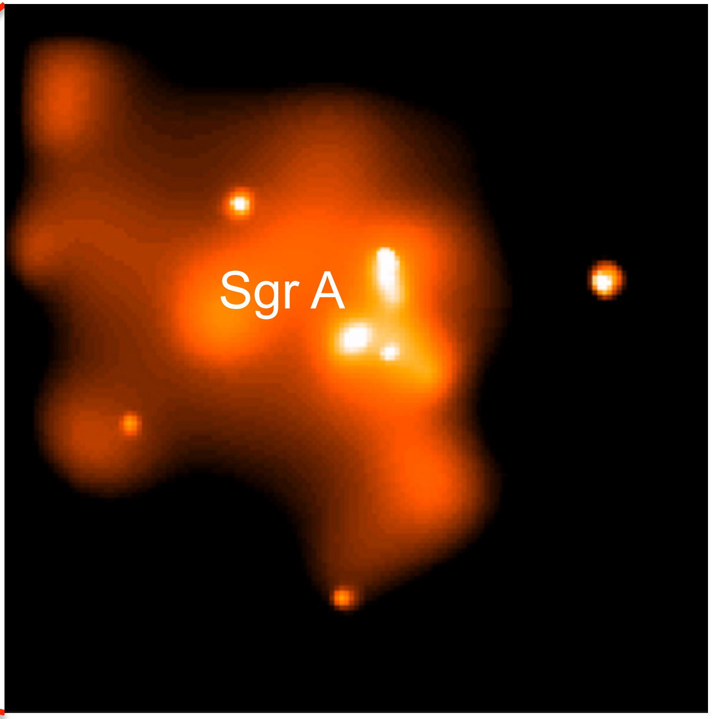


L~100 kpc λ_{mfp}~1 kpc $r_{L,i}$~ 10^{-9} pc

The galactic center



Innermost 3 pc (X-ray, Chandra)



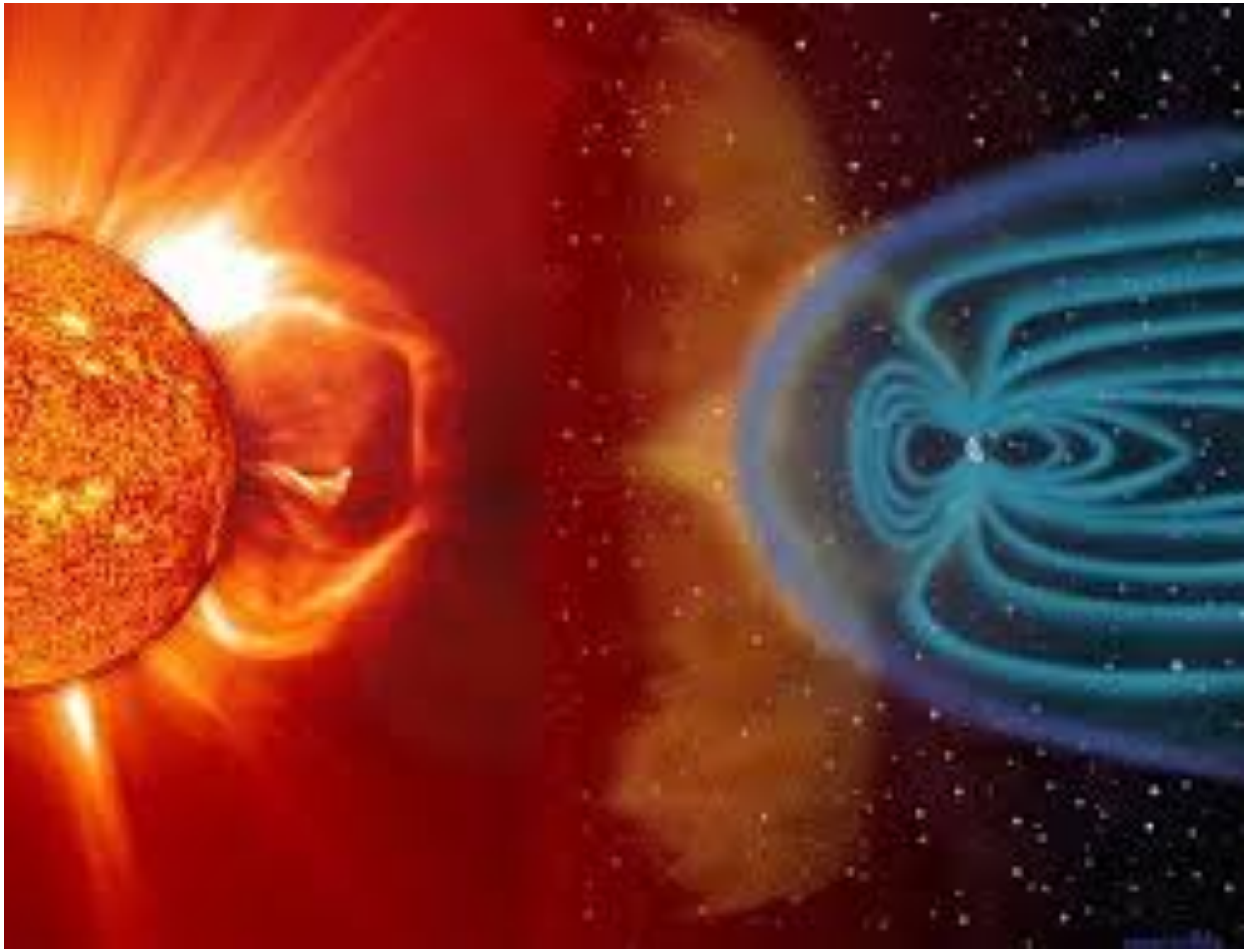
At Bondi radius:

$L\sim 0.1$ pc $\sim 10^5 R_g$

$\lambda_{mfp}\sim 0.01$ pc

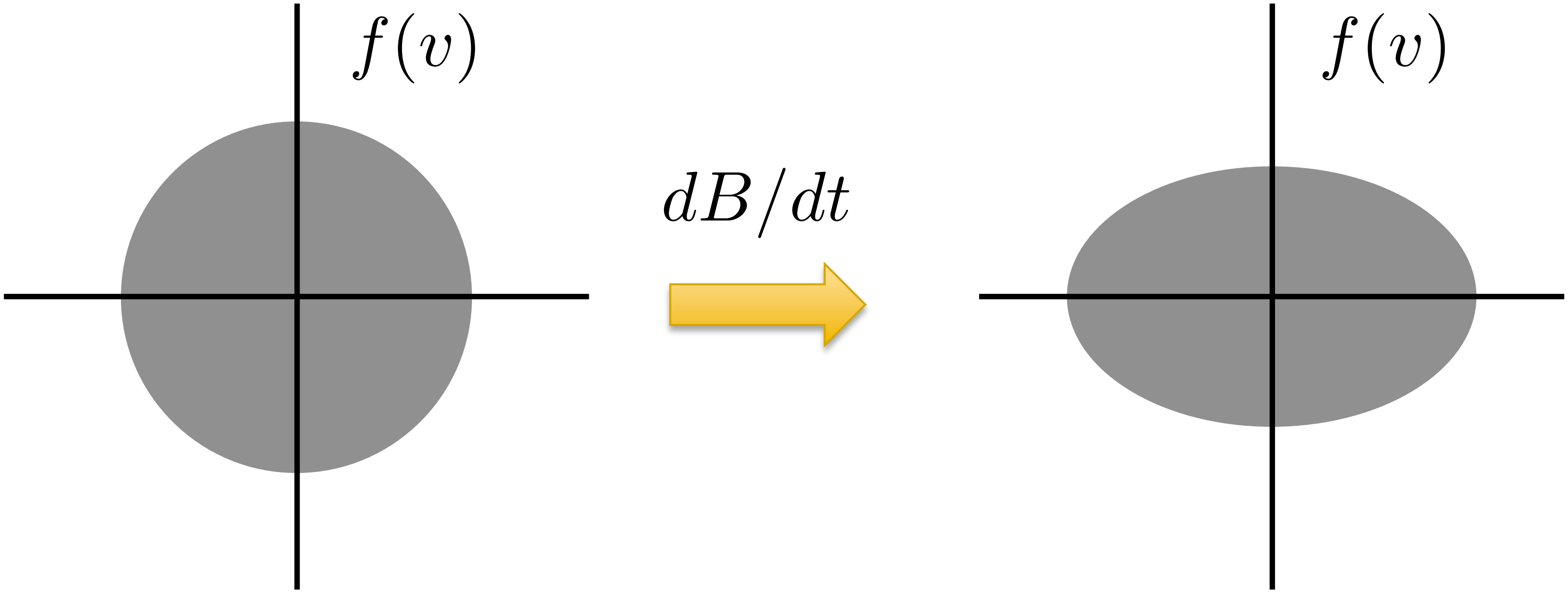
$r_{L,i}\sim 10^{-12}$ pc

Solar wind



At ~1 AU: L~1 AU, λ_{mfp}~1 AU, $r_{L,i}$~ 10^{-6} AU

Development of pressure anisotropy



Adiabatic evolution: $P_\perp \propto nB$ and $P_{||} \propto n^3/B^2$

Driving source:

- Shear motion: e.g., accretion disks
- Expansion (e.g., solar wind) or compression
- Turbulence (nearly everywhere): has all of the above.

How much pressure anisotropy?

In the limit of very small anisotropy (Braginskii MHD):

$$\frac{P_\perp - P_\parallel}{P} \sim \boxed{\frac{1}{\nu_i}}\boxed{\frac{d}{dt}\left(\ln\frac{B^3}{n^2}\right)} \sim \frac{\text{Collision time}}{\text{Dynamical time}}$$

Collisional relaxation

Adiabatic driving

In intracluster medium:

$$\frac{|P_\perp - P_\parallel|}{P} \sim \frac{V}{v_{\rm th}}\frac{\lambda_{\rm mfp}}{L}$$

$$\sim \text{a few} \times 10^{-2}$$

Not particularly large, but:

sufficient to modify dynamics appreciably (more later)

In collisionless accretion disks (e.g., in radiatively inefficient accretion flows):

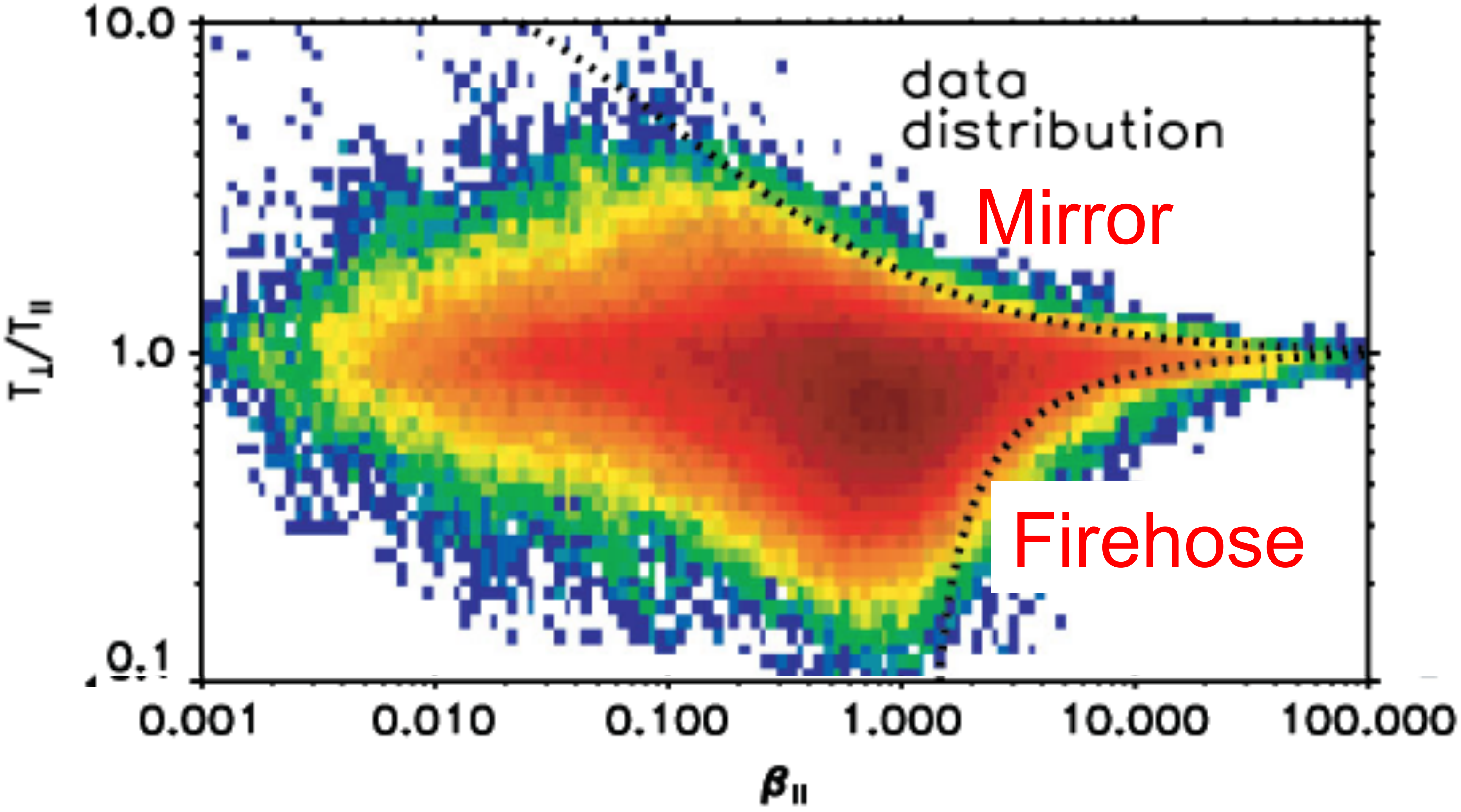
Collision time >> shearing time scale

Pressure likely be highly anisotropic!

