Magnetorotational Instability and Simulations

Lin Ming

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Advisor:Bai Xuening

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Disks: the incomplete list

Galactic disk	spiral	
	elliptical	NGC 4278
Supermassive BH	Quasar	3C 273
	Seyfert	MCG -6-30-15
	LINER	NGC 4258
	LLAGN	Sgr A*
	TDE	Swift J1644+57

Disks: the incomplete list

Stellar mass BH	microquasar	GRS 1915+ 105
	gamma-ray burst	long bursts?
Neutron star	LMXB	Aql X-1
	HMXB	Cyg X-1
	gamma-ray burst	short bursts?
White dwarf	dwarf nova	SS Cyg
	nova	RS Oph
Protostar	protoplanetary	HL Tau
	debris	Fomahaut
Planet	protolunar disk	Earth/moon
	planetary rings	Saturn

Keplerian disks

- What is a Keplerian disk?
- A disk of material that obeys Kepler's laws of motion due to the dominance of the central massive body

- The azimuthal and angular velocity of a fluid parcel with a distance *R* from the center are

$$v_\phi = \sqrt{\frac{GM}{R}} \quad \Omega = \sqrt{\frac{GM}{R^3}}$$

Keplerian disks

- Hydrostatic equilibrium: the gradient of the pressure equals the gravity

$$\rho = \rho_0 \exp(-\Omega^2 z^2/2c_s^2) \equiv \rho_0 \exp(-z^2/2H^2)$$

- C_s is the isothermal sound speed, and the characteristic scale height H is defined as $H = c_s/\Omega$, and then $H/R = c_s/v_K$

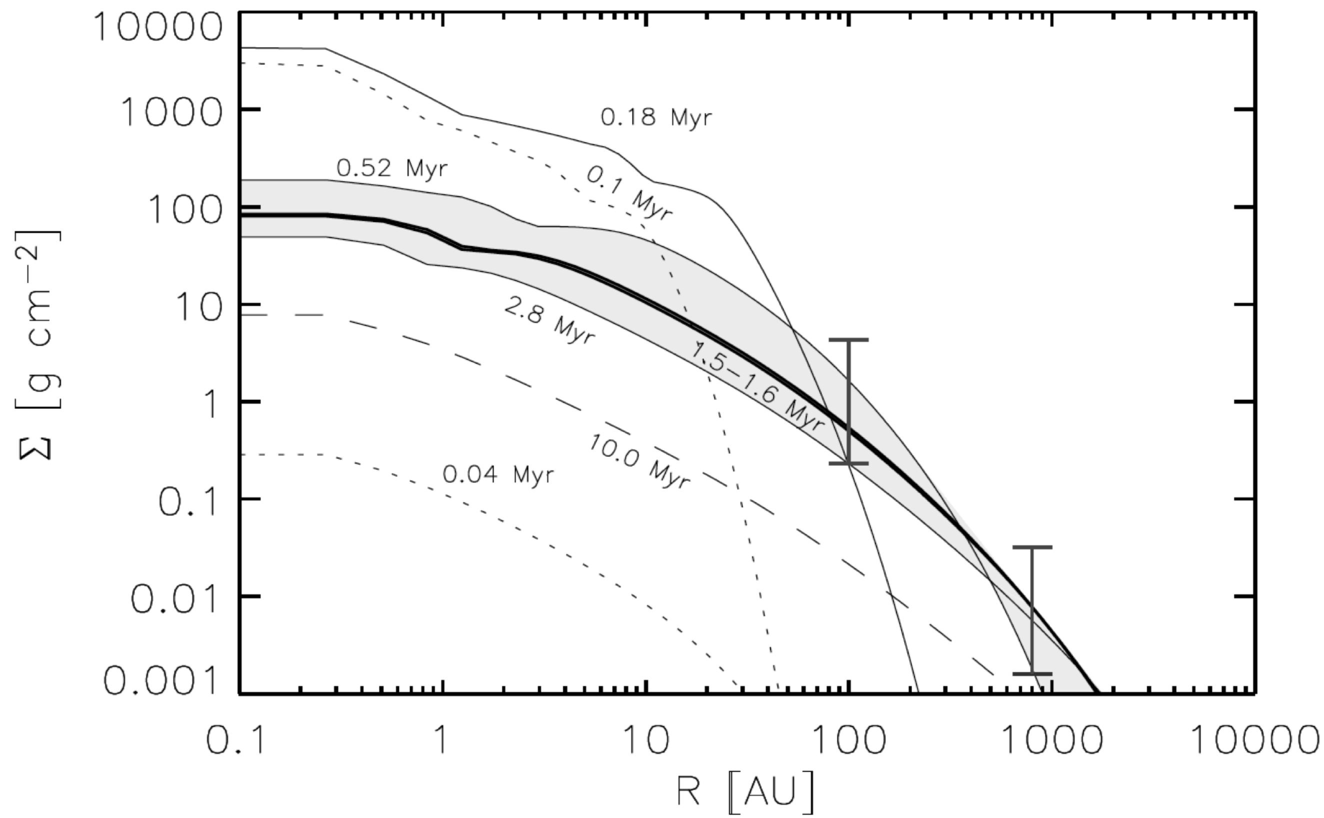
- Thin disk approximation requires $H \ll R$, which means that the local rotating speed is much larger than the sound speed

