



# THICK ACCRETION DISK

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2017/11/10

GUIDED BY BAI XUENING

# CONTENT



- **Basic model**
- **Thick disk**
- **Properties**
- **Discussion**

The background is a dark blue gradient with a field of small, light blue stars. On the right side, there are several technical diagrams. One is a large circular scale with numerical markings from 80 to 210 and a dashed arrow pointing counter-clockwise. Below it is another circular diagram with a dashed arrow pointing clockwise. In the bottom left corner, there are more circular diagrams, including one with a dashed arrow pointing clockwise.

PART I

# BASIC MODEL OF AN ACCRETION DISK

# WHAT IS ACCRETION DISK

- Central massive body
  - Protostars
  - Compact objects (white dwarf, Neutron star and black holes)
- Surrounding material

# HOW TO WORK?



By NASA

## SCALE (H/R)

Thin disk  $\ll 1$

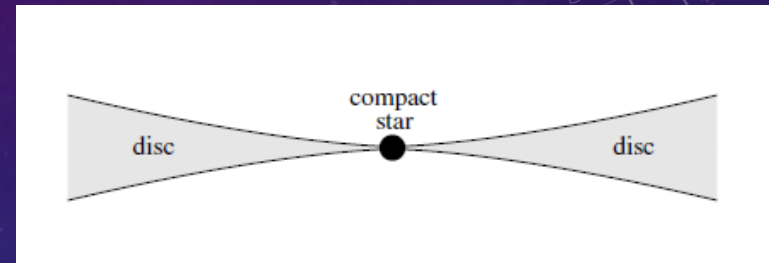
Thick disk  $\sim 1$

$\Rightarrow \rho = \rho_0 \exp\left(-\frac{z^2}{2H^2}\right)$  and  $H = \frac{c_s}{\Omega} \sim$  vertical scale height.

$\Omega_k = \sqrt{\frac{GM}{r^3}} \sim$  Keplerian angular velocity

$$\frac{H}{R} = \frac{c_s}{\Omega R} \sim \frac{c_s}{v_k} \sim \frac{v_{th}}{v_k}$$

$c_s$  is the velocity of sound.



# BASIC FUNCTIONS

- Continuity equation
- Momentum equilibrium at radius
- Angular momentum equilibrium

# CONTINUITY EQUATION

- Mass flux conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$



## CONT.

- Cylindrical coordinate:

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial R} (\Sigma v_r R) = 0$$

Where  $\Sigma = \int \rho dz = 2\rho H$  is defined as surface density.

- For a stable system:  $\frac{\partial}{\partial t} = \frac{\partial}{\partial \varphi} = 0$
- For the simplified equation is shown as:

$$\frac{\partial}{\partial R} (\rho R H v) = 0$$

# MOMENTUM EQUILIBRIUM AT R DIRECTION

- Cylindrical coordinate
- Thin disk :  $\Omega^2 R = \Omega_k^2 R$
- Thick disk:

$$v \frac{dv}{dR} - \Omega^2 R = -\Omega_k^2 R - \frac{1}{\rho} \frac{d}{dR} (\rho c_s^2)$$

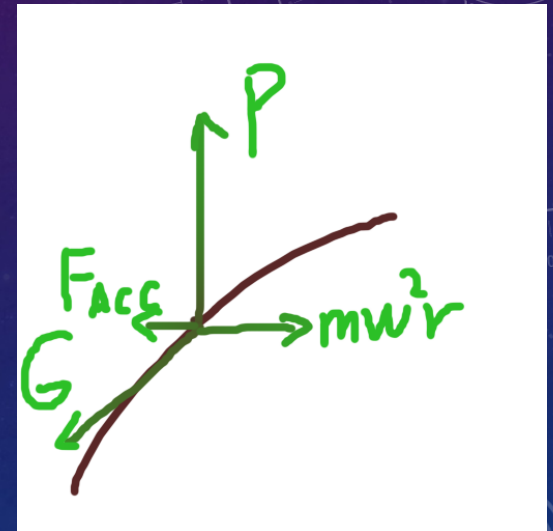
Accretion

centrifugal force

gravity

pressure

- Advection-dominated accretion disk(ADAF)



# ANGULAR MOMENTUM EQUILIBRIUM

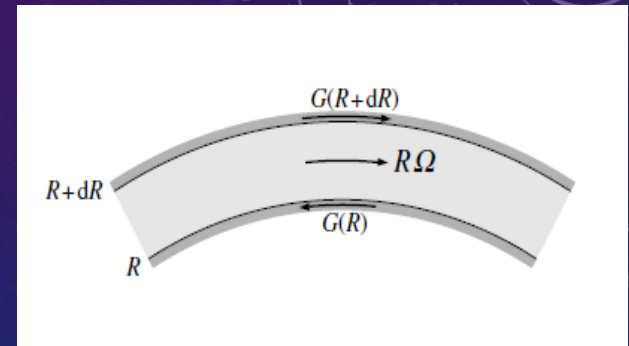
- Cylindrical with  $\phi$  (AM by viscosity)

$$\Sigma v R \frac{\partial}{\partial R} (\Omega R^2) = \frac{\partial}{\partial R} (\Sigma v R \frac{R \partial \Omega}{\partial R} R)$$