Pressure anisotropy: is there a limit?

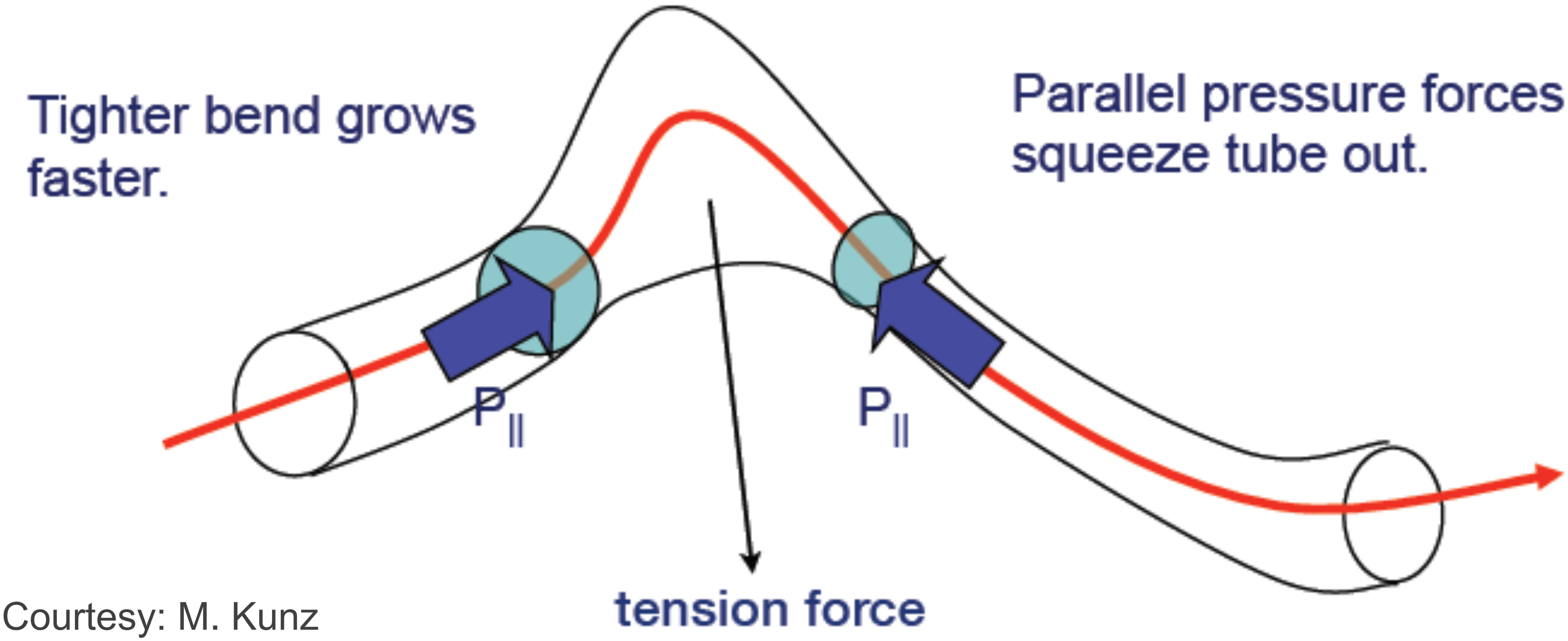
In the solar wind, expectation from adiabatic expansion: $\frac{T_\perp}{T_\parallel} \propto r^{-2}$ (assuming $B \propto r^{-2}$, before the Parker-spiral develops)

In-situ measurement of proton temperature (at ~1 AU):



Bale et al. 2009 PRL

Firehose instability: physical mechanism



Courtesy: M. Kunz

Alfven waves become unstable to the firehose instability when:

$$P_\| - P_\perp \gtrsim \frac{B^2}{4\pi}$$

Firehose instability: a quick derivation

Starting point: the momentum equation

$$\rho \frac{d\boldsymbol{v}}{dt} = -\nabla\left(P_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\boldsymbol{bb}\left(P_\perp - P_\| + \frac{B^2}{4\pi}\right)\right]$$

A few lines of algebra (see handout)...

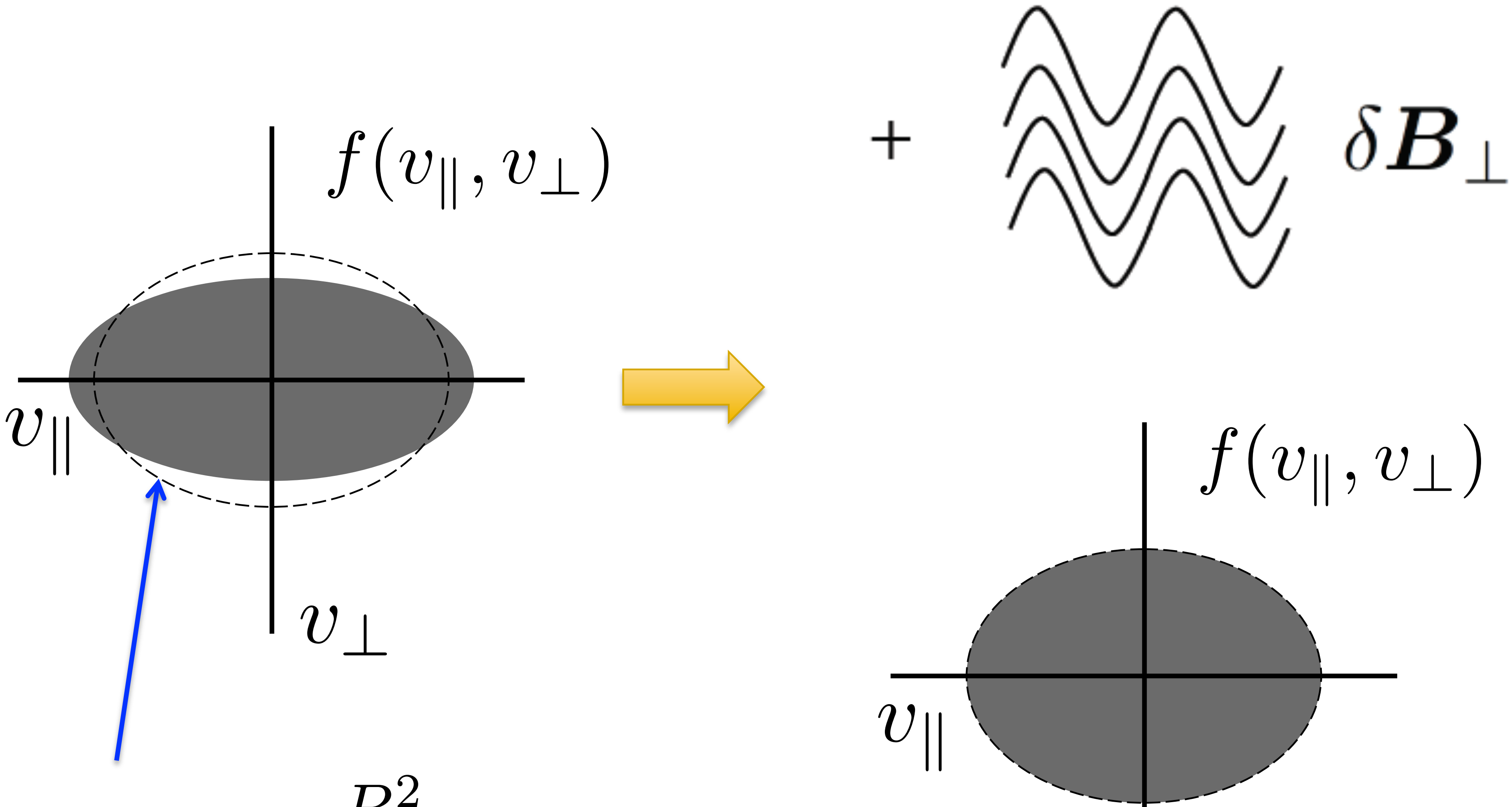
Result: $\omega^2 = k_\|^2\left[v_A^2 + \frac{1}{\rho}(P_\perp - P_\|)\right]$ (Recall: $v_A^2 = \frac{B^2}{4\pi\rho}$)

This instability is MHD in nature, and high-beta plasmas are more susceptible to the firehose instability. However, when unstable, we encounter the UV catastrophe:

Growth rate $\gamma \propto k_\|$

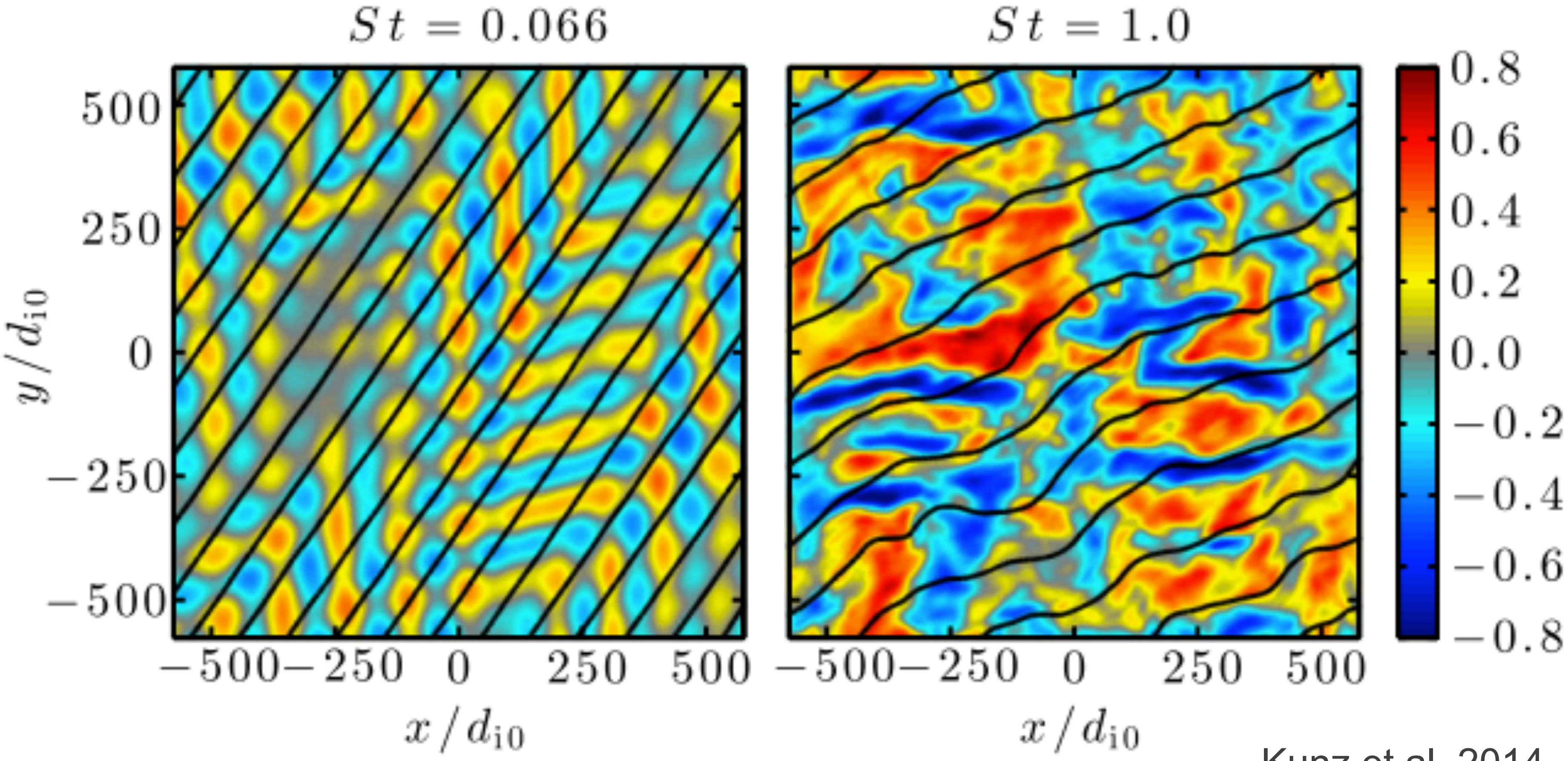
In reality, MHD breaks down at Larmor radius scale, where the growth rate peaks.

