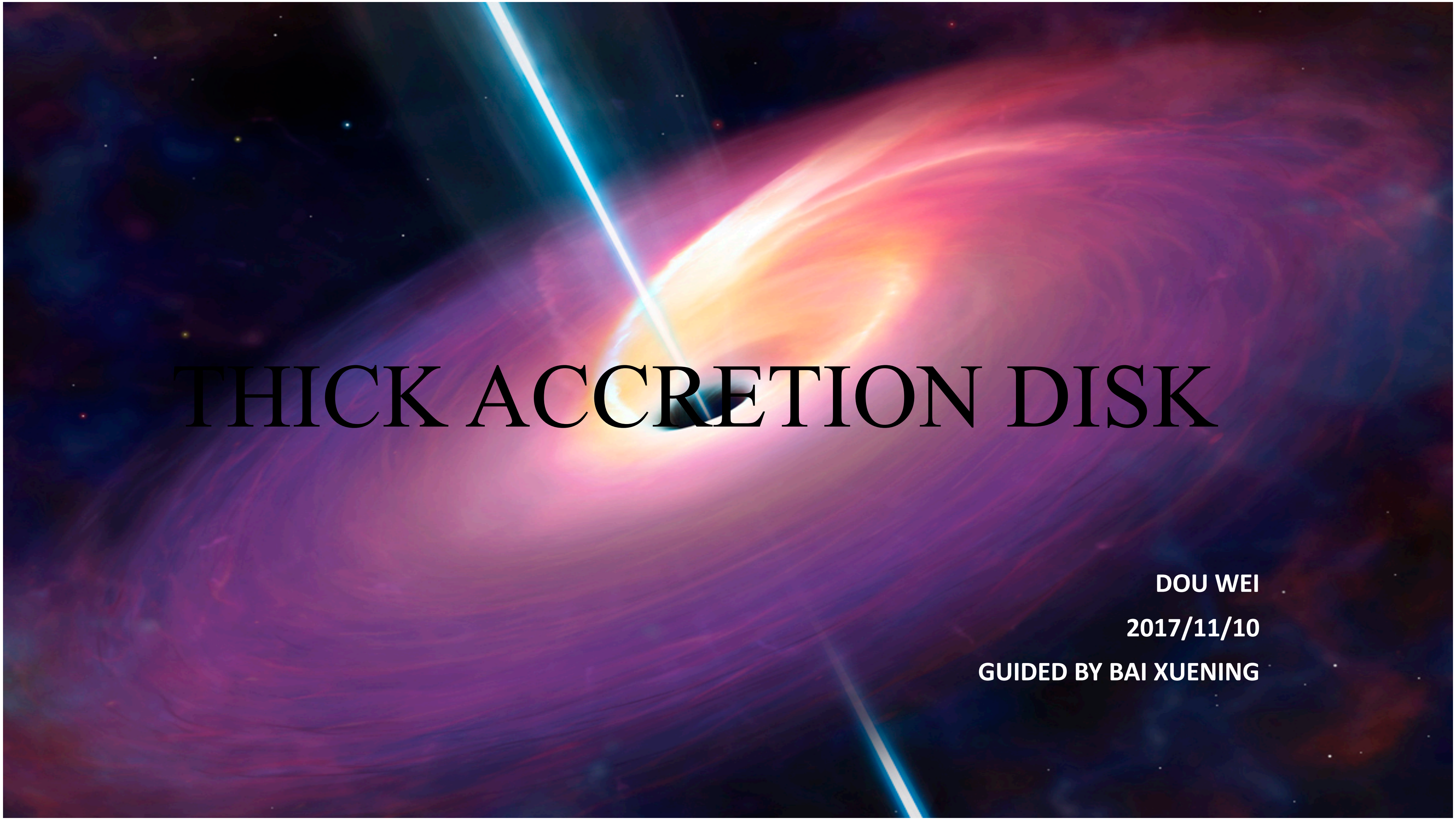