α model disks

- Shakura & Sunyaev(1973), Lynden-Bell & Pringle (1974)
- Ignore external torques, infall/winds, variation in α
- Molecular viscosity $\nu \sim v_{th}\lambda$ is too small for a reasonable accretion rate
- Turbulent viscosity is thus introduced $\nu_t = \alpha c_s H$,usually $\alpha \leq 1$
- Or in another form $w_{R\phi} = \alpha\rho c_s^2$, the stress tensor $w_{R\phi} = \rho\delta v_R \delta v_\phi - \frac{B_R B_\phi}{4\pi}$

α model disks

- Radial profile of the surface density using α disk model , Hueso & Guillot (2005)



Turbulence in Disks

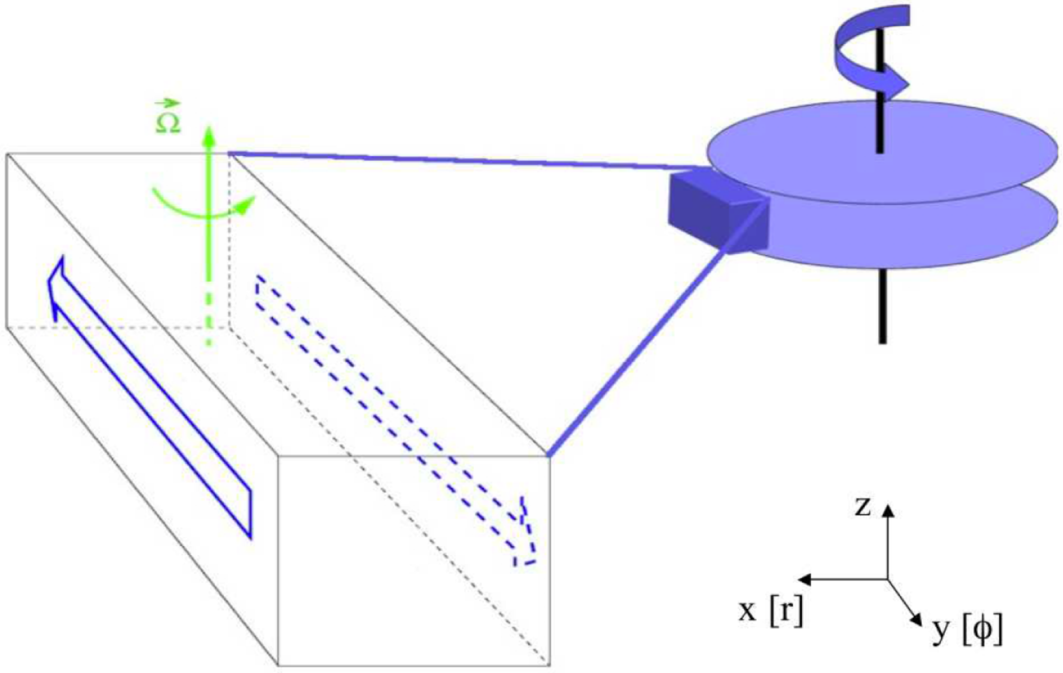
- What generates turbulence?
- Possibilities:
- - magnetorotational instability (linear)
- - gravitational instability
- - zombie vortex instability (unique in PPDs)
- - subcritical baroclinic instability (PPDs)
- - vertical shear instability (PPDs)