Firehose instability: quasi-linear evolution



$$P_\parallel - P_\perp \gtrsim \frac{B^2}{4\pi}$$

Firehose instability: non-linear saturation



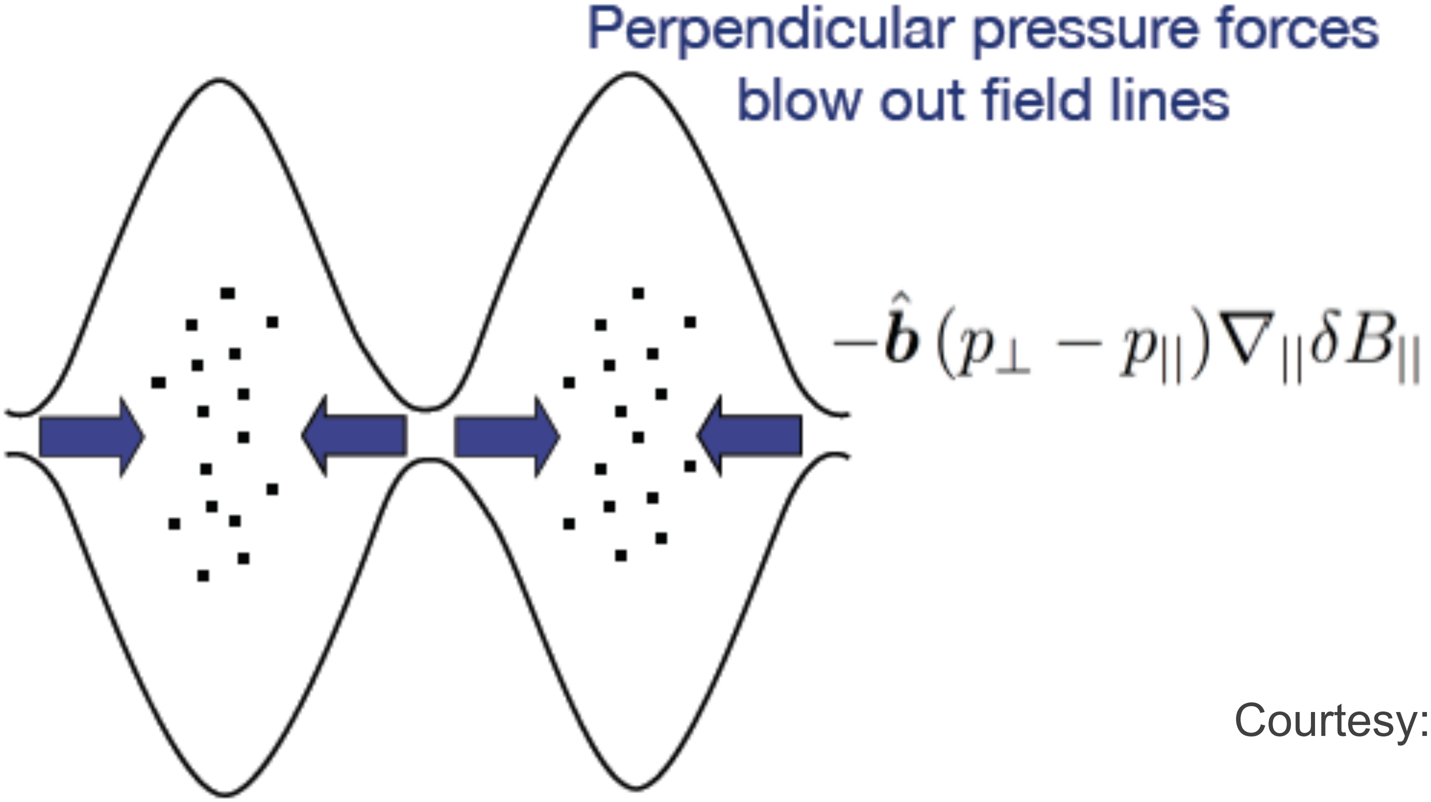
Kunz et al. 2014, PRL

Magnetic field evolution driven by shear.

Instability grows at Larmor-radius scale.

Saturation achieved by particles scattering off at Larmor-radius scale fluctuations.

Mirror instability: physical mechanism



Perpendicular pressure forces blow out field lines

$-\hat{\boldsymbol{b}}\,(p_\perp - p_{||})\nabla_{||}\delta B_{||}$

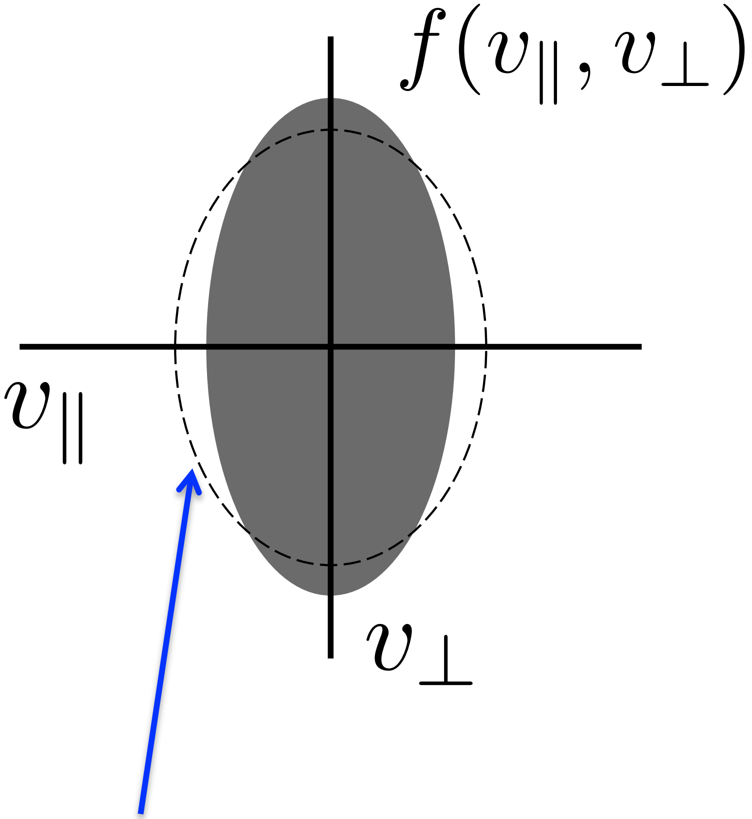
Courtesy: M. Kunz

Not quite an MHD instability, but MHD gives about the correct instability threshold:

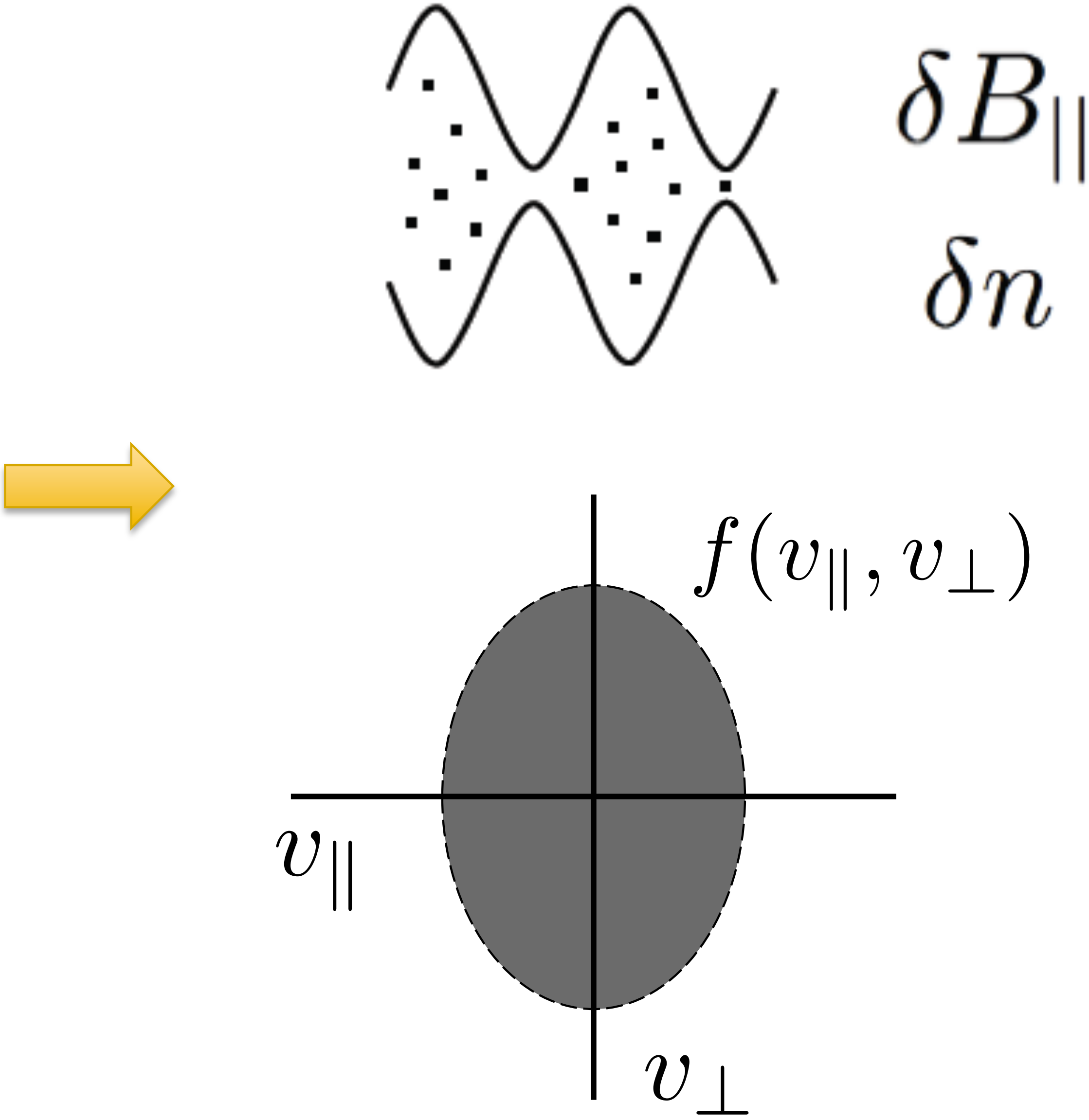
$$P_\perp - P_{||} \gtrsim \frac{B^2}{8\pi}$$

Produces almost non-propagating, oblique magnetic-mirror structures.

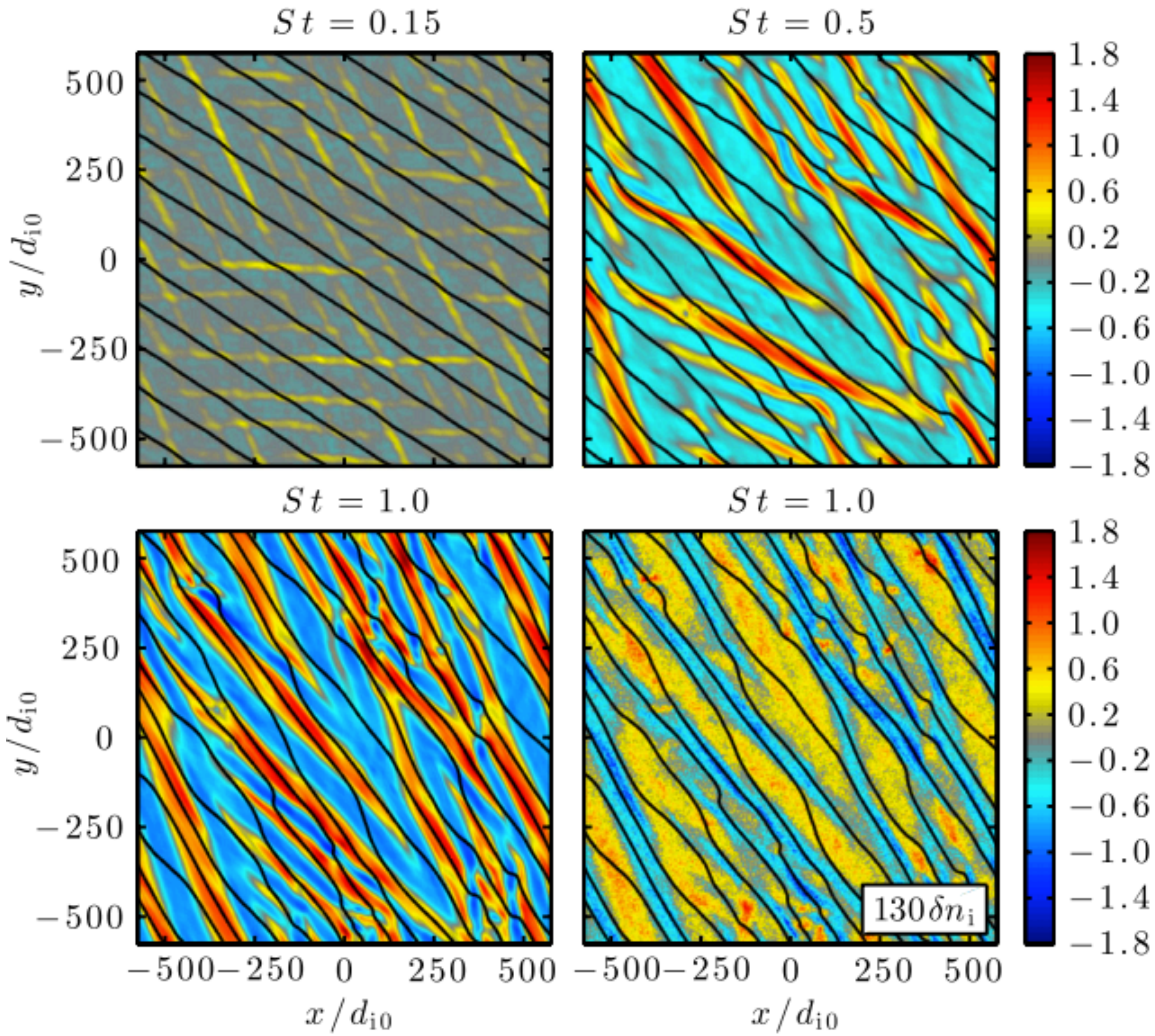
Mirror instability: quasi-linear evolution



