

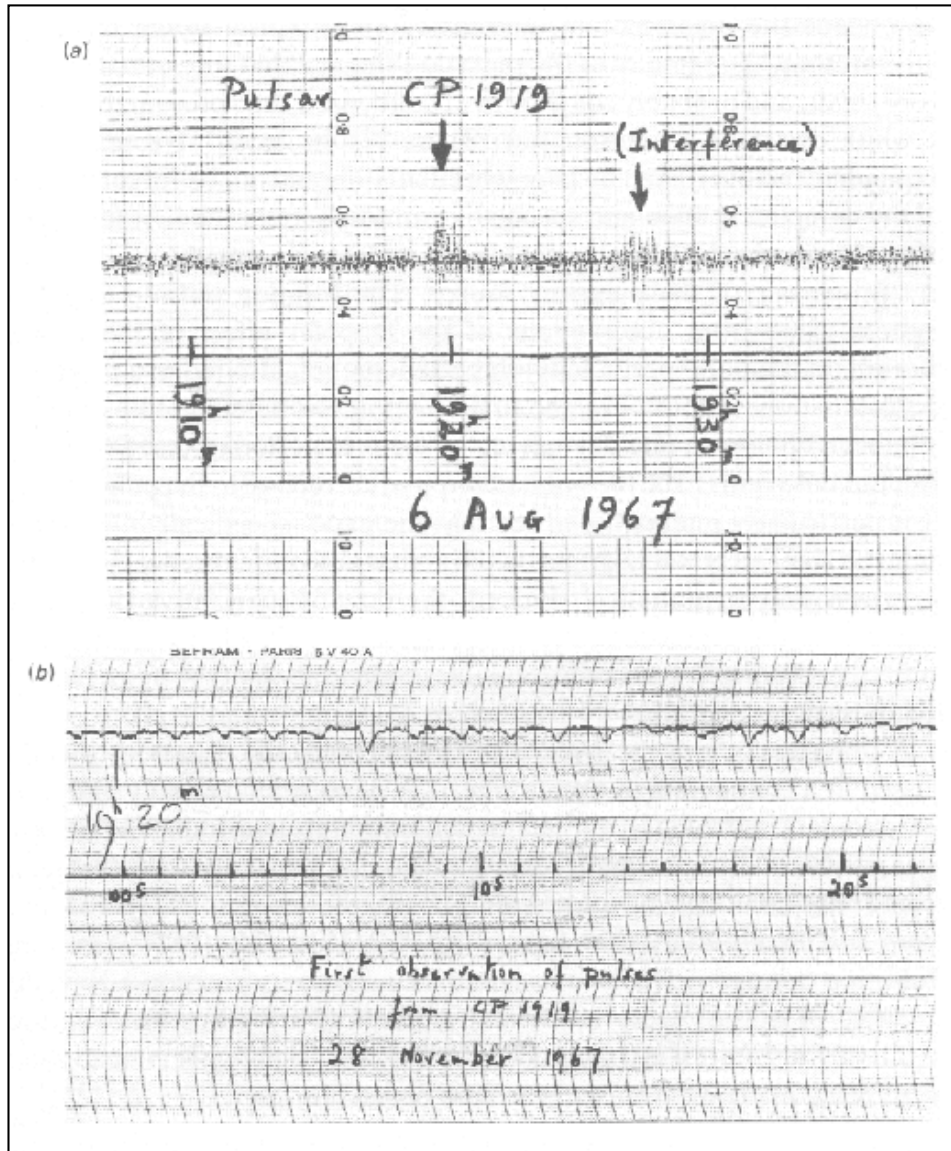
PULSAR ELECTRODYNAMICS

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- **Electric & Magnetic Fields around Pulsar**
- **Inclined Axis Properties**
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- **Summary**

Pulsar ABC



- First discovery: PSR B1919+21, November 28, 1967, by Jocelyn Bell Burnell and Antony Hewish.
- 1974 Nobel Prize