The magnetorotational instability (MRI)

- Balbus & Hawley(1991)
- A powerful ***linear*** instability that displays enormous growth rates
- *Ideal* MHD model is applied for MRI (frozen-in, infinite conductivity)

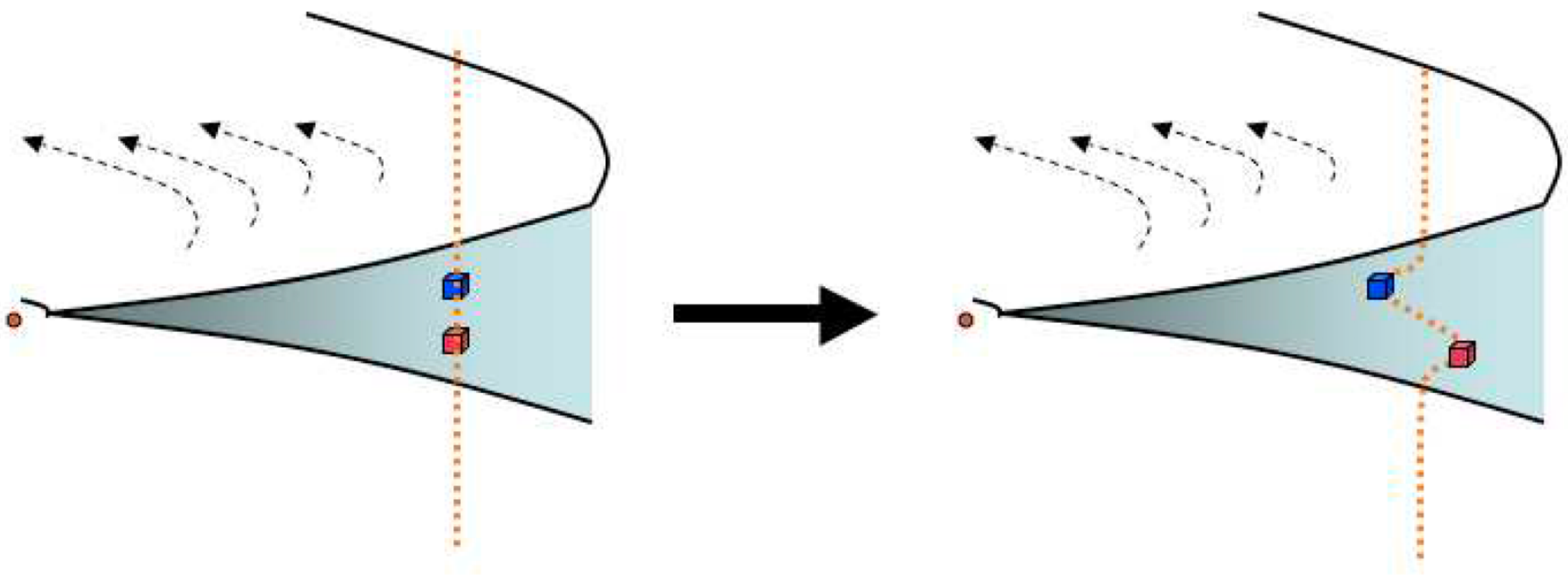
The magnetorotational instability (MRI)

- We use local Cartesian coordinate corotating with the disk



The magnetorotational instability (MRI)