$$P_\perp - P_\parallel \gtrsim \frac{B^2}{8\pi}$$



Mirror instability: non-linear saturation



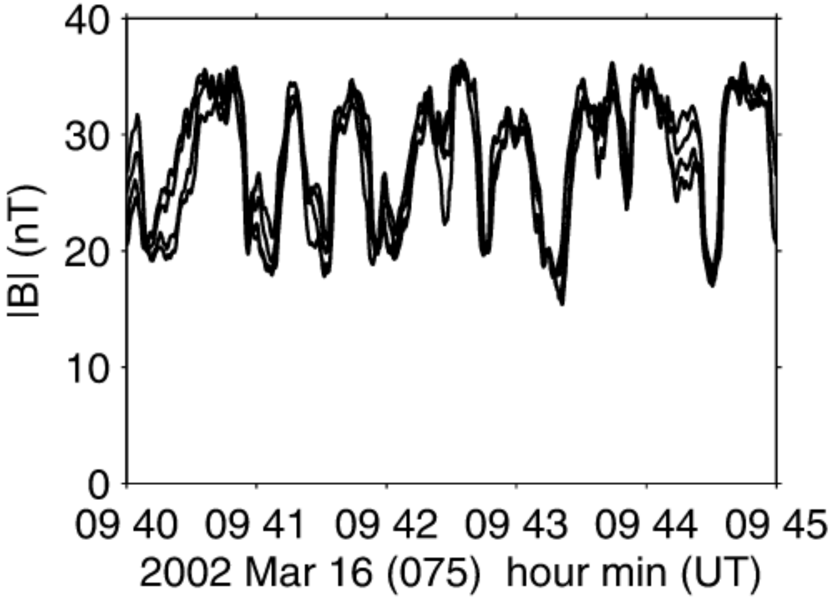
Particle trapping in magnetic mirrors

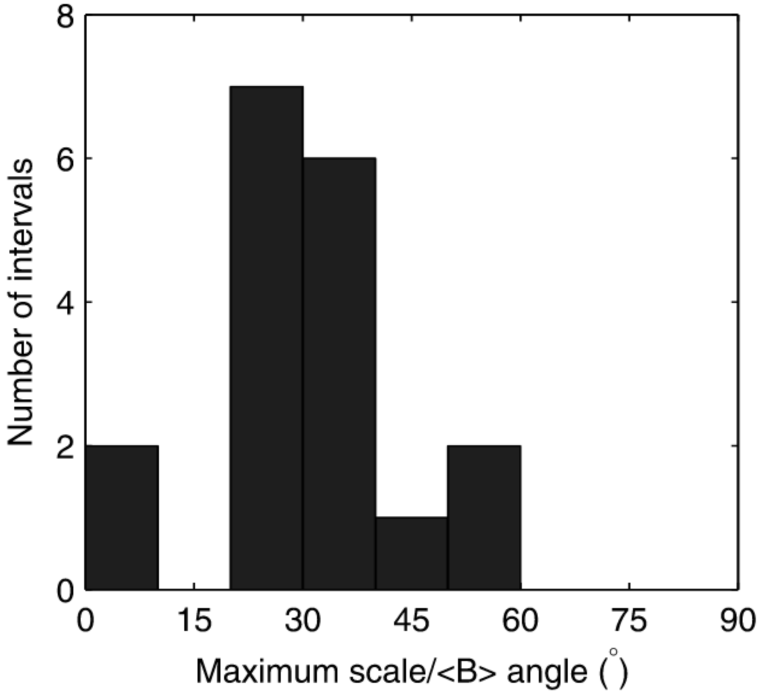
+

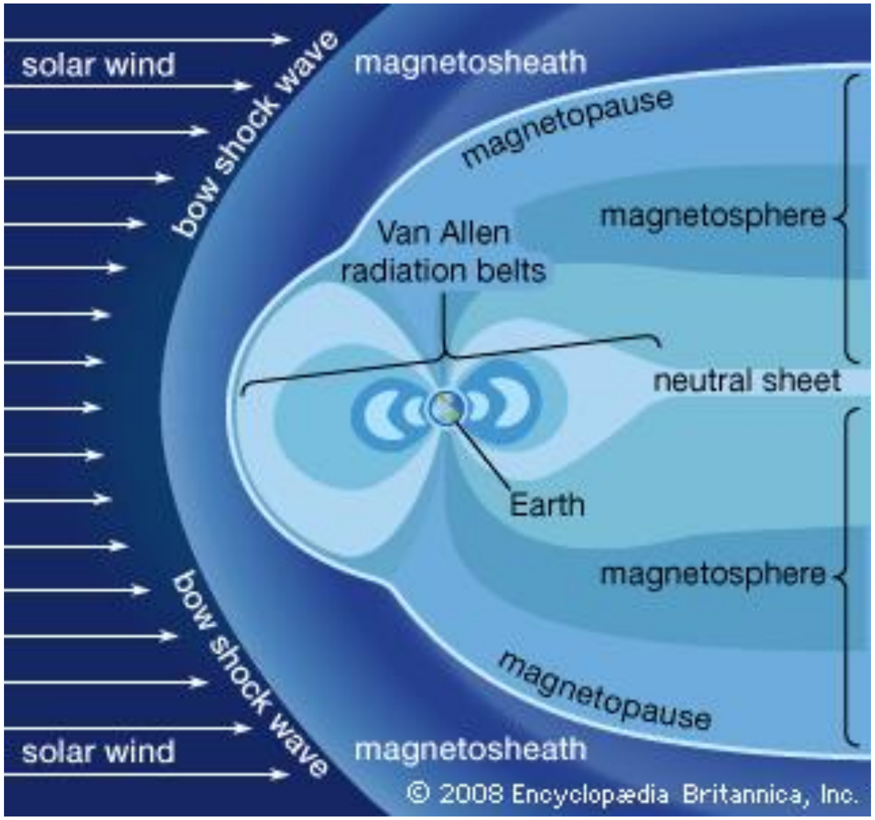
scattering off from Larmor-radius scale fluctuations

Kunz et al. 2014, PRL

Mirror modes in the magnetosheath







© 2008 Encyclopædia Britannica, Inc.

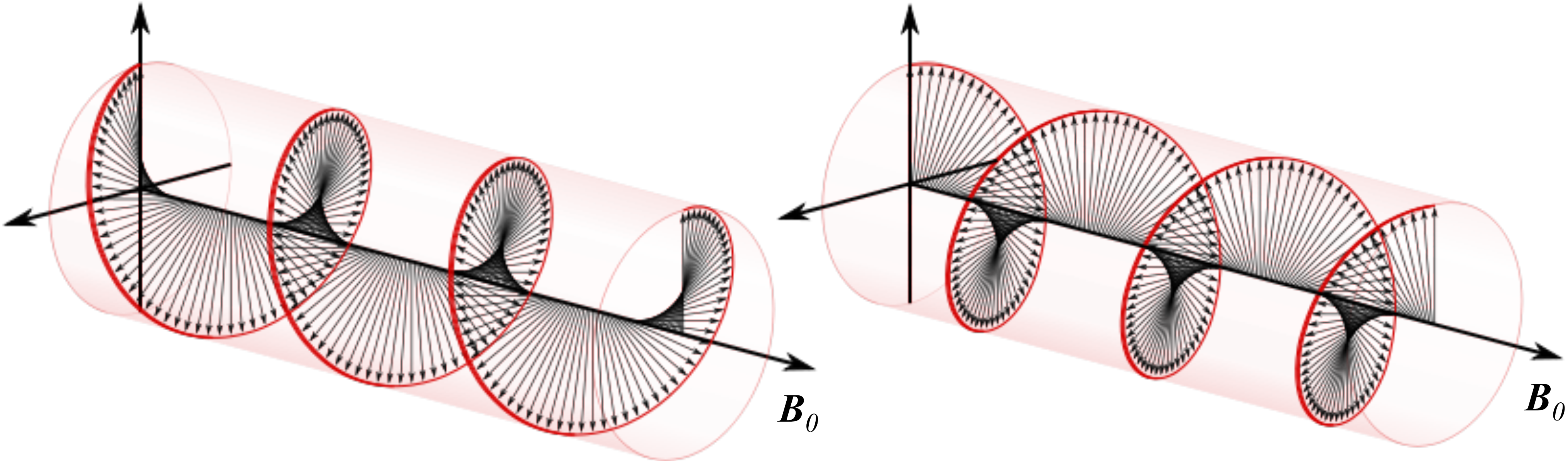
Magntosheath is being compressed: $T_\perp > T_\|$

Mirror mode constantly observed, with long-axis oriented ~30deg with <B>

Horbury & Lucek 2009, JGR

Ion cyclotron instability

Right polarization (whistler): Left polarization (ion-cyclotron):



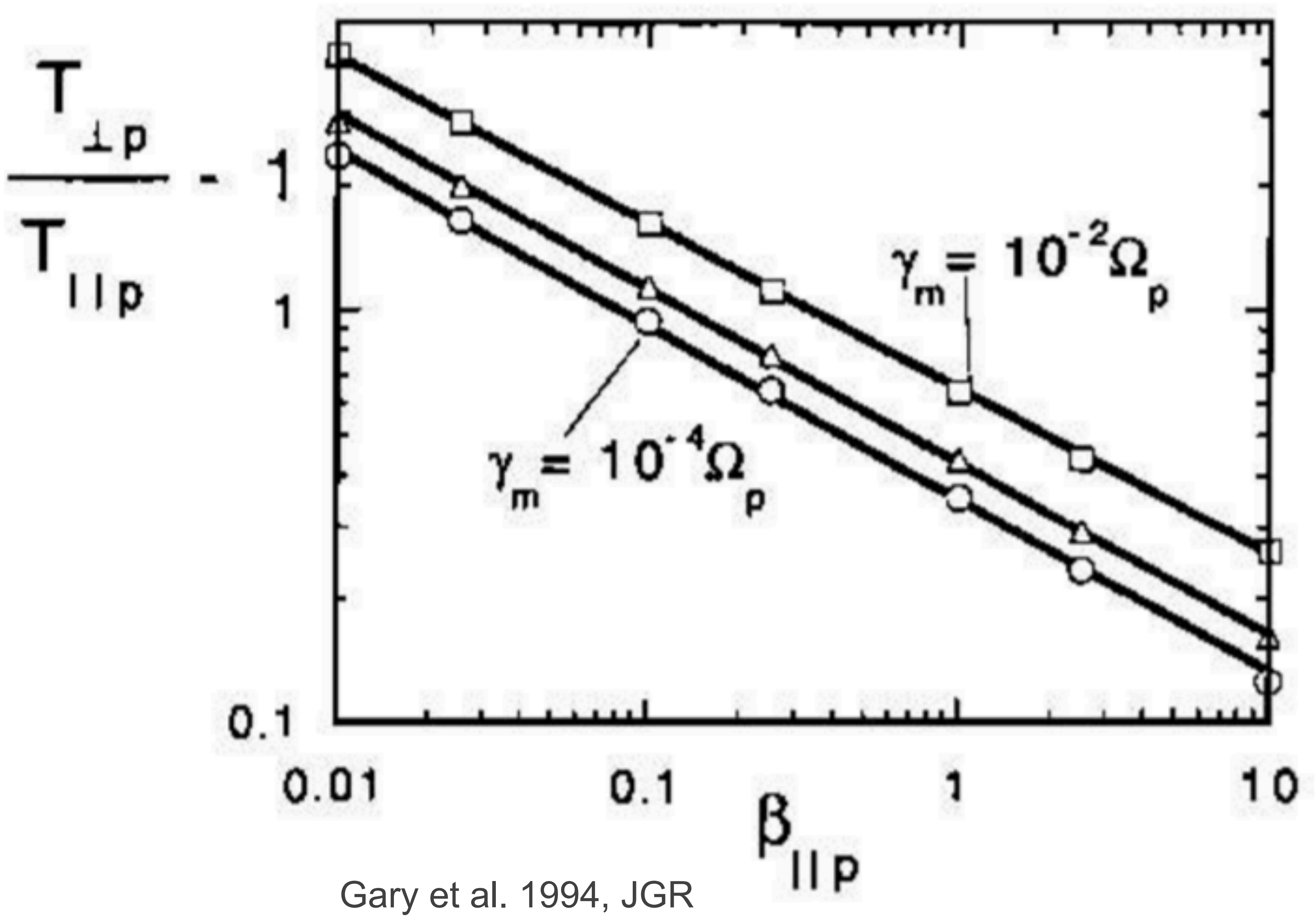
Resonant with forward-traveling electrons. Resonant with forward-traveling ions.