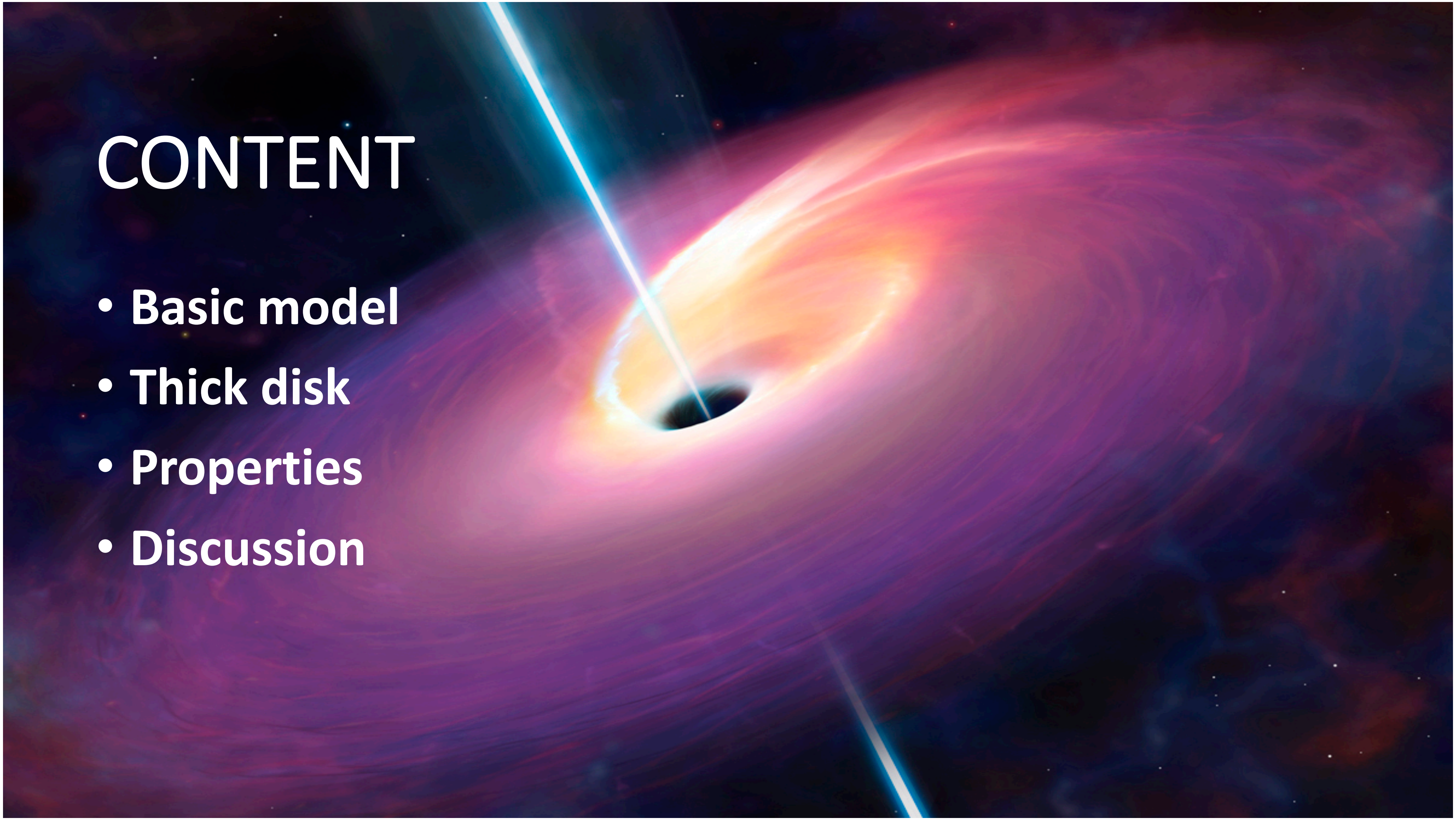