- Physical interpretation: two fluid element sitting on a magnetic field line in analogy to two masses in orbit on a spring



The magnetorotational instability (MRI)

- Dispersion relation of MRI

$$\omega^4 - \omega^2[2k^2v_A^2 + \kappa^2] + k^2v_A^2[k^2v_A^2 - 2q\Omega^2] = 0$$

- Epicyclic frequency is given by $\kappa^2 = 2(2-q)\Omega^2$

- Critical wavenumber $k_c^2 = 2q\Omega^2/v_A^2$ $\quad q = -d\ln\Omega/d\ln R$

- All MRI modes are incompressible

The magnetorotational instability (MRI)

- Rayleigh Criterion: accretion disks with angular momentum increasing outward are linearly stable (q<=2, with no magnetic field)

$$d(R^2\Omega)/dR > 0$$

- MRI instability criterion

$$d\Omega/dR < 0$$

- Also $k < k_c$ or $\lambda > \lambda_c$

- Fast growing mode is the most unstable mode, also known as channel mode:

$$k_{max}^2 v_A^2 = \frac{q}{4}(4-q)\Omega^2$$

Simulations: from the beginning

- Why do we simulate?
- -To understanding MHD turbulence as an angular momentum transport process
- -To study the nonlinear evolution or nonlinear regime of the MRI as it grows
- -To explore the conditions under which MHD turbulence occurs

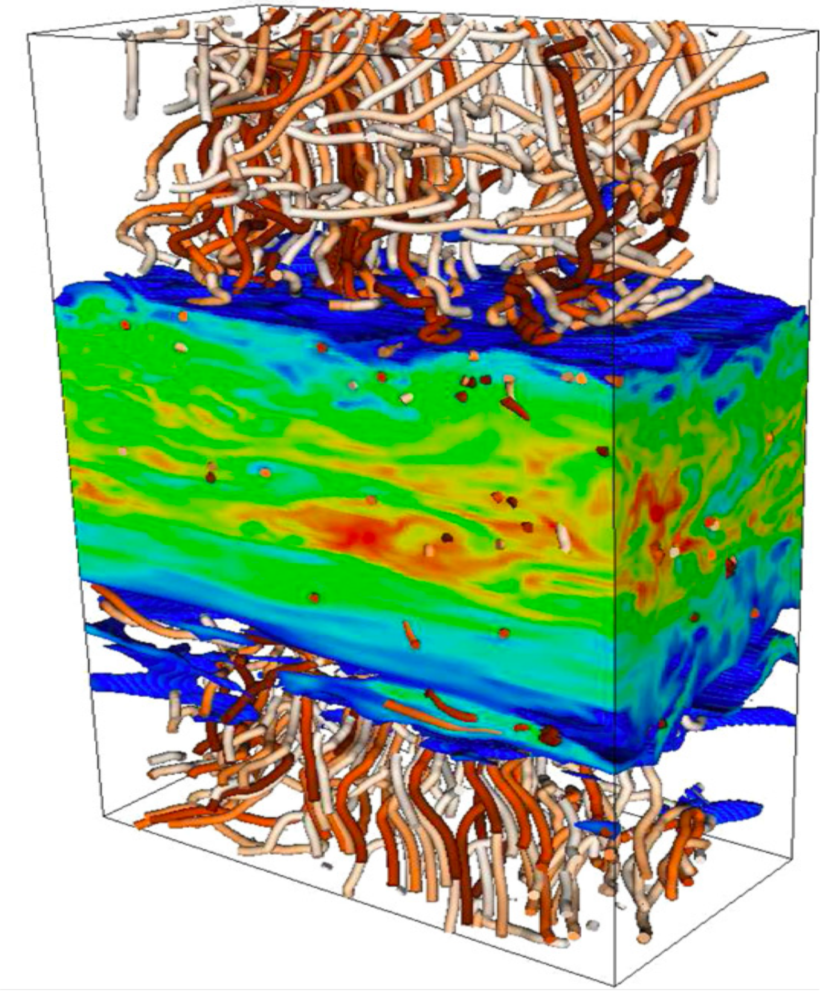
Simulations: from the beginning

- Catetogories
- -2D or 3D
- -local or global
- -stratified or unstratified
- -compressible or incompressible
- -ideal MHD limit or dissipation MHD (viscosity, resistivity)
- -zero magnetic flux, weak magnetic flux or strong magnetic flux