Gyro resonance: $\omega\text{-}kv_z = \pm\Omega$

Ion-cyclotron wave (parallel propagating) becomes unstable when perpendicular pressure exceed parallel pressure.

No analytical criterion, but somewhat similar than mirror.

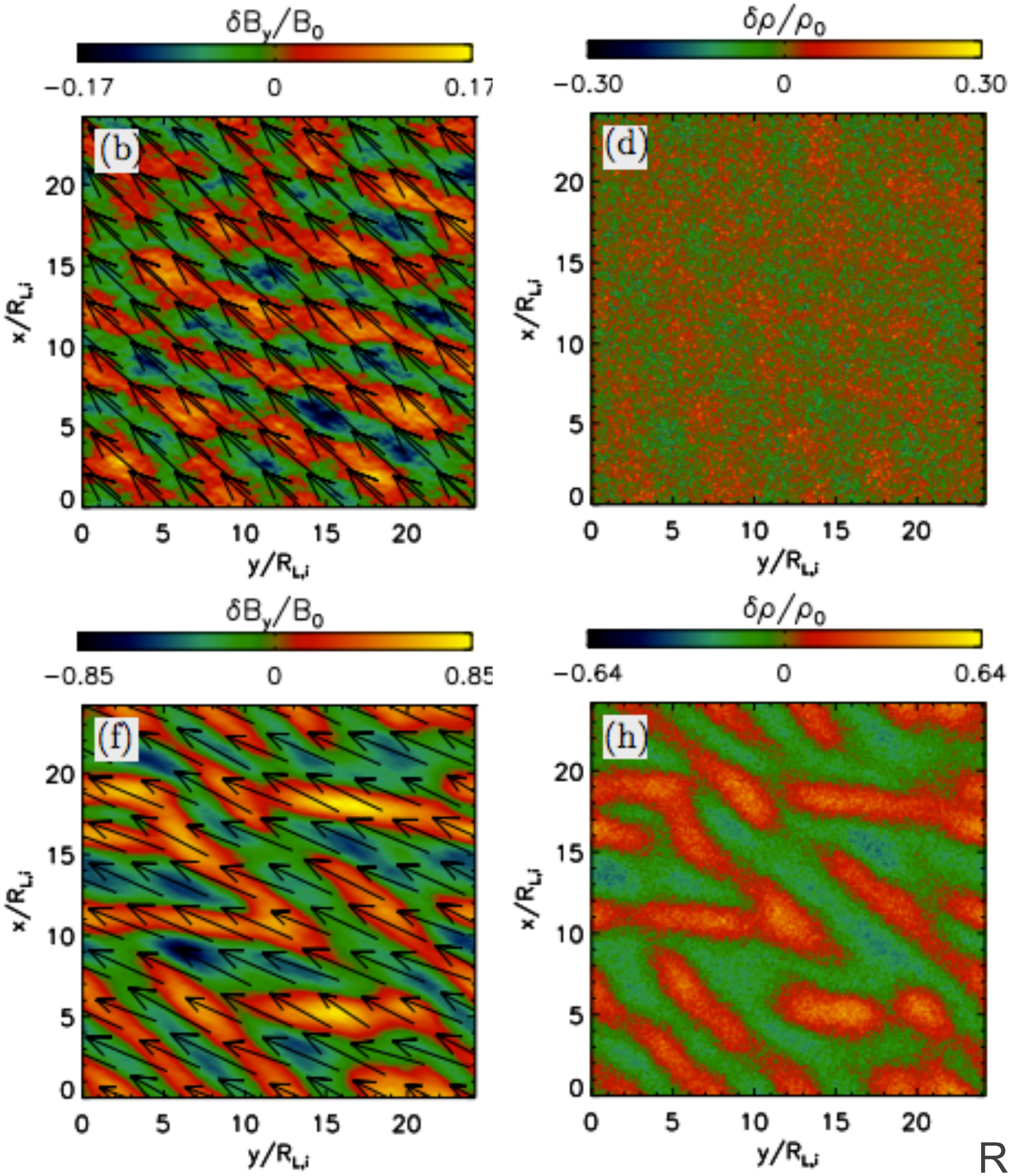
Ion cyclotron instability: linear growth rate



instability threshold at a given growth rate.

Gary et al. 1994, JGR

Ion cyclotron vs. mirror: which one dominates?



Early

Late

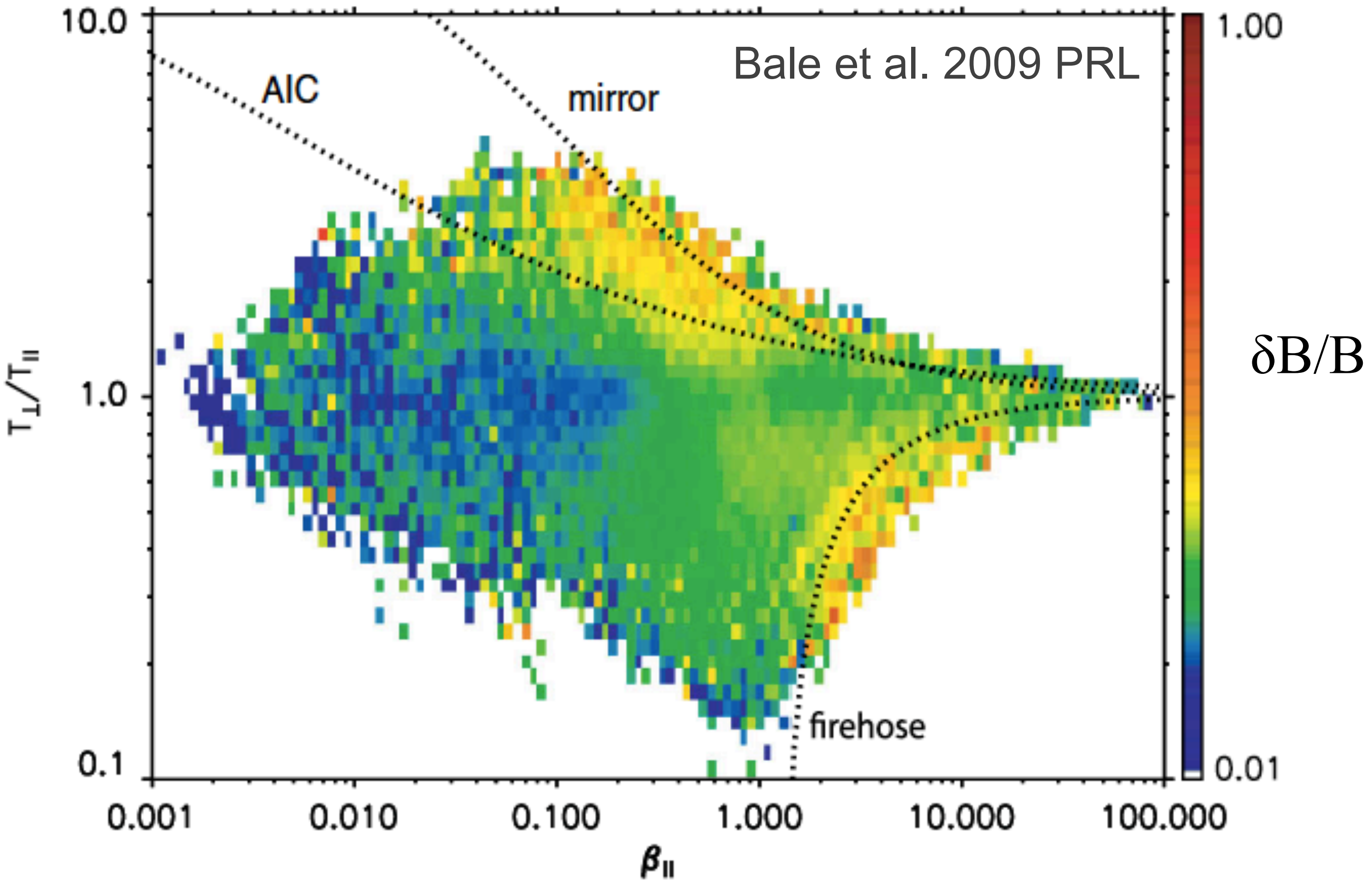
Full PIC simulation with equal mass ratio.

IC grows faster in low-beta regime.

However, even when IC grows faster initially, mirror can dominate at later times!

Riquelme et al. 2014

Solar wind observations

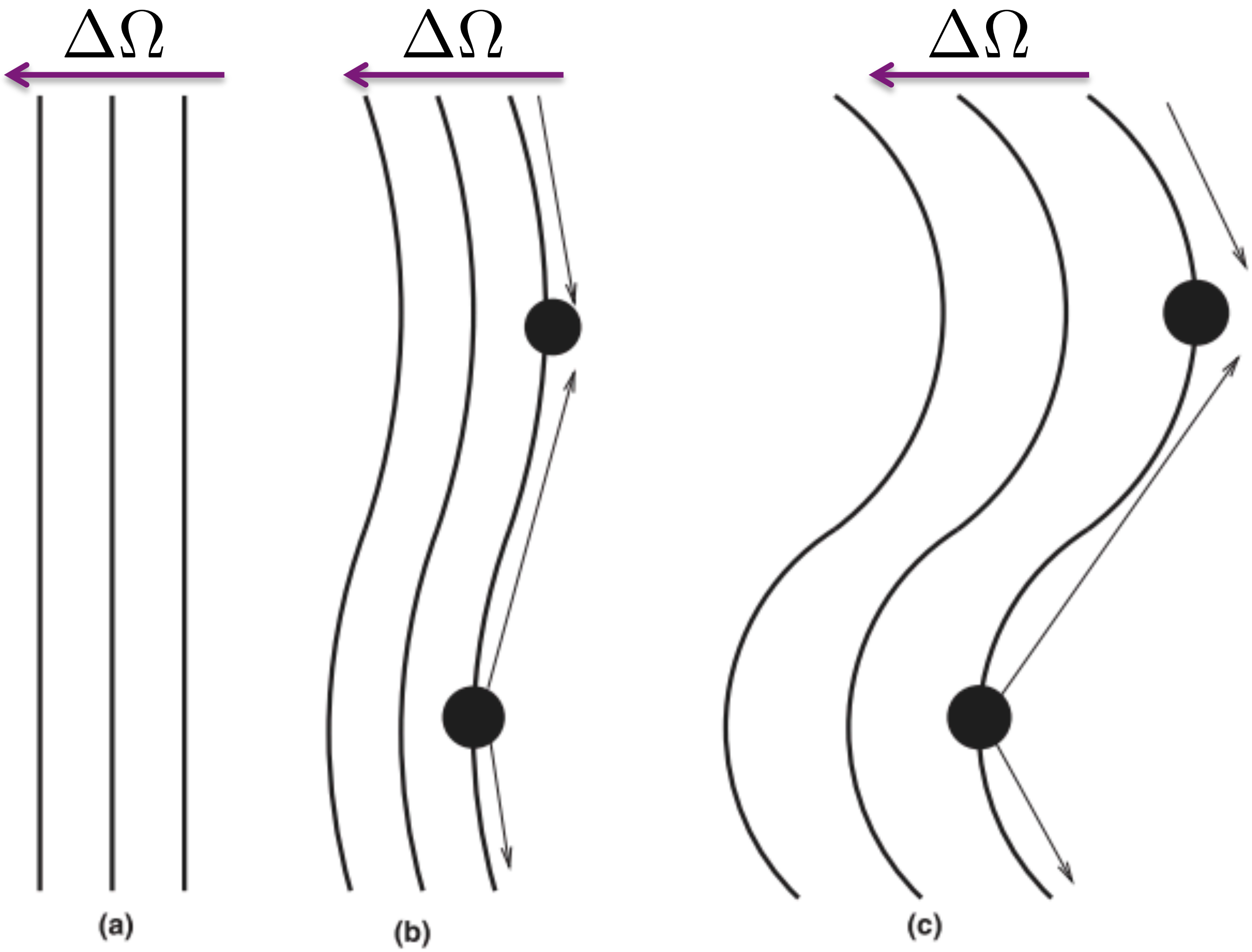


In the solar wind, pressure anisotropy is regulated by firehose and mirror (but not the ion cyclotron) instabilities.