Angular momentum flux changed in  $2\pi R dR$  and viscosity

We also use constant  $\alpha$  describing the effect of viscosity

$$v = \alpha c_s H = \frac{\alpha c_s^2}{\Omega_k}$$





PART II

**THICK DISK**

# ENERGY EQUATION

$$\Sigma v T \frac{ds}{dR} = Q^+ - Q^-$$

- Thin disk

$$Q^- = 4\pi R F^-$$

when optically thick and geometry thin

optically thin?  $\Rightarrow \tau \ll 1$

# ENERGY EQUATION

Optically thin->

- Free-free emission (braking radiation)
- Emissivity  $\sim N_e^2 \sim \rho^2 \sim \Sigma^2$

Optically thick->

- Blackbody radiation
- Emissivity  $\sim \sigma T^4 \sim \dot{M} \sim \Sigma$

# ENERGY EQUATION

If no cooling, just viscosity!

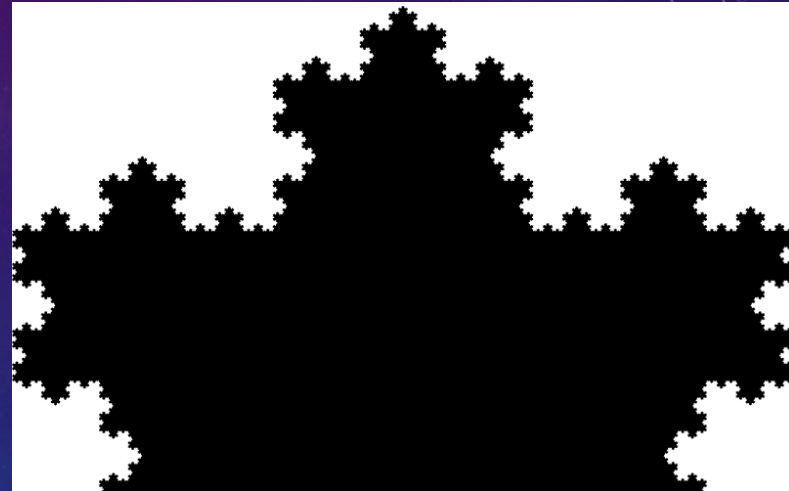
$$\Sigma v T \frac{ds}{dR} = Q^+ = \nu \left( R \frac{d\Omega}{dR} \right)^2$$

Using  $Tds = C_V dT + pdV$

$$\frac{3 + 3\epsilon}{2} 2\rho H \nu \frac{dc_s^2}{dR} - 2c_s^2 H \nu \frac{d\rho}{dR} = Q^+$$

# SOLVE : SELF-SIMILAR SOLUTION

- $\frac{\partial}{\partial R}(\rho R H v) = 0$
- $v \frac{dv}{dR} - \Omega^2 R = -\Omega_k^2 R - \frac{1}{\rho} \frac{d}{dR}(\rho c_S^2)$
- $\Sigma v_r R \frac{\partial}{\partial r}(\Omega^2 R) = \frac{\partial}{\partial r}(\Sigma v R^3 \frac{\partial \Omega}{\partial r})$
- $T \frac{ds}{dR} = v \left( r \frac{d\Omega}{dr} \right)^2$



By wiki

Solve the equations with varieties relative to  $R$ , we now have

$$\rho \propto R^{-\frac{3}{2}} \quad v \propto R^{-\frac{1}{2}} \quad \Omega \propto R^{-\frac{3}{2}} \quad c_S^2 \propto R^{-1}$$



# INTERESTING RESULTS

## Thick Disk

- $v \approx -\frac{3\alpha}{5} v_k$
- $\Omega \approx 0$
- $\frac{c_s^2}{v_k^2} \approx \frac{2}{5}$
- $\dot{M} \approx -4\pi R H v \rho = \text{const}$

## Thin Disk

- $v \sim \alpha \left(\frac{H}{R}\right)^2 v_k$
- $\Omega \approx \Omega_k$
- $\frac{c_s^2}{v_k^2} = \left(\frac{H}{R}\right)^2 \ll 1$