Simulations: codes and algorithms

- ZEUS (Stone & Norman 1992;Fromang et al. 2007I,II)
- PENCIL CODE (Brandenburg et al. 1995)
- ATHENA(Stone et al. 2008)
- PLUTO Godnunov (Mignone et al. 2007; Flock et al. 2011)
- NIRVANA-III(Ziegler 2004,2008)
- RAMSES (Teyssier 2002; Fromang et al. 2006)
- SNOOPY (publicly available)

Simulations: from the beginning



- Bai & Stone (2013)
- isosurface blue to red
- streamlines white to dark white

Simulations: formalism

- Ideal magnetohydrodynamic(MHD) equatinos

$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\frac{1}{\rho}\nabla(P + \frac{B^2}{8\pi}) + \frac{(\boldsymbol{B} \cdot \nabla)\boldsymbol{B}}{4\pi\rho} - 2\boldsymbol{\Omega} \times \boldsymbol{v}$$
$$+2q\Omega^2 x \boldsymbol{e}_x - \Omega^2 z \boldsymbol{e}_z$$

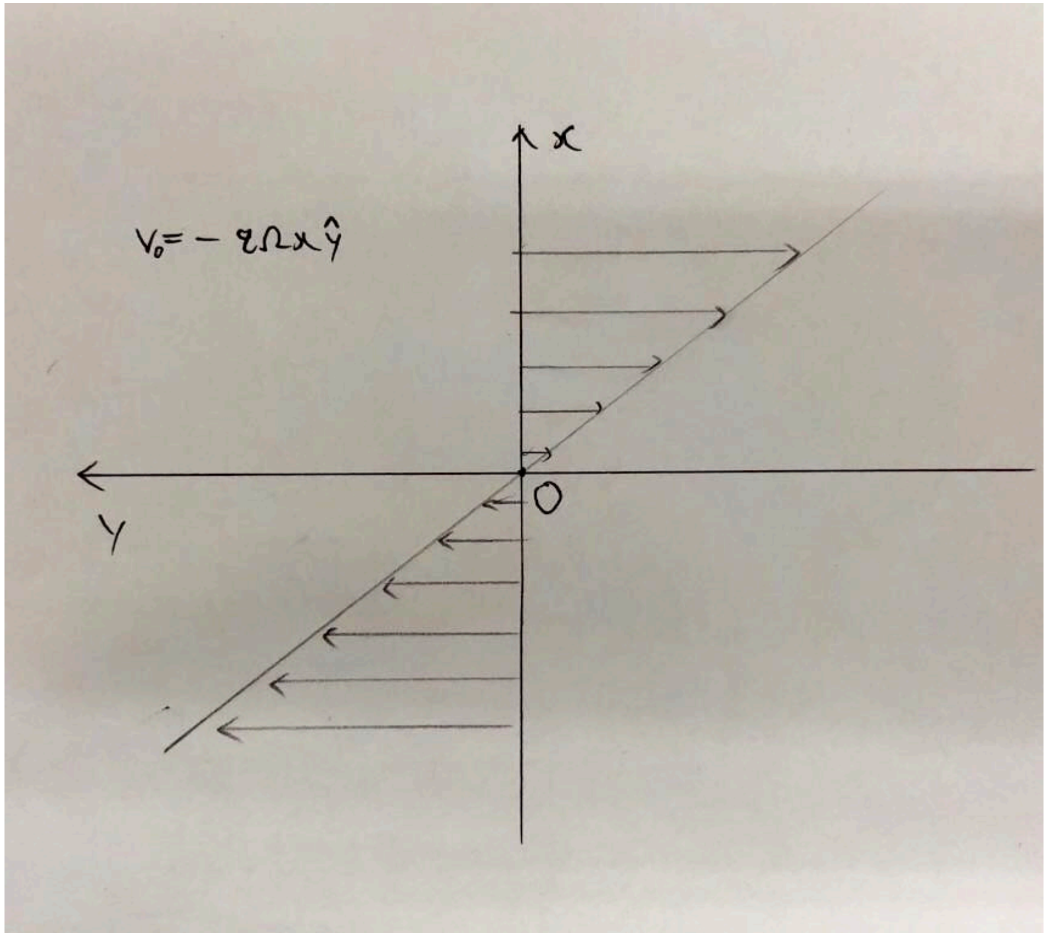
$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