Enhanced magnetic fluctuations near the instability threshold.

MRI in dilute plasmas

Magnetoviscous instability (Balbus, 2004)

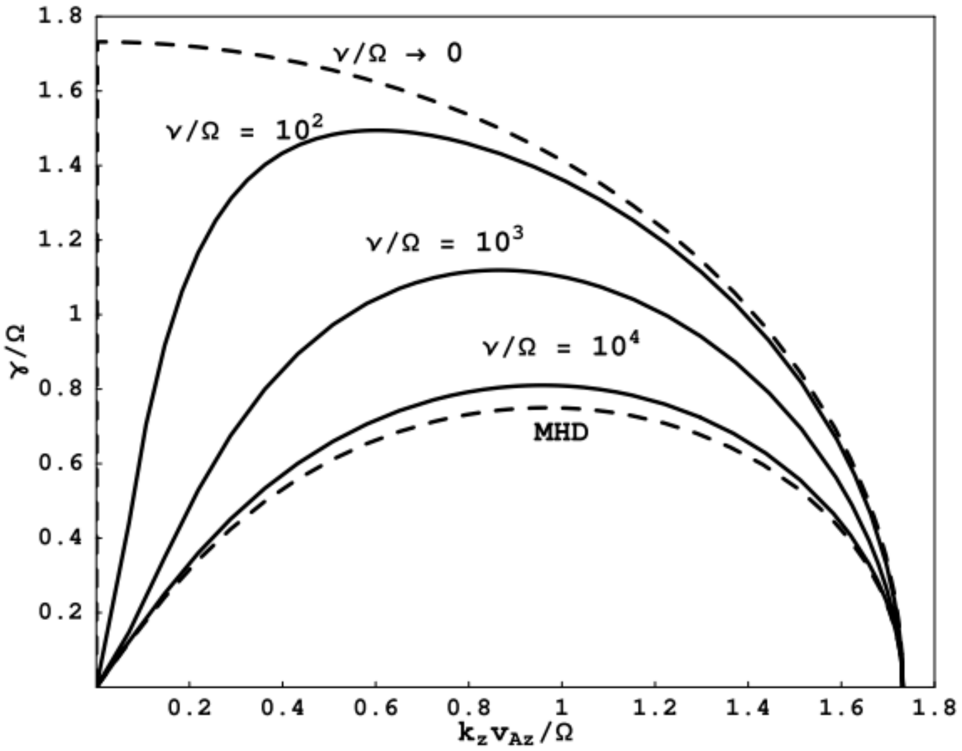


Anisotropic (Braginskii) viscous transport tends to enforce constant Ω along field lines.

System is distabilized even without magnetic tension (as in the MRI)!

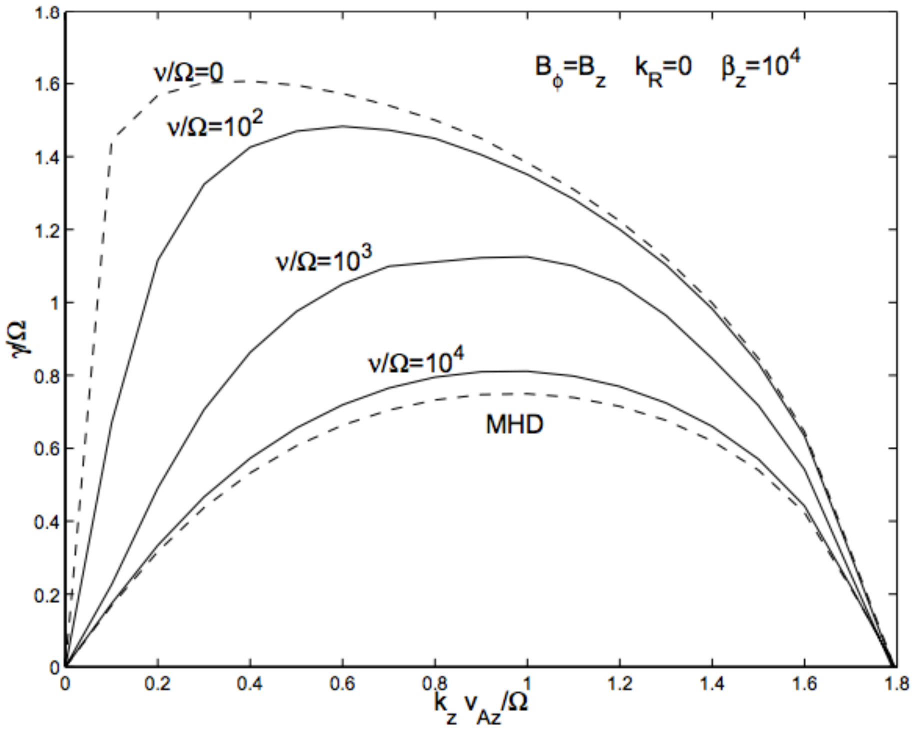
Linear MRI at different collisionalities

Braginskii treatment (MVI):



Islam & Balbus, 2005

Kinetic treatment:



Sharma et al. 2003

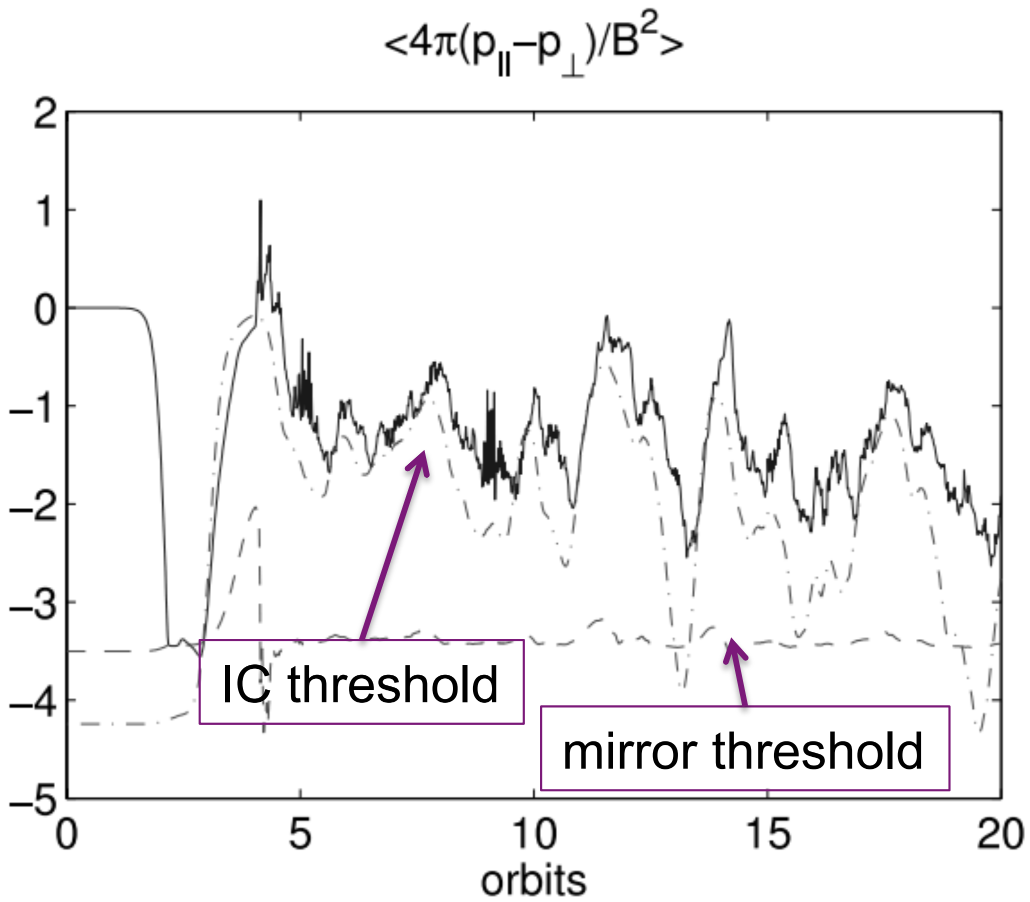
In a weakly collisional system, the MRI can grow faster!

(Quataert et al. 2002)

Kinetic MRI: non-linear evolution

Angular momentum transport:

$$T_{R\phi} = \rho v_R \delta v_\phi - \hat{b}_R \hat{b}_\phi \left(\frac{B^2}{4\pi} + P_\perp - P_\parallel \right)$$



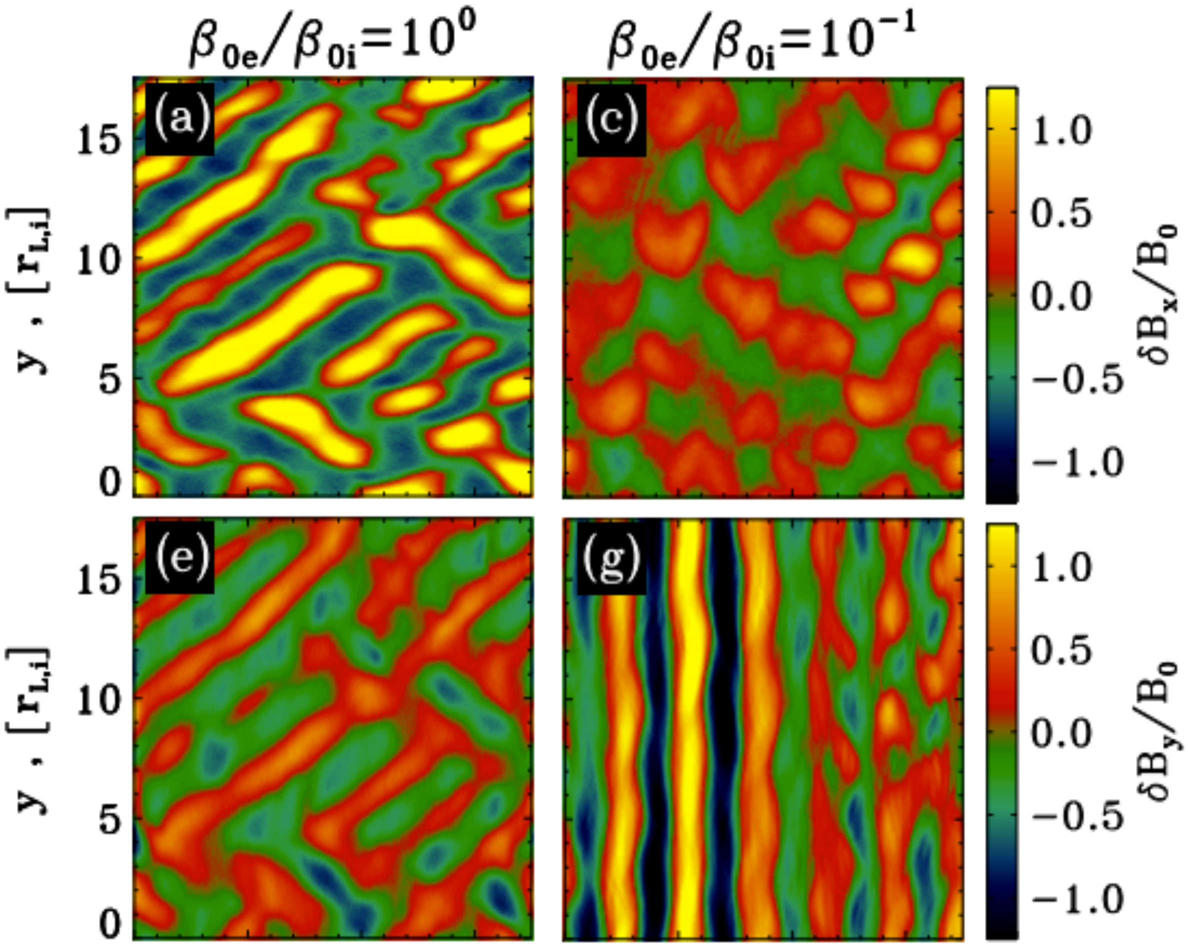
Sharma et al. 2006

Shearing-box kinetic MHD simulations with Landau fluid closure.