THICK ACCRETION DISK

DOU WEI

2017/11/10

GUIDED BY BAI XUENING


CONTENT
• Basic model
• Thick disk
• Properties
• Discussion

PART I

BASIC MODEL OF AN ACCRETION DISK

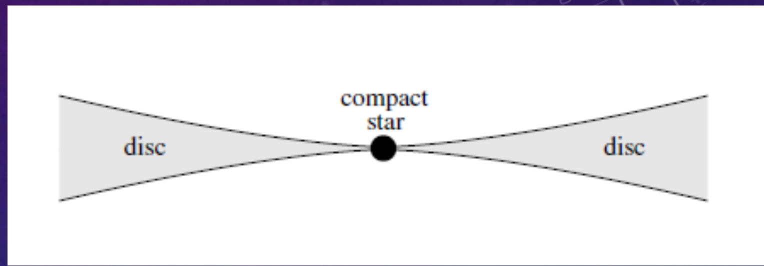
WHAT IS ACCRETION DISK

- Central massive body
  - Protostars
  - Compact objects (white drawf, Neutron star and black holes)
- Surrounding material

HOW TO WORK?

By NASA

SCALE (H/R)



Thin disk << 1

Thick disk ~ 1

$\Rightarrow \rho = \rho_0 \exp(-\frac{z^2}{2H^2})$ and $H = \frac{c_s}{\Omega} \sim$ vertical scale height.

$\Omega_k = \sqrt{\frac{GM}{r^3}}$ ~Keplerian angular velocity

$$\frac{H}{R} = \frac{c_s}{\Omega R} \sim \frac{c_s}{v_k} \sim \frac{v_{th}}{v_k}$$

c_s is the velocity of sound.

BASIC FUNCTIONS

- Continuity equation
- Momentum equilibrium at radius
- Angular momentum equilibrium

CONTINUITY EQUATION

- Mass flux conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

CONT.

- Cylindrical coordinate:

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial R}(\Sigma v_r R) = 0$$

Where $\Sigma = \int \rho dz = 2\rho H$ is defined as surface density.

- For a stable system: $\frac{\partial}{\partial t} = \frac{\partial}{\partial \varphi} = 0$
- For the simplified equation is shown as:

$$\frac{\partial}{\partial R}(\rho R H v) = 0$$

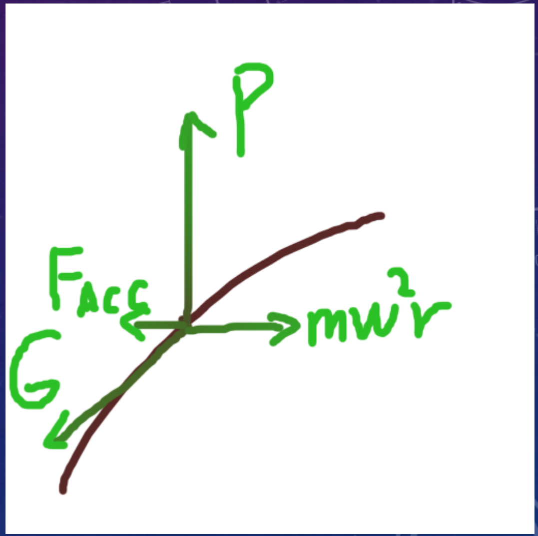
MOMENTUM EQUILIBRIUM AT R DIRECTION

- Cylindrical coordinate
- Thin disk : $\Omega^2 R = \Omega_k^2 R$
- Thick disk:

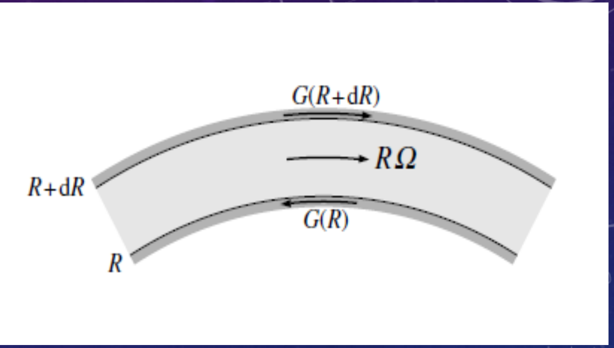
$$v\frac{\mathrm{d}v}{\mathrm{d}R} - \Omega^2 R = -\Omega_k^2 R - \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}R}(\rho c_S^2)$$

Accretion centrifugal force gravity pressure

- Advection-dominated accretion disk(ADAF)



ANGULAR MOMENTUM EQUILIBRIUM



- Cylindrical with ϕ (AM by viscosity)

$$\Sigma v R \frac{\partial}{\partial R}(\Omega R^2) = \frac{\partial}{\partial R}\left(\Sigma \nu R \frac{R\partial\Omega}{\partial R} R\right)$$

Angular momentum flux changed in $2\pi R \mathrm{d}R$ and viscosity

We also use constant α describing the effect of viscosity

$$\nu = \alpha c_s H = \frac{\alpha c_s^2}{\Omega_k}$$

PART II

THICK DISK

ENERGY EQUATION

$$\Sigma \nu T \frac{\mathrm{d}s}{dR} = Q^{+} - Q^{-}$$

- Thin disk

$$Q^{-} = 4\pi R F^{-}$$

when optically thick and geometry thin

optically thin? $\Rightarrow \tau \ll 1$

ENERGY EQUATION

Optically thin->

- Free-free emission (braking radiation)
- Emissivity $\sim N_e^2 \sim \rho^2 \sim \Sigma^2$

Optically thick->

- Blackbody radiation
- Emissivity $\sim \sigma T^4 \sim \dot{M} \sim \Sigma$

ENERGY EQUATION

If no cooling, just viscosity!