Pulsar ABC

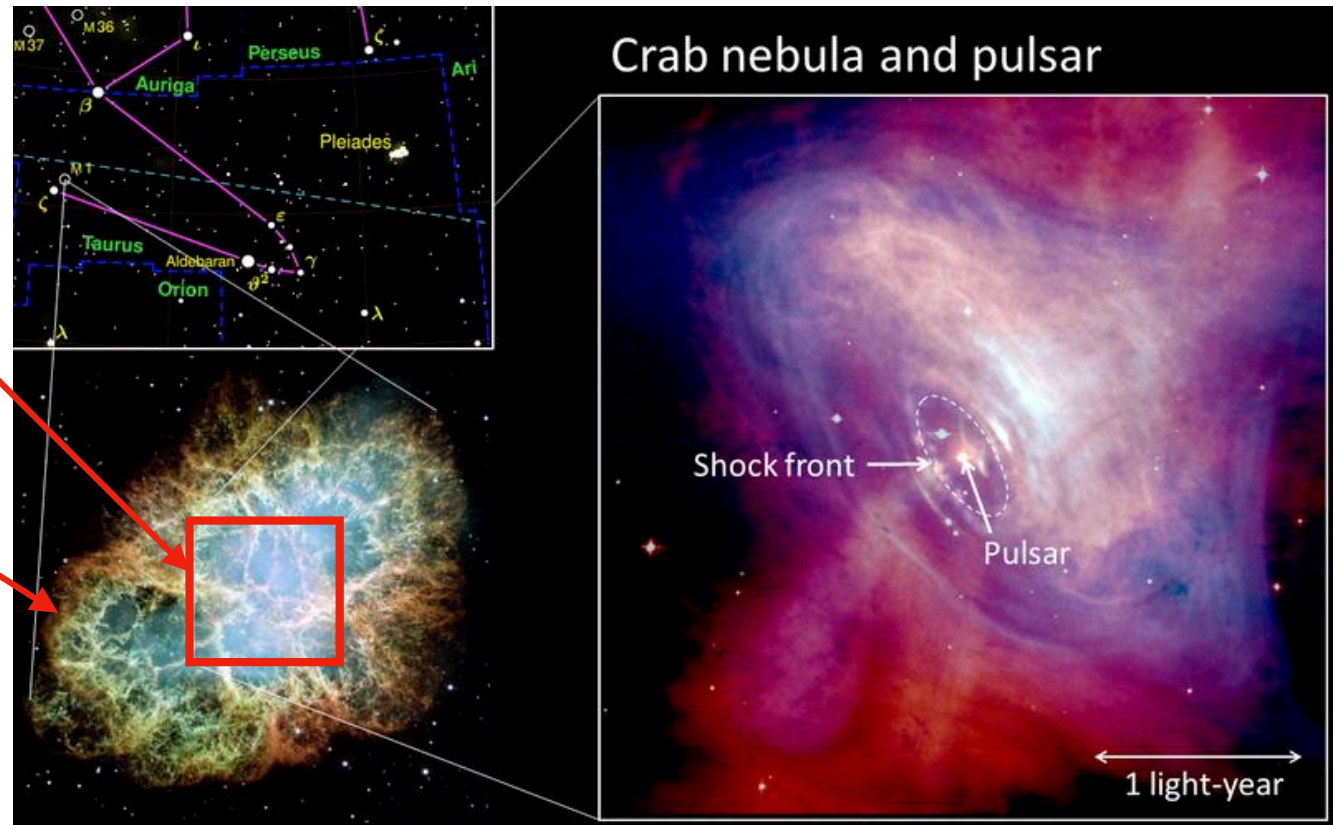
- **Pulsar Formation & Structure**

- Supernova -> neutron star -> pulsar



Pulsar Wind Nebula

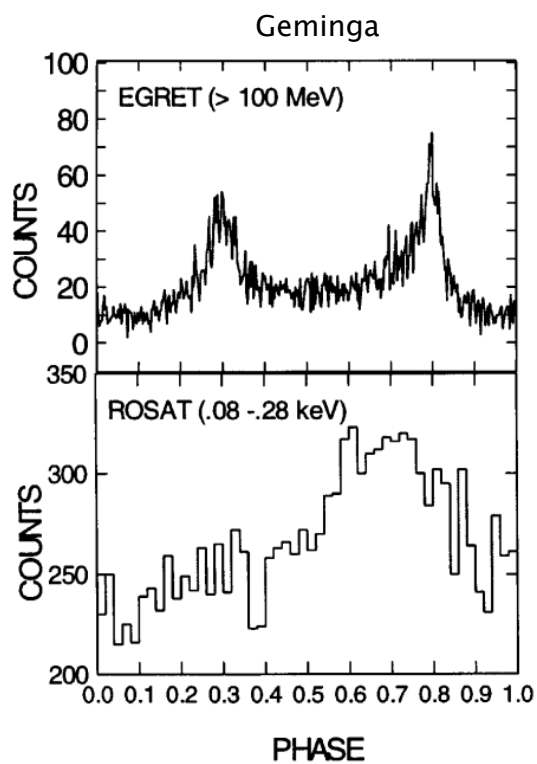
Supernova Remnant



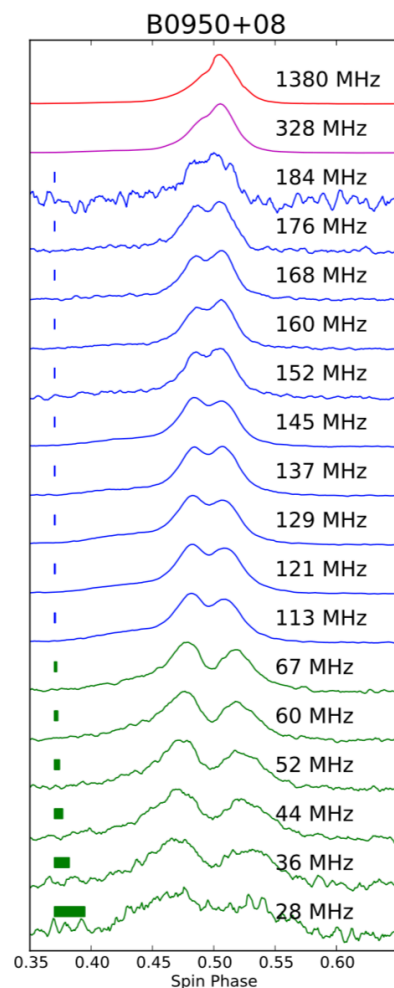
(Hubble & Chandra)

Pulsar ABC

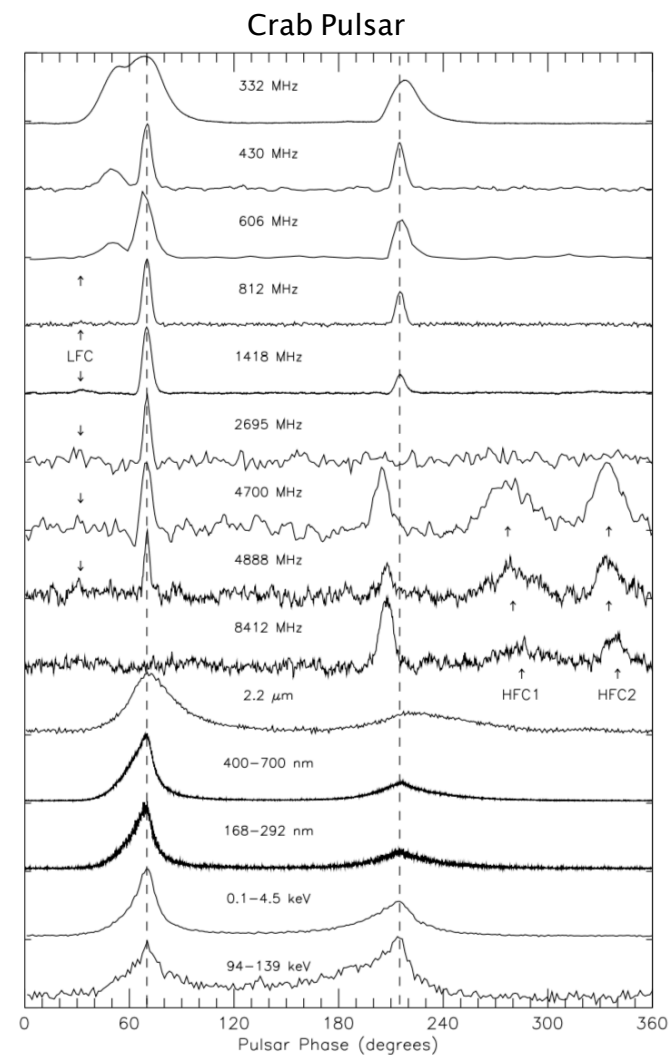
- Radio & Gamma Ray Profile



(Alice K. Harding and Alexander G. Muslimov, 1998)



(M. Pilia, J. W. T. Hessels, et.al, 2015)

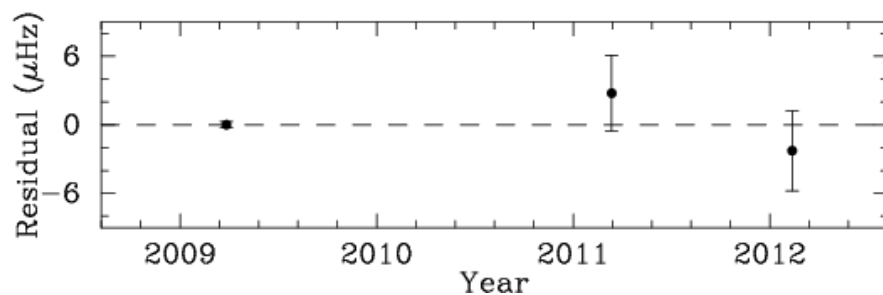
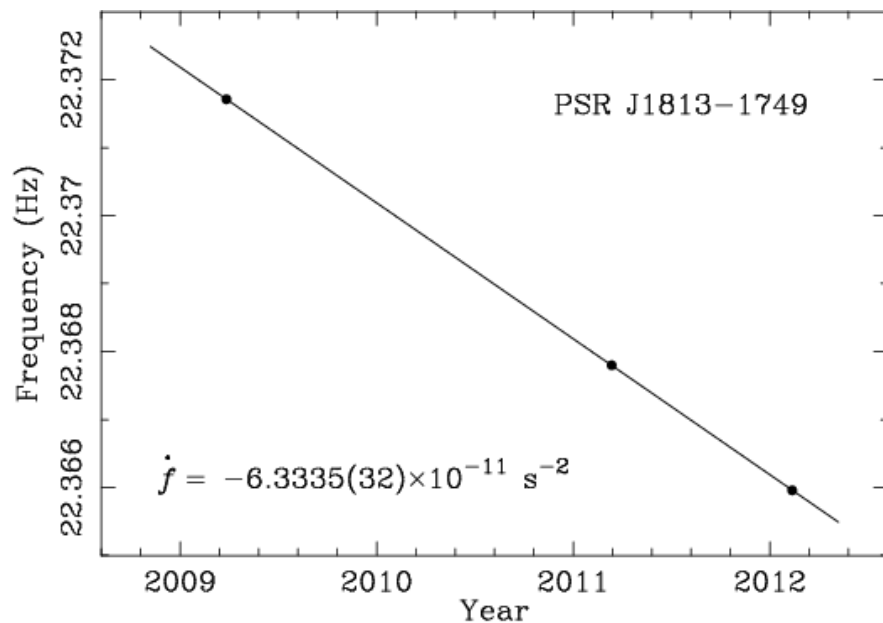


(J. A. Eilek & T. H. Hankins)

Pulsar ABC

- Spin down and pulsar timing

Crab Pulsar $P = 33.5028583$ ms,
 $P' = 3.7e-10$ Hz/s



(J.P. Halpern, E.V. Gotthelf, F. Camilo, 2012)

**Magnetic Dipole
Radiation Power**

$$P = \frac{\mu_0 \Omega^4 m_0^2}{12\pi c^3}$$

**Estimate Pulsar
Radiation Power or
Surface
Magnetic Field**

Pulsar ABC

- **Motivation: Why Pulsar?**

The most extraordinary physics laboratories in the Universe

density, gravity, magnetic and electric fields.