# INTERESTING RESULTS

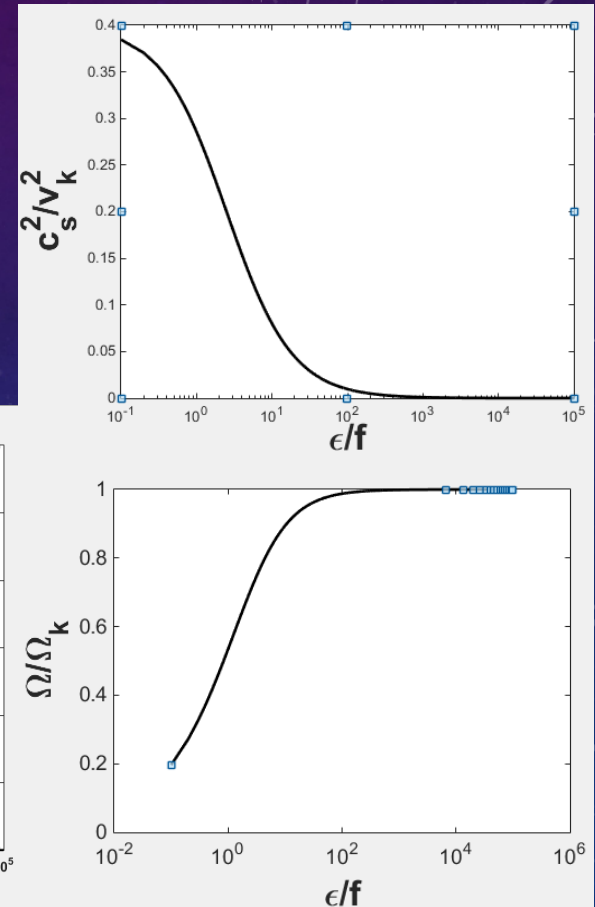
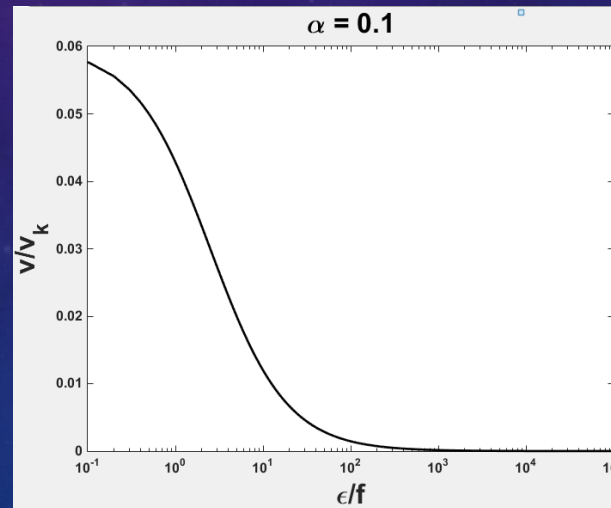
Cooling does exist!

- $Q^+ - Q^- = fQ^+$ ,  $f = 0$  represent efficient cooling and  $f = 1$  is no radiative cooling.
- $C_V$  is relativistic! Use a function of  $\epsilon = 0$  represent no relativity and  $\epsilon = 1$  represent relativity.
- Using  $\epsilon' = \frac{\epsilon}{f} \sim$  a critical role in determining the nature of flow.

# INTERESTING RESULTS

- $v \approx -\frac{3\alpha}{5+2\epsilon'} v_k$
- $\Omega \approx \frac{2\epsilon'}{5+2\epsilon'} \frac{1}{2} \Omega_k$
- $c_s^2 \approx \frac{2\epsilon'}{5+2\epsilon'} v_k^2$
- $\frac{H}{R} \sim 1 = \frac{c_s}{v_k}$

- Thin disk  $\epsilon' \rightarrow \infty$
- ADAF  $\epsilon' < 1$



The background is a dark blue gradient with a starry field of small white dots. On the right side, there are several technical diagrams. One is a large circular scale with numbers from 80 to 210 and a dashed arrow pointing clockwise. Another is a smaller circular scale with numbers from 100 to 140 and a dashed arrow pointing clockwise. There are also some dashed lines and other faint circular elements scattered across the background.

PART III

PROPERTIES OF THICK DISK

# BERNOULLI CONSTANT

- Normlized Bernoulli constant (Energy conserved)

$$b = \frac{1}{v_k^2} \left( \frac{1}{2} v^2 + \frac{1}{2} \Omega^2 R^2 - \Omega_k^2 R^2 + \frac{\gamma}{\gamma - 1} c_s^2 \right) = \frac{3\epsilon - \epsilon'}{5 + 2\epsilon'}$$

$b > 0 \rightarrow$  gas reach infinity ( $\Omega \sim R^{-\frac{3}{2}}, v \sim R^{-\frac{1}{2}}$ )

$b < 0 \rightarrow$  cannot spontaneously escape

With  $f > 1/3, \gamma < \frac{5}{3} \Rightarrow b > 0!$

- Winds? Jets?
- See more in Stone, Pringle(1999)