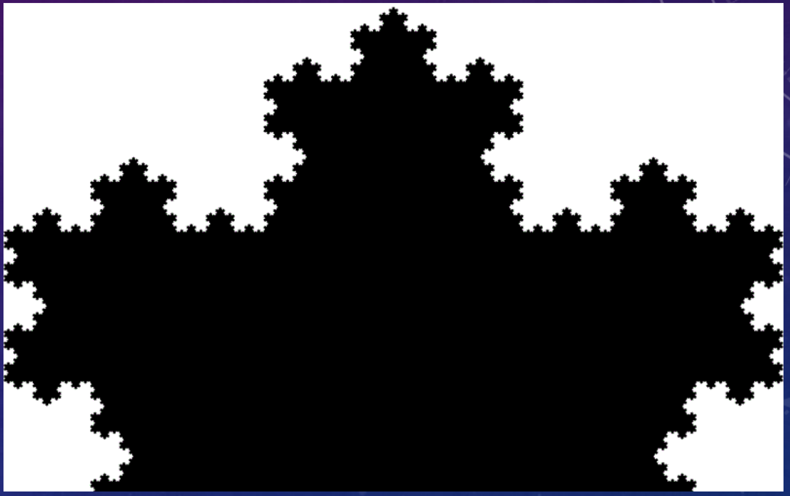
$$\Sigma v T \frac{\mathrm{d}s}{dR} = Q^{+} = \nu \left(R \frac{\mathrm{d}\Omega}{\mathrm{d}R} \right)^{2}$$

Using $T\mathrm{d}s = C_V \mathrm{d}T + p\mathrm{d}V$

$$\frac{3+3\epsilon}{2} 2\rho H v \frac{dc_s^2}{dR} - 2c_s^2 H v \frac{d\rho}{dR} = Q^{+}$$

SOLVE : SELF-SIMILAR SOLUTION

- $\frac{\partial}{\partial R}(\rho R H v) = 0$
- $v\frac{\mathrm{d}v}{\mathrm{d}R} - \Omega^2 R = -\Omega_k^2 R - \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}R}(\rho c_s^2)$
- $\Sigma v_r R \frac{\partial}{\partial r}(\Omega^2 R) = \frac{\partial}{\partial r}(\Sigma \nu R^3 \frac{\partial \Omega}{\partial r})$
- $T\frac{\mathrm{d}s}{dR} = \nu\left(r\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right)^2$



By wiki

Solve the equations with varieties relative to R, we now have

$$\rho \propto R^{-\frac{3}{2}} \qquad v \propto R^{-\frac{1}{2}} \qquad \Omega \propto R^{-\frac{3}{2}} \qquad c_s^2 \propto R^{-1}$$

INTERESTING RESULTS

Thick Disk

- $v \approx -\frac{3\alpha}{5} v_k$
- $\Omega \approx 0$
- $\frac{c_s^2}{v_k^2} \approx \frac{2}{5}$
- $\dot{M} \approx -4\pi R H v \rho = const$

Thin Disk

- $v \sim \alpha \left(\frac{H}{R}\right)^2 v_k$
- $\Omega \approx \Omega_k$
- $\frac{c_s^2}{v_k^2} = \left(\frac{H}{R}\right)^2 \ll 1$