Colllisionless effect enhances angular momentum transport from pressure anisotropy! (by a factor of ~2)

Energy dissipation and electron heating

Radiatively inefficient accretion flow can be largely collisionless: electrons and ions are collisionally decoupled -> electrons cool



Mirror

IC

Sironi & Narayan, 2015

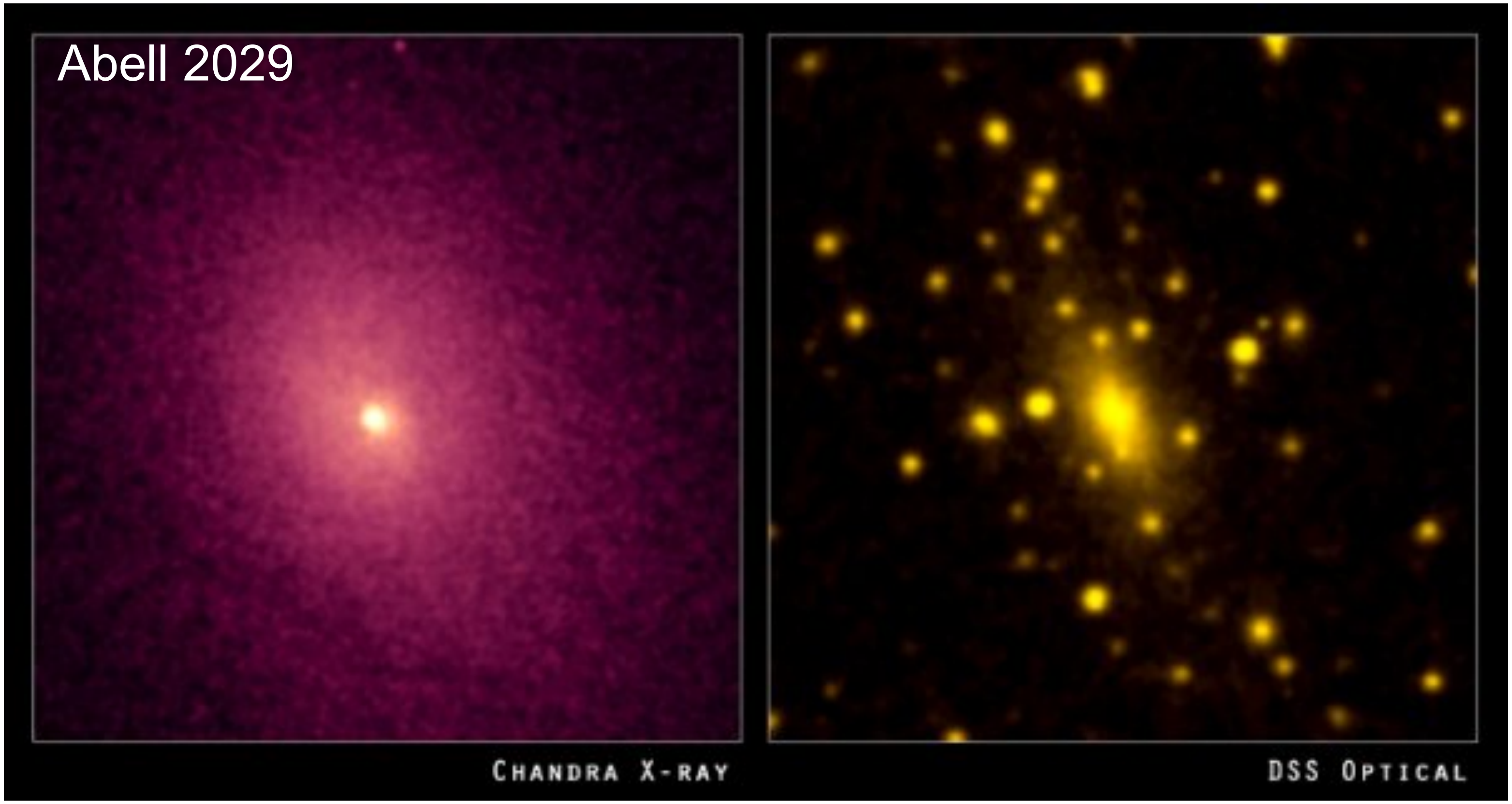
Energy dissipation from turbulence: how much goes to heat e^-/ions?

Full PIC, expanding-box simulations:

High T_e ($\sim T_i$): mirror dominates

Low T_e ($\sim 0.1 T_i$): IC dominates -> leads to electron heating

Plasma physics of the intracluster medium



~90%: dark matter

~10%: hot plasmas

~1%: galaxy

L~ Mpc

H~100 kpc

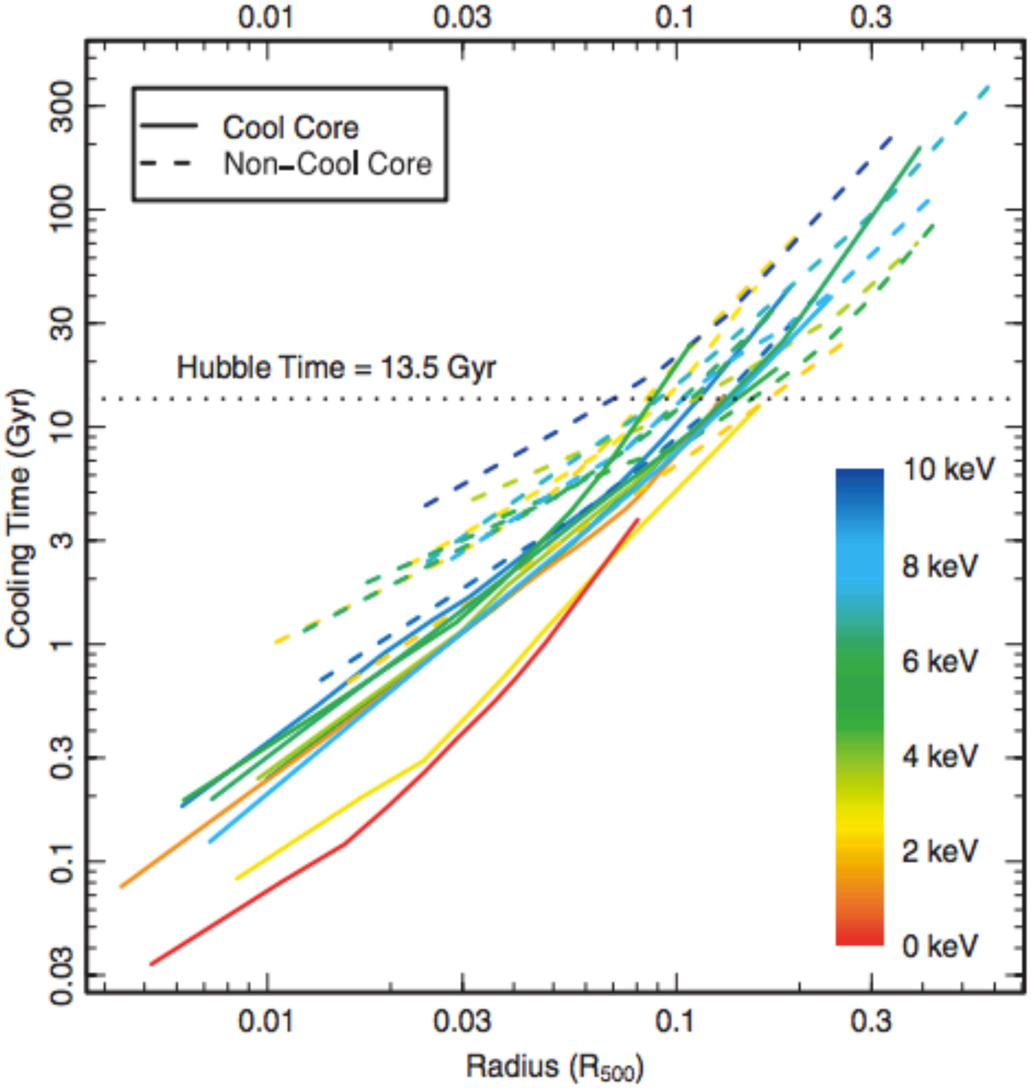
T~3-10 keV

λ_{mfp}~1 kpc

B~ 10^{-6} G

$r_{L,i}$~ 10^{-9} pc

The cooling flow problem (e.g., Fabian 1994, ARA&A)



Sanderson et al. 2006, MNRAS

Cooling rate (Bremsstrahlung):

$$\rho\mathcal{L} \propto n^2 T_e^{1/2}$$

Cooling time:

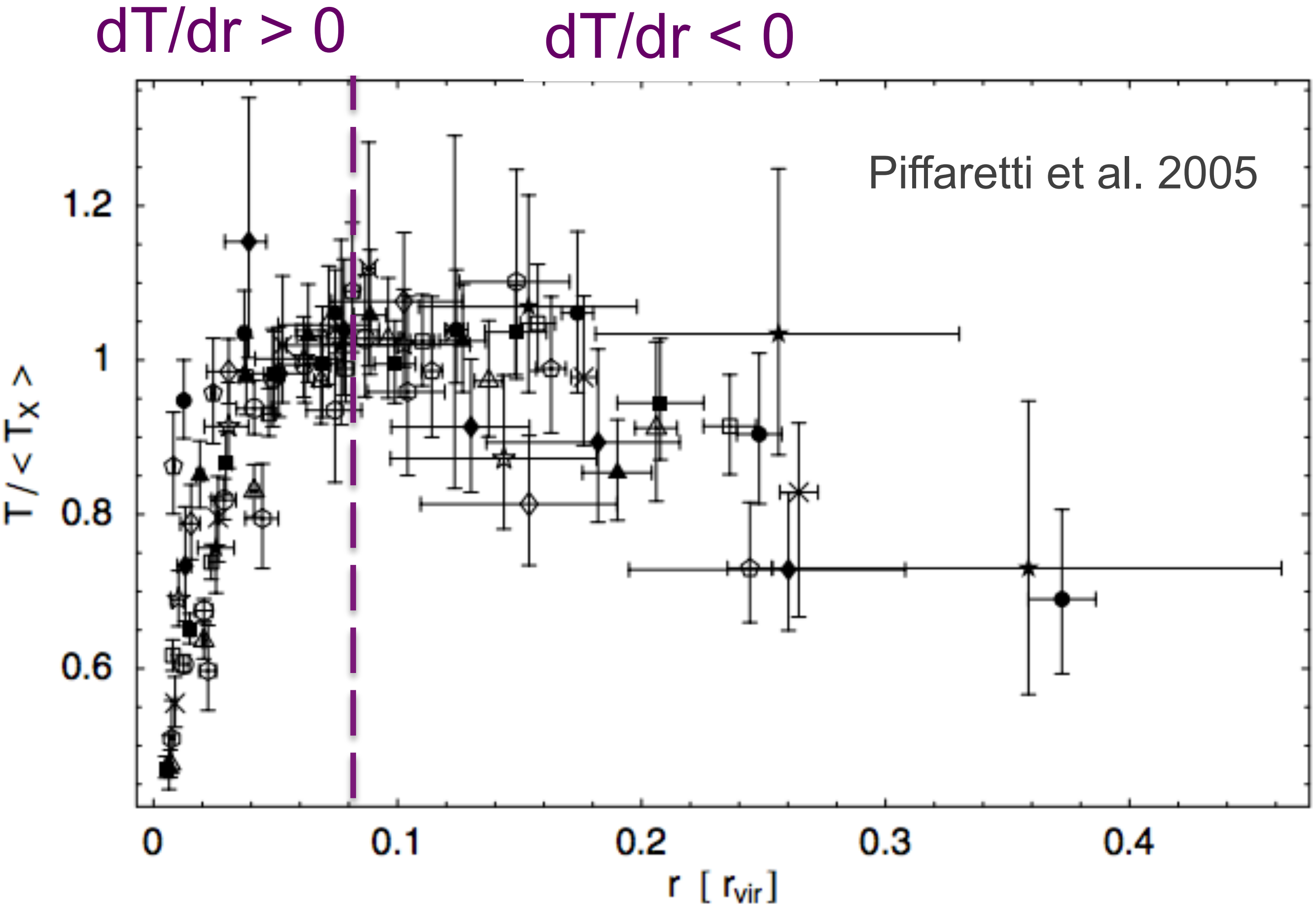
$$t_{\rm cool} \sim \frac{nkT}{\rho\mathcal{L}} \propto \frac{T^{1/2}}{n}$$

Runaway: the more it cools, it cools faster!

Observations infer short cooling time, but no strong cooling flows!