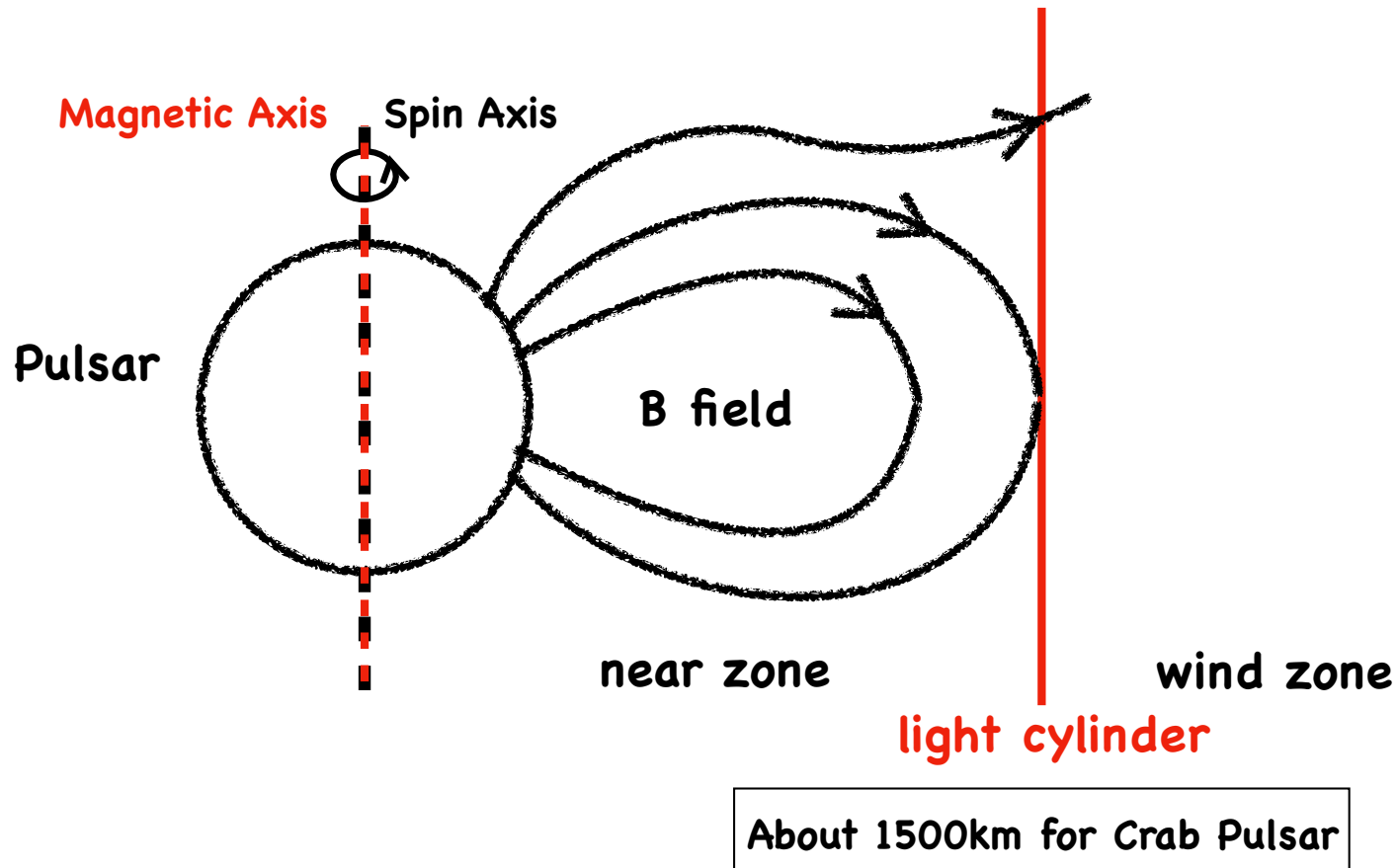
High Power & Luminosity Radiation

Open questions like: extreme equation of states, physics of strongly magnetized plasmas, radiation mechanisms, etc.

Gravitational Waves!

Electric & Magnetic Fields around Pulsar

- Structure of a Axisymmetric Pulsar



Electric & Magnetic Fields around Pulsar

- **Outside of Pulsar:**
 - interior boundary condition

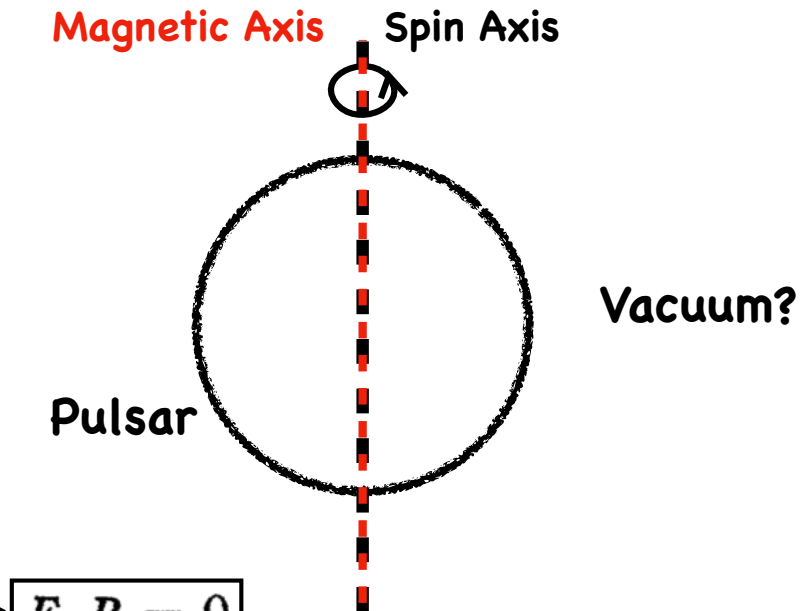
$$E + \frac{(\boldsymbol{\Omega} \times \mathbf{r})}{c} \times \mathbf{B} = 0$$

Outer Space Electric Potential

$$\Phi = \frac{-B_0 \Omega R^5}{3cr^3} P_2(\cos \theta)$$

~~Electric quadrupole induced by the Magnetic dipole~~

~~$$E \cdot B = - \left(\frac{\Omega R}{c} \right) \left(\frac{R}{r} \right)^7 B_0^2 \cos^3 \theta$$~~



(Peter Goldreich & William EL Julian, 1969)

Electric & Magnetic Fields around Pulsar

- Compare the EM force and the gravitational force

- for an electron:

$$8 \times 10^{11} B_{12} R_6^3 \cos^2 \theta / PM$$

e.g. For the **crab nebula**, B_{12} is $4 \times 10^{12}(\text{gauss})/10^{12}$, R_6 is about $10^6 \text{cm}/10^6$, for an **electron** $\sim 7 \times 10^{14}$.

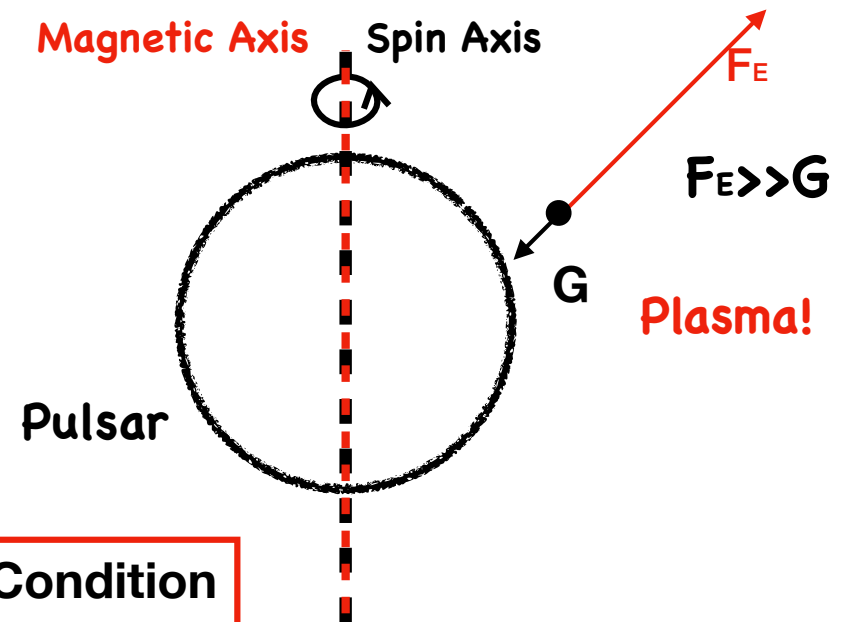
Conclusion:

Pulsar must be surrounded by plasma.