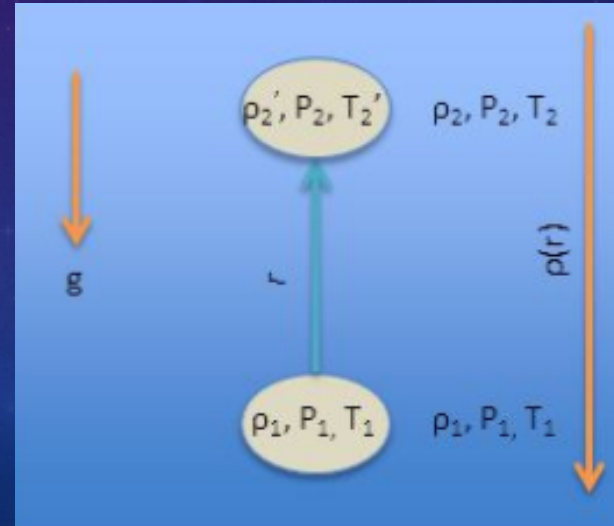
# BRUNT -VÄISÄLÄ FREQUENCY

- $N \sim$  Brunt-Väisälä frequency

$$\rho_0 \frac{\partial^2 z'}{\partial t^2} = -g(\rho(z) - \rho(z + z')) = -g \frac{\partial \rho(z)}{\partial z} z'$$

$$\Rightarrow z' = z'_0 e^{\sqrt{-N^2}t} \text{ and } N = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}}$$

$$\frac{\partial \rho}{\partial z} > 0 \rightarrow \text{not stable !}$$



# INSTABILITY

- Dynamical convective instability :

$$N_{eff}^2 = N^2 + \kappa^2$$

$N \sim$  Brunt- Väisälä frequency

$\kappa \sim$  epicyclic frequency

- Self-similar:  $N_{eff}^2 = \frac{10\epsilon' + 6\epsilon\epsilon' - 15\epsilon}{(5+3\epsilon)(5+2\epsilon')} \Rightarrow f > \frac{2}{3} + \frac{2}{5}\epsilon$
- Double diffusive instability



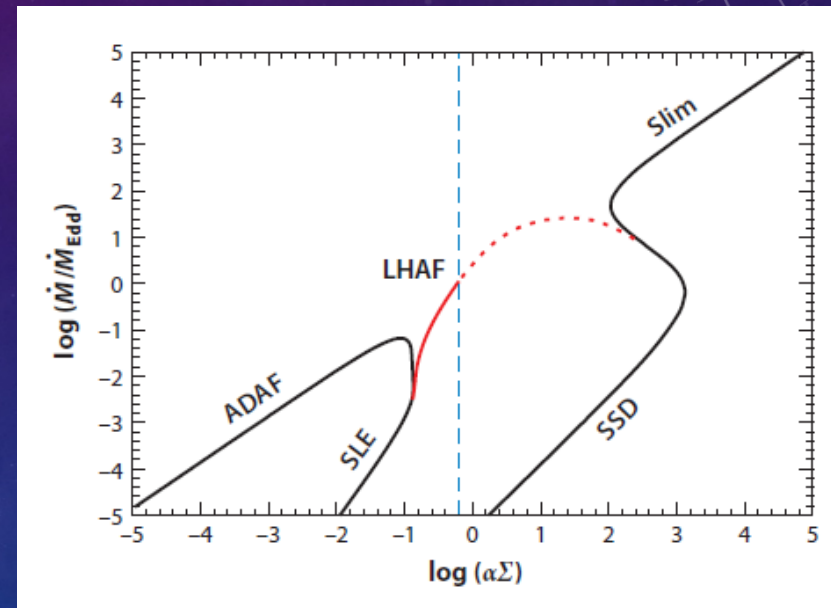
PART IV

**DISCUSSION**



# $\dot{M} > \dot{M}_E \Rightarrow \text{SLIM DISK}$

- $P = P_{gas} + P_{rad}$
- $Tds = dE_{viscosity} - dE_{radiation}$



• Yuan (2003)

# SUMMARY

- Non-standard model – ADAF is an optically thin geometry thick model and cooled by free-free emission which is not efficient comparing to the blackbody radiation.
- We use a self-similar solution which is only based on the radius to get the function of velocity, density, angular velocity and sound speed.
- There may be some instability in the simulation but the observation are all stable up to now.

The background is a dark blue gradient with a subtle starry or particle effect. On the right side, there are faint, light blue technical diagrams. One prominent diagram is a circular gauge or scale with numerical markings from 80 to 210. Other diagrams include concentric circles and dashed lines with arrows, suggesting a technical or scientific theme.

**THANK YOU!**