(but see McDonald et al. 2012, nature)

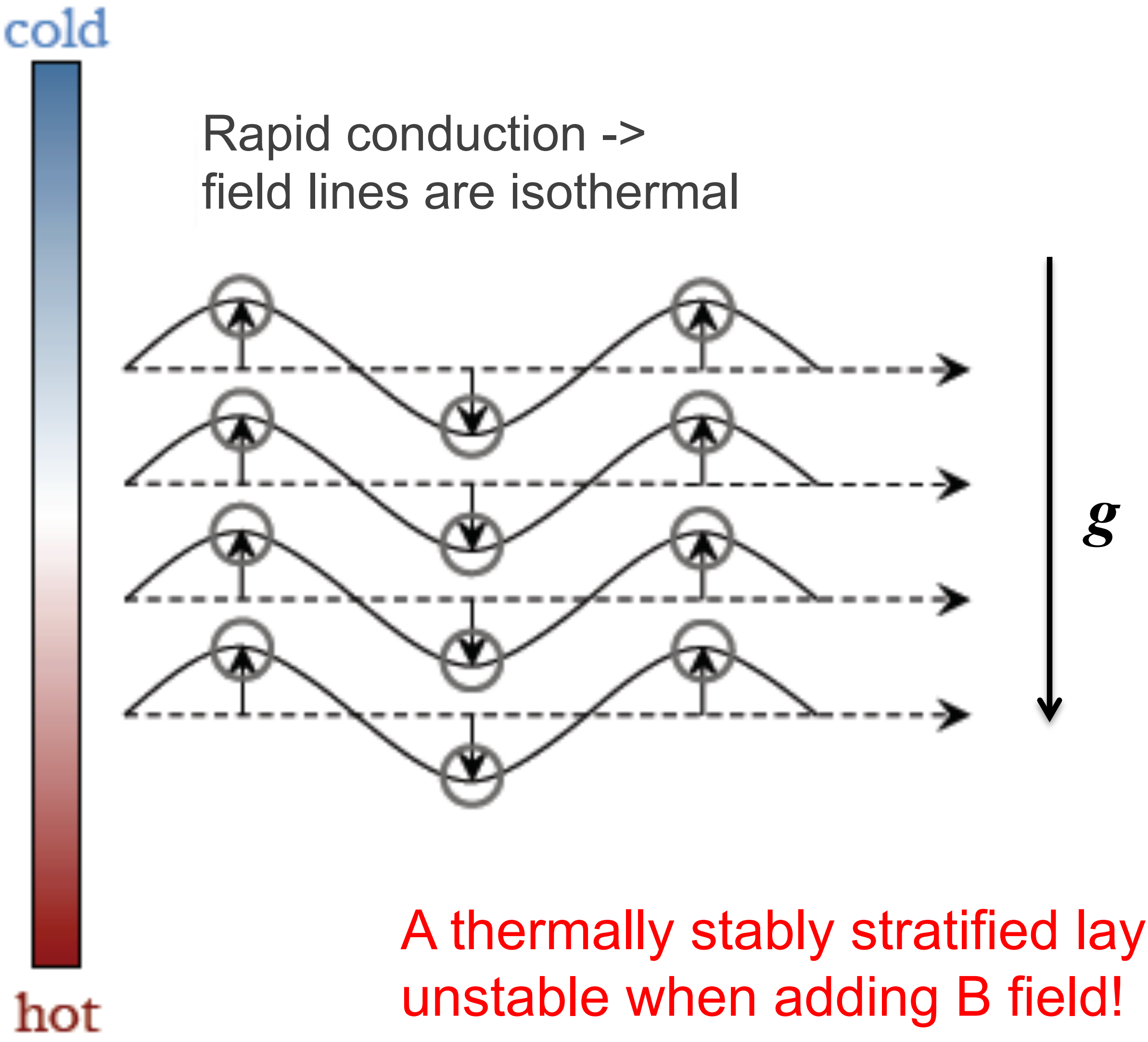
Temperature profiles (cool-core clusters)



Expect: anisotropic heat conduction $\boldsymbol{Q} = -\kappa \boldsymbol{b}\boldsymbol{b} \cdot \nabla T$

Magneto-thermal instability (MTI)

Balbus, 2000, 2001



Rapid conduction ->
field lines are isothermal

Courtesy: M. Kunz

Unstable when:

$$g\frac{d\ln T}{dz} > 0$$

Growth timescale:

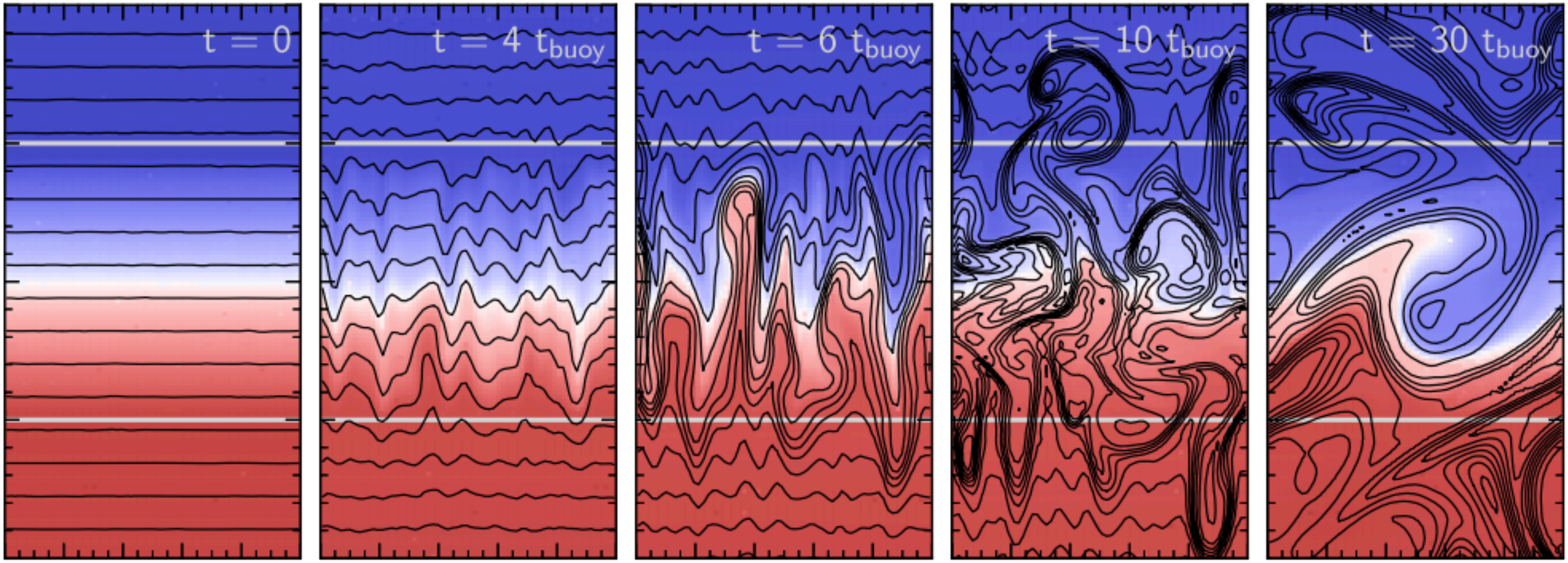
$$t_{\rm buoy} = \left(g\frac{\partial \ln T}{\partial z}\right)^{-1/2}$$

A thermally stably stratified layer becomes buoyantly unstable when adding B field!

Applicable to the outer region of galaxy clusters.

Saturation of the MTI

Local simulations with anisotropic heat conduction (see also Parrish & Stone, 2005)



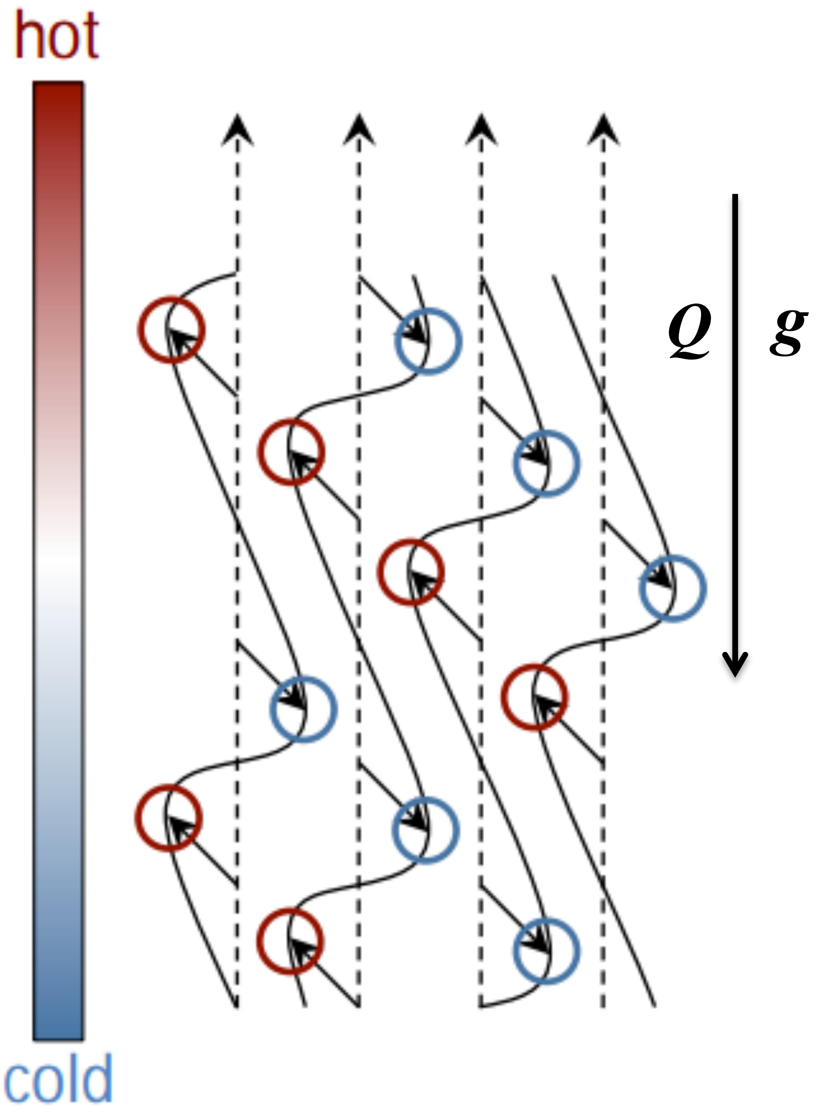
McCourt et al. 2011

Leads to sonic turbulence and convection with efficient heat transport.

In reality, the outcome should depend on the global thermal state not captured in local simulations.

Heat-flux buoyancy instability (HBI)

Quataert 2008



Downward displaced fluid sees field line (and hence heat flux) diverging -> cools; and vice versa

Unstable when:

$$g\frac{d\ln T}{dz} < 0 \quad \text{+ vertical field}$$

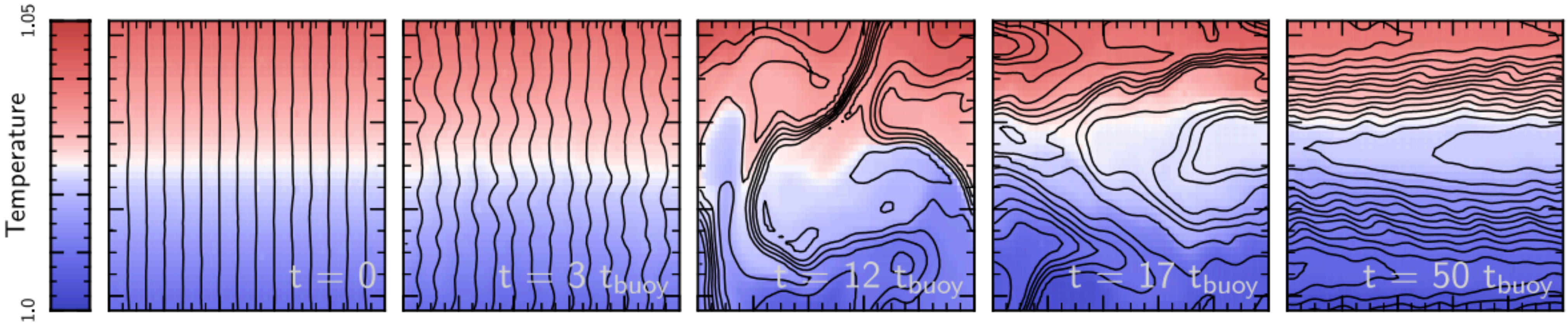
Growth timescale: same as MTI

Courtesy: M. Kunz

Applicable to the core region of cool-core clusters.

Saturation of the HBI

Local simulations with anisotropic heat conduction (see also Parrish & Quataert, 2008)



McCourt et al. 2011

Saturation by re-orienting magnetic field lines to preferentially horizontal configuration.

➡ Heat conduction is suppressed -> self-quenching of the HBI

Implication: HBI makes the cooling flow problem even more serious…

ICM physics: further complications

- Braginskii viscosity + anisotropic thermal conduction:

  HBI is suppressed, MTI is strengthened. (Kunz 2011)

- The ICM is turbulent.
- Braginskii MHD not quite applicable in the outer ICM

- Feedback from the central AGN? (radiation, wind, jet, bubbles, etc.)
- Mass accretion from outskirt
- Role of galaxy cluster mergers?
- Role of cosmic-rays?

Summary:

- A lot of astrophysical plasmas are weakly collisional.
- Development of pressure anisotropy from μ conservation.
- Micro-instabilities from anisotropic pressure

  Firehose when parallel pressure dominates

  Mirror and/or ion-cyclotron when perpendicular pressure dominates

- Properties of the MRI at low collisionalities

  Magneto-viscous instability: grows faster than the MRI at small scale

  Enhanced angular momentum transport.

- Instabilities in the intracluster medium driven by anisotropic heat conduction.

  Magneto-thermal instability when T decreases with height

  Heat-flux driven buoyancy instability when T increases with height