Force Free Condition

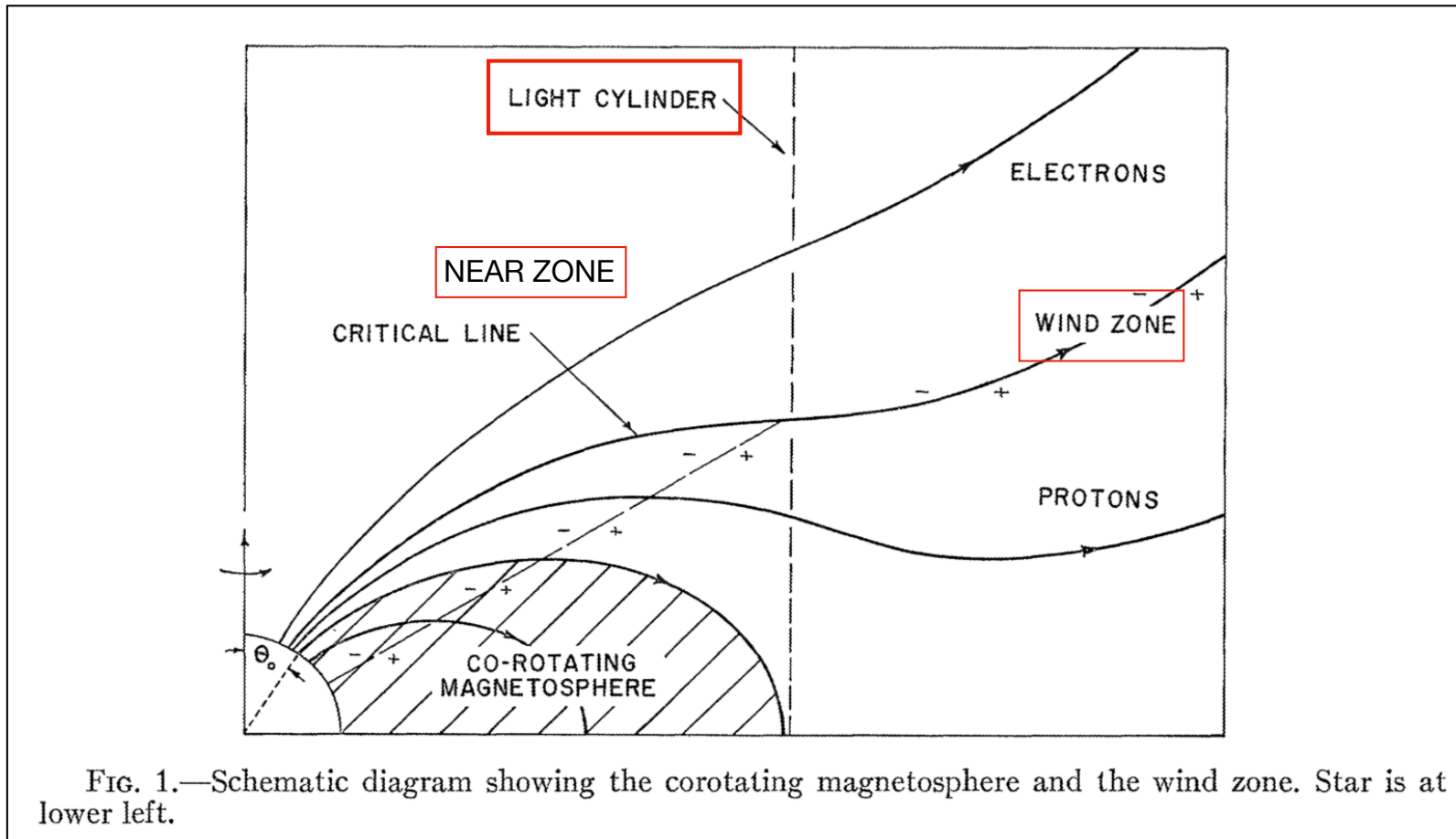
$$\rho \mathbf{E} + \frac{1}{c} \mathbf{j} \times \mathbf{B} = 0$$

$$\mathbf{E} \cdot \mathbf{B} = 0$$



Electric & Magnetic Fields around Pulsar

- **Near Zone and Wind Zone Field Lines**



Electric & Magnetic Fields around Pulsar

- **Motion Equations:** $m_e \dot{\beta} = e(\mathbf{E} + \beta \times \mathbf{B}) + m_e g$

omitted

force free

Steady-state & Axial Symmetry

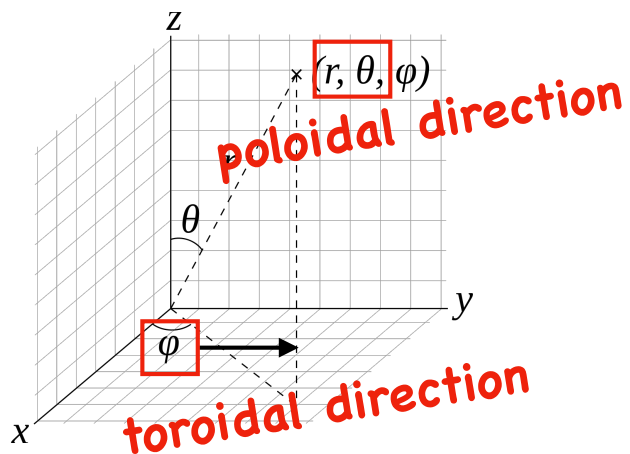
+

$\mathbf{E} \cdot \mathbf{B} = 0$

=

Electric Field
 $\mathbf{E} + \frac{(\boldsymbol{\Omega} \times \mathbf{r})}{c} \times \mathbf{B} = 0$

Charged Particle Motion Condition
 $\beta_p = \kappa B_p \text{ \& } \beta = v/c$



Charged Particle Motion Solution
 $\beta = \kappa B + (\Omega r/c) \sin \theta \phi$

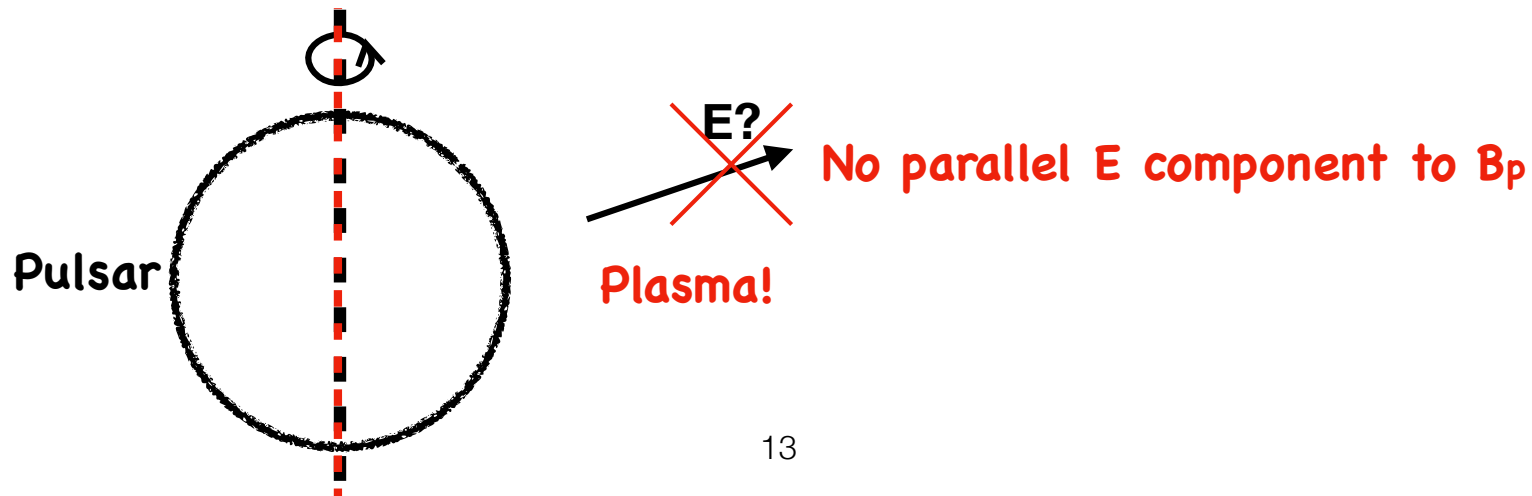
Electric & Magnetic Fields around Pulsar