INTERESTING RESULTS

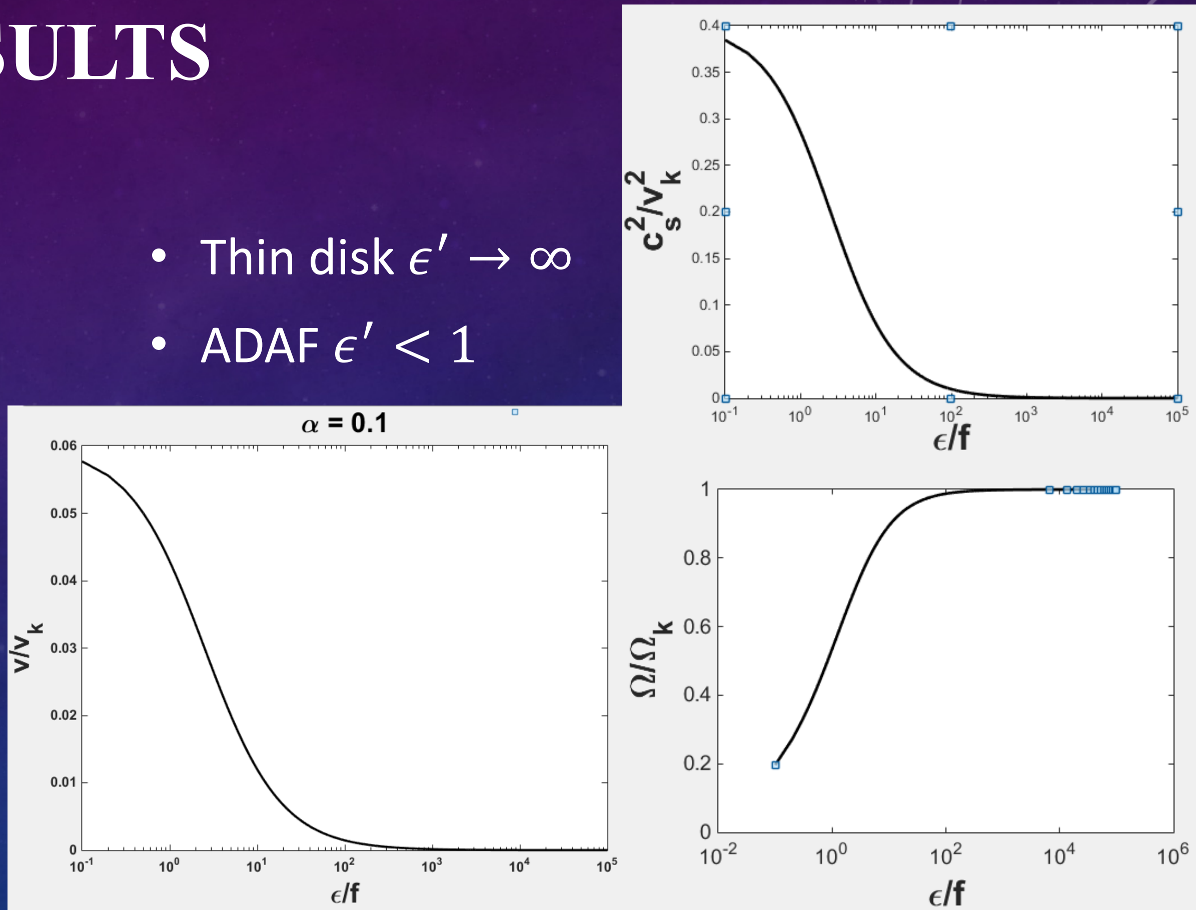
Cooling does exist!

- $Q^+ - Q^- = fQ^+, f = 0$ represent efficient cooling and $f = 1$ is no radiative cooling.
- C_V is relativistic! Use a function of $\epsilon = 0$ represent no relativity and $\epsilon = 0$ represent relativity.
- Using $\epsilon' = \frac{\epsilon}{f} \sim$ a critical role in determining the nature of flow.

INTERESTING RESULTS

- $v \approx -\frac{3\alpha}{5+2\epsilon'} v_k$
- $\Omega \approx \left(\frac{2\epsilon'}{5+2\epsilon'}\right)^{\frac{1}{2}} \Omega_k$
- $c_s^2 \approx \frac{2\epsilon'}{5+2\epsilon'} v_k^2$
- $\frac{H}{R} \sim 1 = \frac{c_s}{v_k}$

- Thin disk $\epsilon' \to \infty$
- ADAF $\epsilon' < 1$



PART III

PROPERTIES OF THICK DISK

BERNOULLI CONSTANT

- Normlized Bernoulli constant (Energy conserved)

$$b = \frac{1}{v_k^2}\left(\frac{1}{2}v^2 + \frac{1}{2}\Omega^2 R^2 - \Omega_k^2 R^2 + \frac{\gamma}{\gamma - 1}c_s^2\right) = \frac{3\epsilon - \epsilon'}{5 + 2\epsilon'}$$

$b > 0 \rightarrow$ gas reach infinity ($\Omega \sim R^{-\frac{3}{2}}, v \sim R^{-\frac{1}{2}}$)

$b < 0 \rightarrow$ cannot spontaneously escape

With $f > 1/3\,, \gamma < \frac{5}{3} \Rightarrow b > 0!$

- Winds? Jets?
- See more in Stone, Pringle(1999)