- **Charge & Current Density**

<p style="text-align: center;">Electric Field</p> $E = -\frac{\Omega r \sin \theta}{c} \phi \times B_p$	<p style="text-align: center;">Goldreich-Julian Charge Density</p> $\rho = \frac{\nabla \cdot E}{4\pi} = \frac{-\Omega \cdot B}{2\pi c} \frac{1}{[1 - (\Omega r/c)^2 \sin^2 \theta]}$
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<p style="text-align: center;">Charged Particle Number Density</p> $n = 7 \times 10^{-2} \frac{B_z}{P} \text{ particles cm}^{-3}$	<p style="text-align: center;">Minimum charge density to shield electric field</p>
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Electric & Magnetic Fields around Pulsar

- near zone

Magnetic Fields

$$B_t = -(\Omega r / c \beta_p) \sin \theta B_p$$

$$B_p = r^{-2} \Psi(\theta)$$

Electric Fields

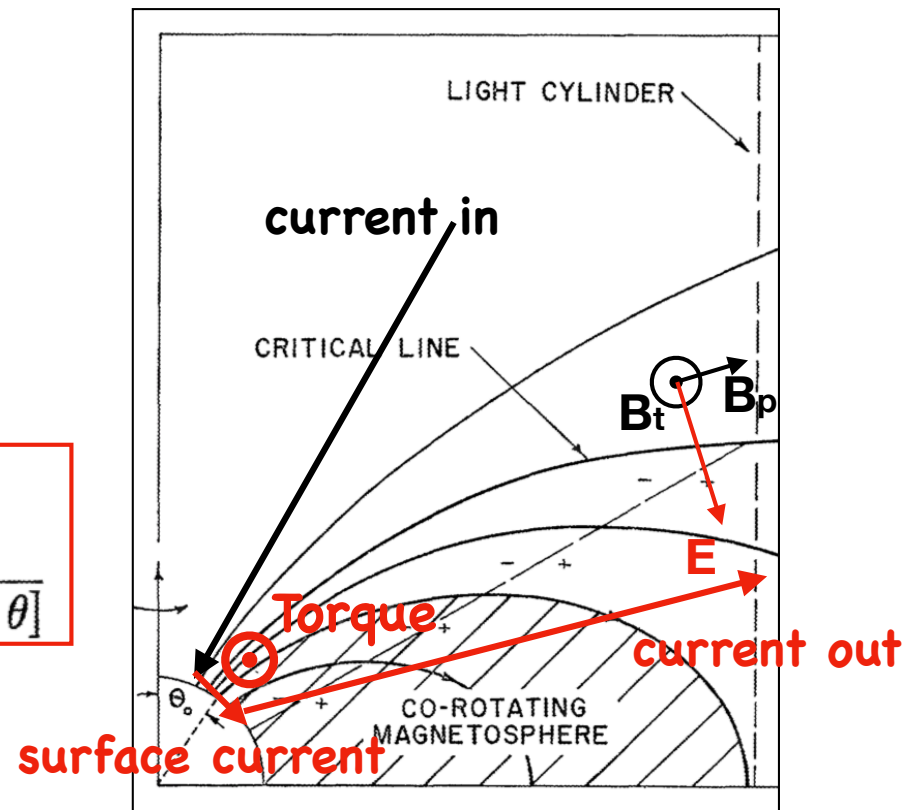
$$E = -\frac{\Omega r \sin \theta}{c} \phi \times B_p$$

Goldreich-Julian Charge Density

$$\rho = \frac{\nabla \cdot E}{4\pi} = \frac{-\Omega \cdot B}{2\pi c} \frac{1}{[1 - (\Omega r / c)^2 \sin^2 \theta]}$$

Goldreich-Julian Current Density

$$J_t = (\Omega r / c) \sin \theta \rho$$



Light Cylinder

Electric & Magnetic Fields around Pulsar

- wind zone

EM Fields

$$B_t = E_\theta = -\frac{1}{2} \left(\frac{\Omega R}{c}\right)^2 \left(\frac{R}{r}\right) B_0 \sin \theta \frac{\Psi(\theta)}{I_1}$$

$$B_p = \frac{1}{2} \left(\frac{\Omega R}{c}\right) \left(\frac{R}{r}\right)^2 B_0 \frac{\Psi(\theta)}{I_1}$$

Note: I_1 is the surface current flowing from north pole to equator

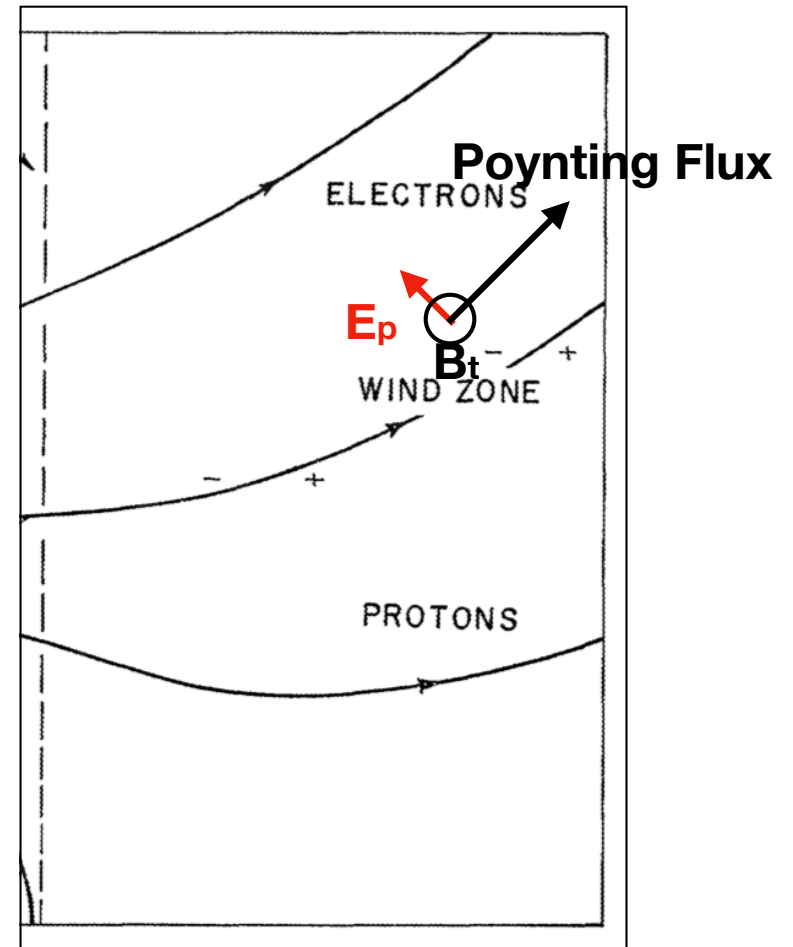
B_t is dominant in this zone

Poynting Vector:

$$S = E \times B = |B_t|^2 = \frac{1}{4} \left(\frac{\Omega r}{c}\right)^4 \left(\frac{R}{r}\right)^2 B_0^2 \sin^2(\theta) \left(\frac{\Phi(\theta)}{I_1}\right)^2 \propto \frac{1}{r^2}$$

Poynting Flux:

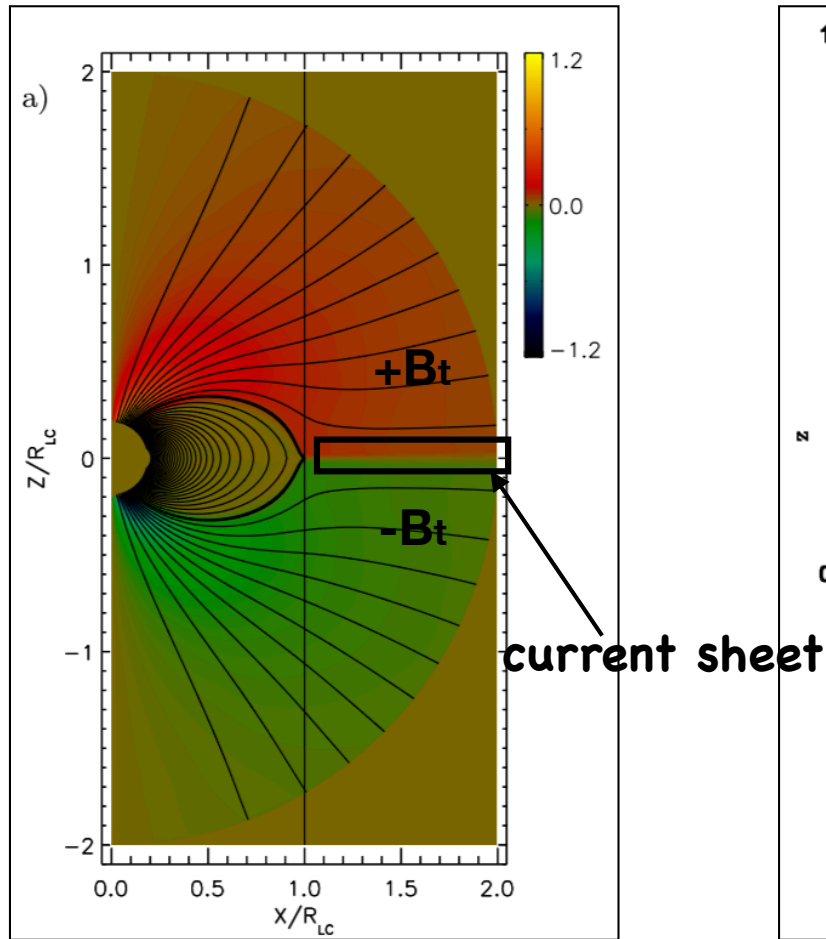
$$L = (1 \pm 0.1) \mu^2 \Omega^4 / c^3$$



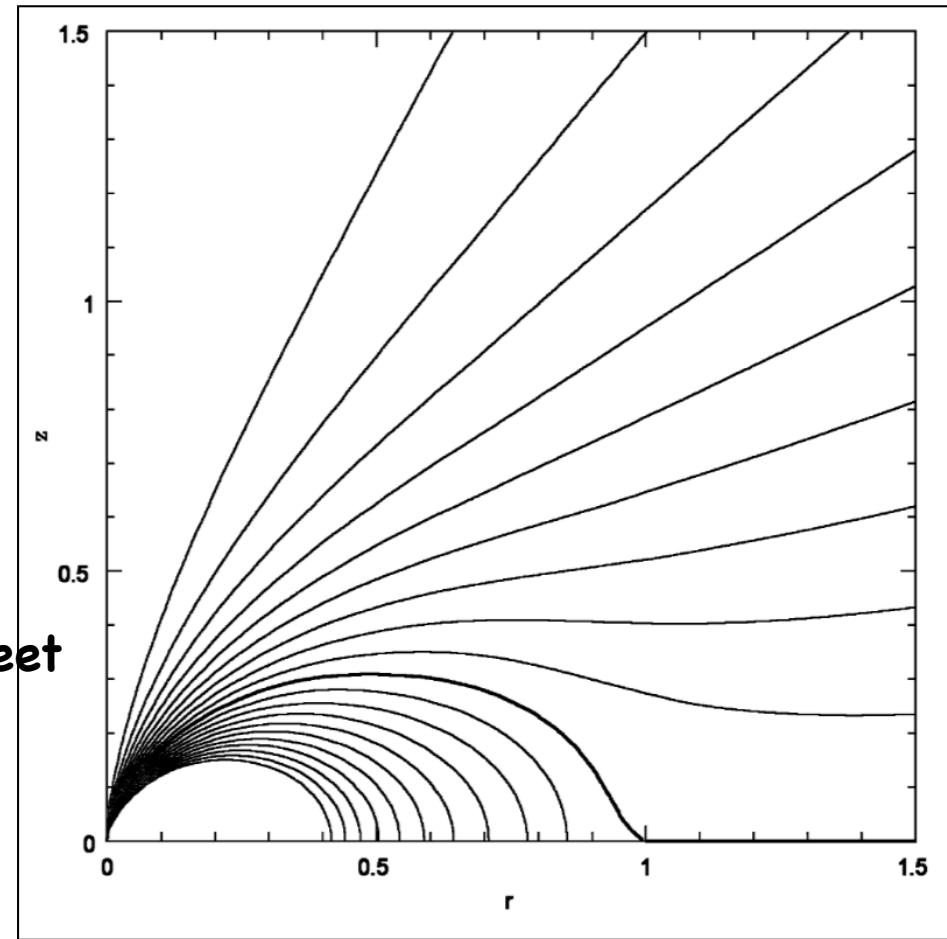
Light Cylinder

Electric & Magnetic Fields around Pulsar

- near zone and wind zone field lines(force free condition)



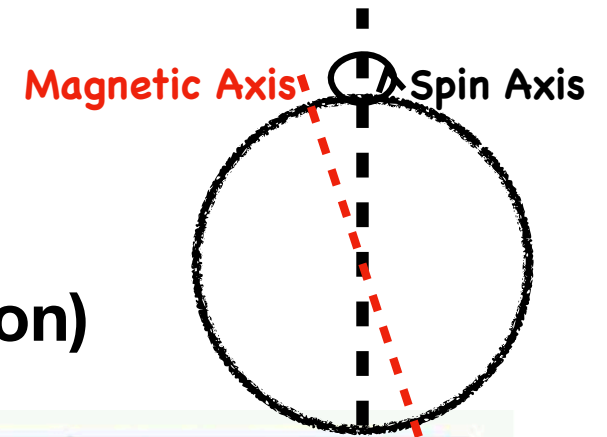
(Anatoly Spitkovsky, 2006)



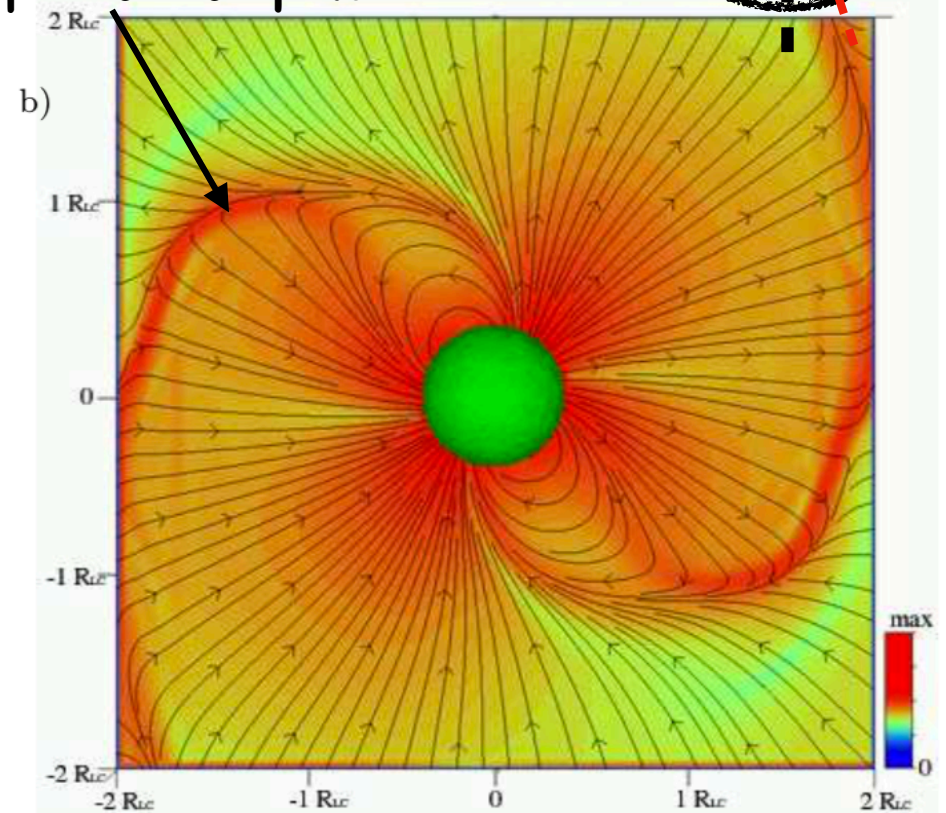
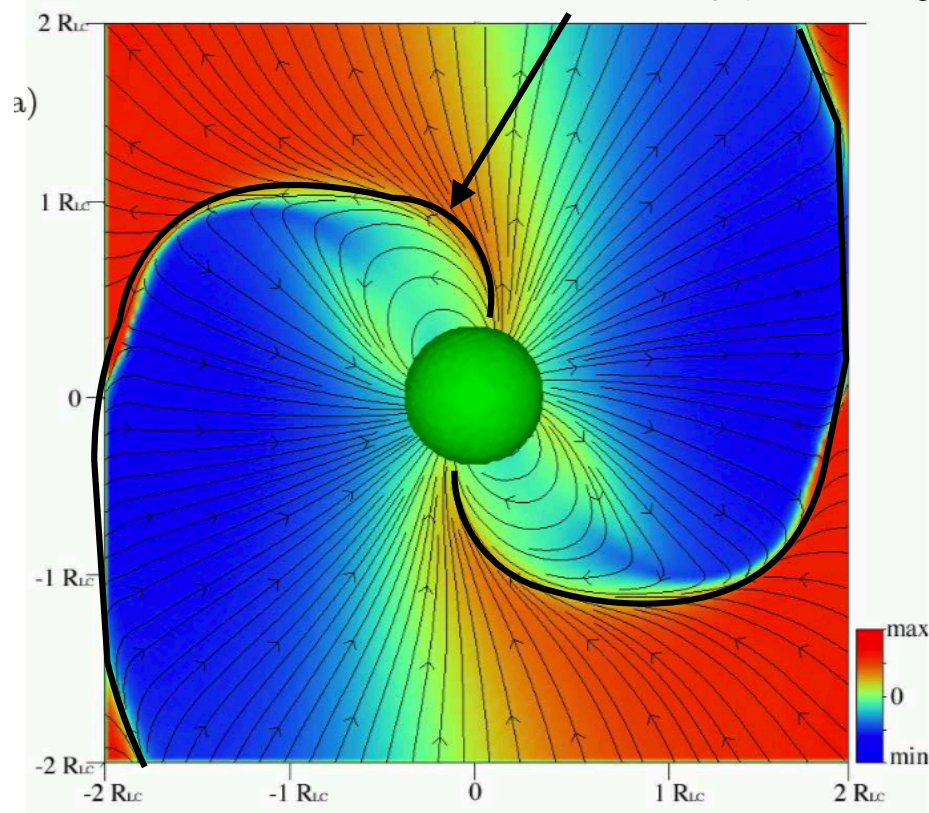
(Andrei Gruzinov, 2005)

Inclined Axis Properties

- Pulsar with Inclined Axis(force free condition)



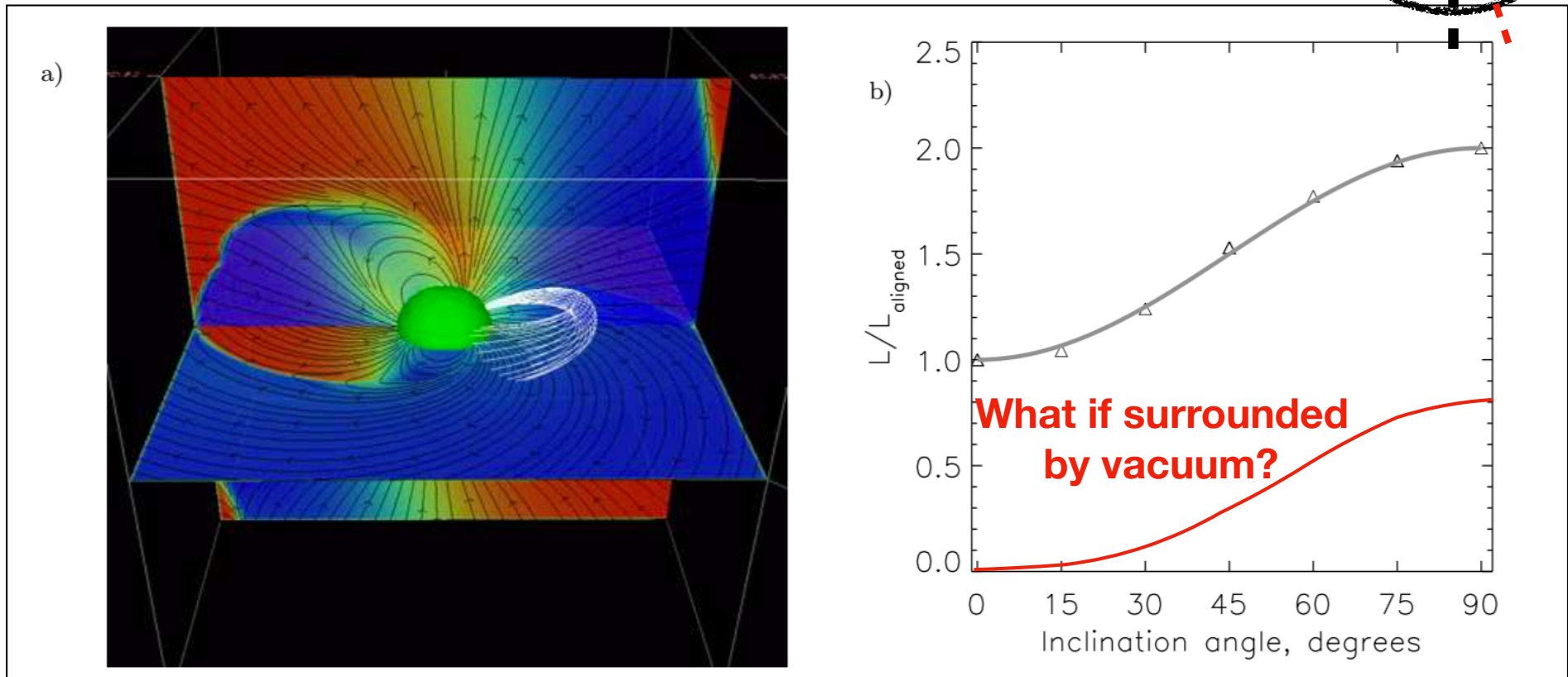
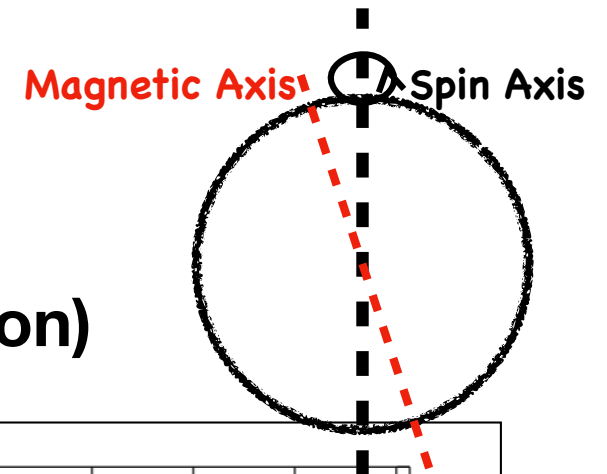
current sheet in oblique rotator pulsar



(Anatoly Spitkovsky, 2006)