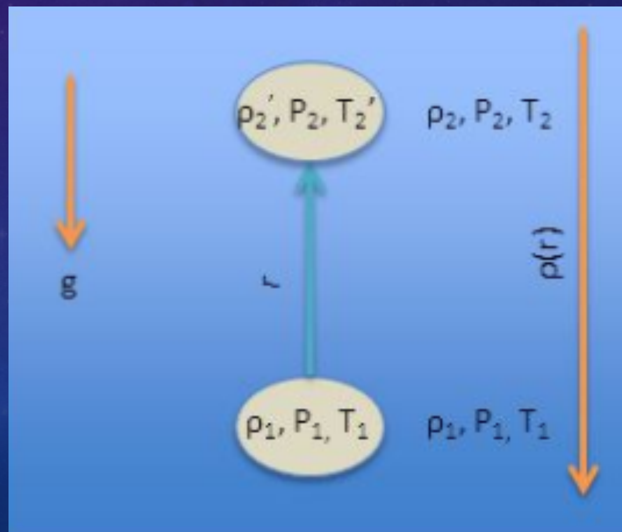
BRUNT -VÄISÄLÄ FREQUENCY

- N~Brunt- Väisälä frequency

$$\rho_0 \frac{\partial^2 z'}{\partial t^2} = -g\big(\rho(z) - \rho(z+z')\big) = -g\frac{\partial \rho(z)}{\partial z} z'$$

$\Rightarrow z' = z_0' e^{\sqrt{-N^2}t}$ and $N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$

$\frac{\partial \rho}{\partial z} > 0$ →not stable !



INSTABILITY

- Dynamical convective instability :

$$N_{eff}^2 = N^2 + \kappa^2$$

$N \sim$Brunt- Väisälä frequency

$\kappa \sim$ epicyclic frequency

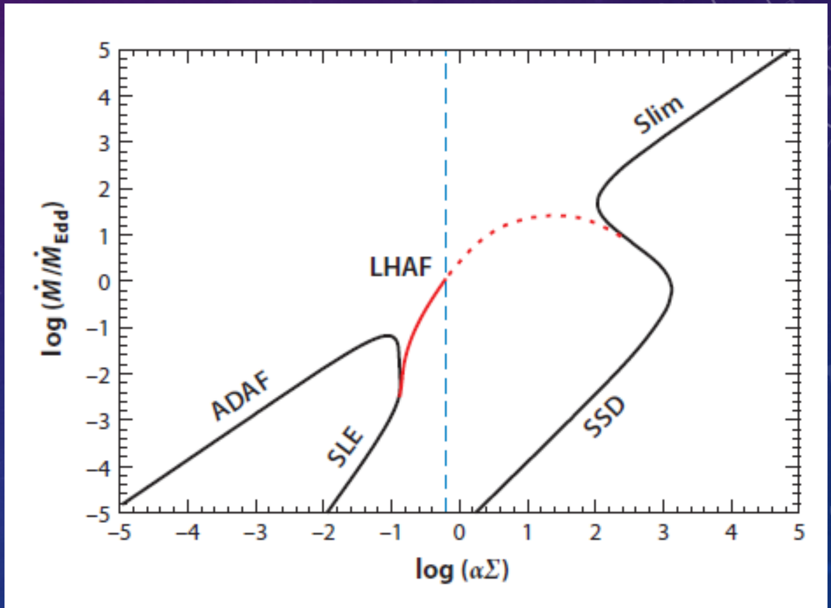
- Self-similar: $N_{eff}^2 = \frac{10\epsilon' + 6\epsilon\epsilon' - 15\epsilon}{(5+3\epsilon)(5+2\epsilon')} \Rightarrow f > \frac{2}{3} + \frac{2}{5}\epsilon$
- Double diffusive instability

PART IV

DISCUSSION

$\dot{M} > \dot{M}_E \Rightarrow$SLIM DISK

- $P = P_{gas} + P_{rad}$
- $Tds = dE_{viscosity} - dE_{radiation}$



- Yuan (2003)

SUMMARY

- Non-standard model – ADAF is an optically thin geometry thick model and cooled by free-free emission which is not efficient comparing to the blackbody radiation.
- We use a self-similar solution which is only based on the radius to get the function of velocity, density, angular velocity and sound speed.
- There may be some instability in the simulation but the observation are all stable up to now.

THANK YOU!