Inclined Axis Properties

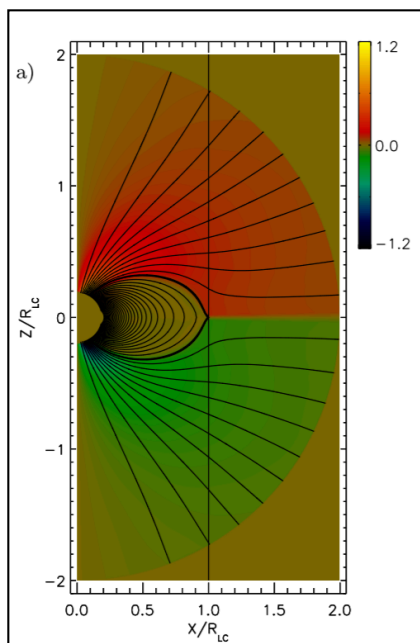
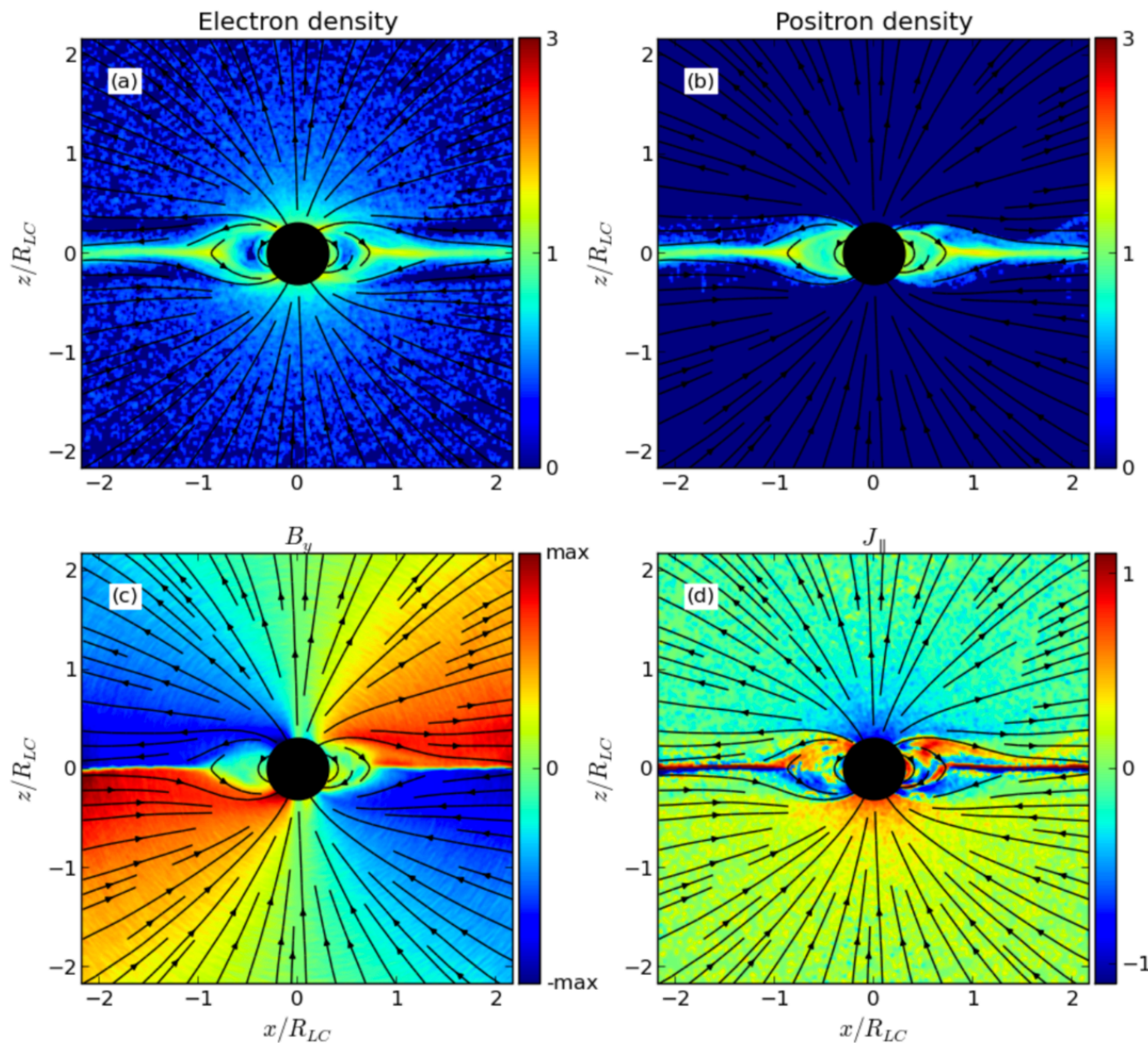
- Pulsar with Inclined Axis(force free condition)



(Anatoly Spitkovsky, 2006)

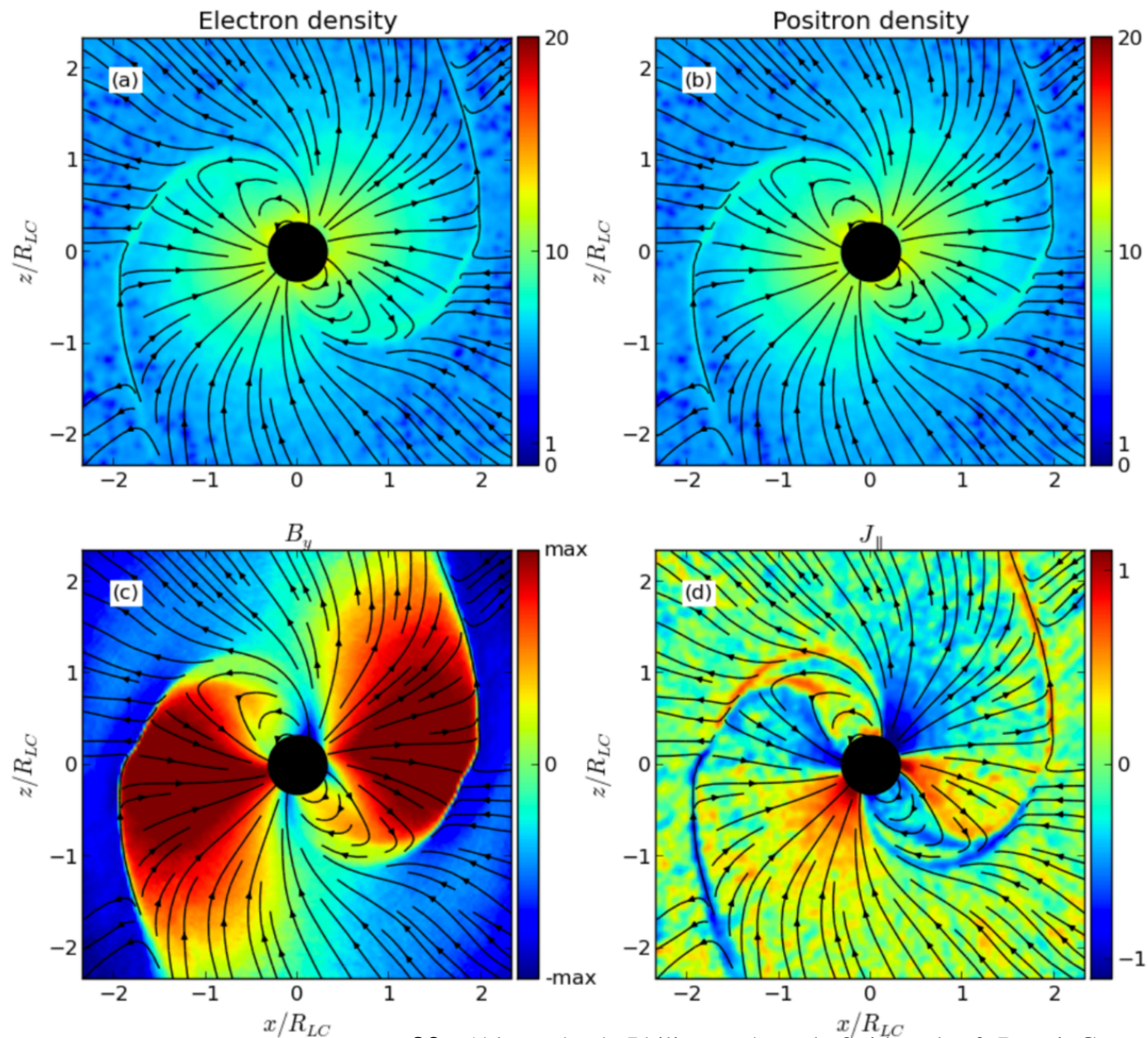
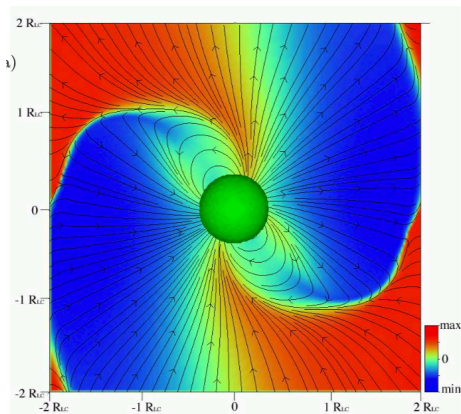
Numerical Simulations of Pulsar

particle-in-cell
simulation with
aligned axes
(first principle)



Numerical Simulations of Pulsar

particle-in-cell
simulation with
inclined axes
(first principle)



Summary

Electro-Magnetic field properties of pulsar

Magnetic dipole field & dipole radiation with aligned axes

Mechanism of pulsar spin down & radiation

Force free & particle in cell simulation of pulsar EM fields