Simulations: shearing box system

- Originated with Hill(1878)
- Local 3D MHD simulations of accretion disk (Hawley et al. 1995)
- It only studies a local patch of the disk in Cartesian form
- Only consider the radial component of the gravity (unstratified)
- Ignore buoyancy effects
- Steady state: a uniform shear flow with $\boldsymbol{v_0} = -q\Omega x \boldsymbol{e}_y$

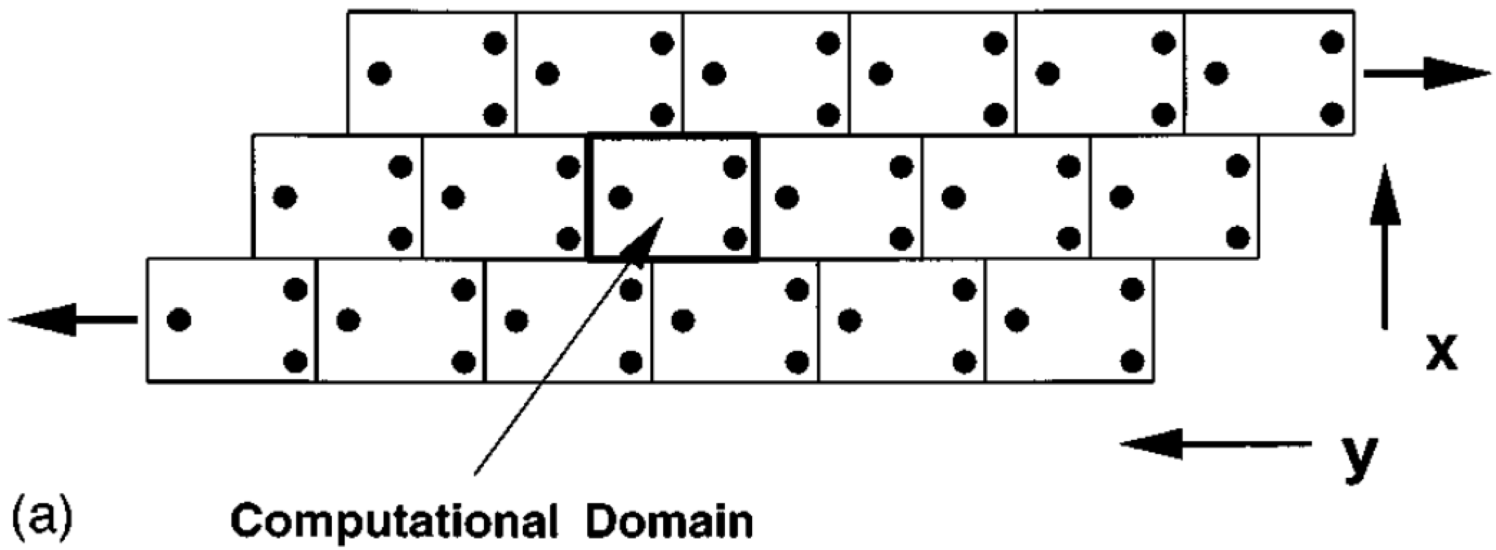
Simulations: shearing box system



- Uniform shearing flow as background

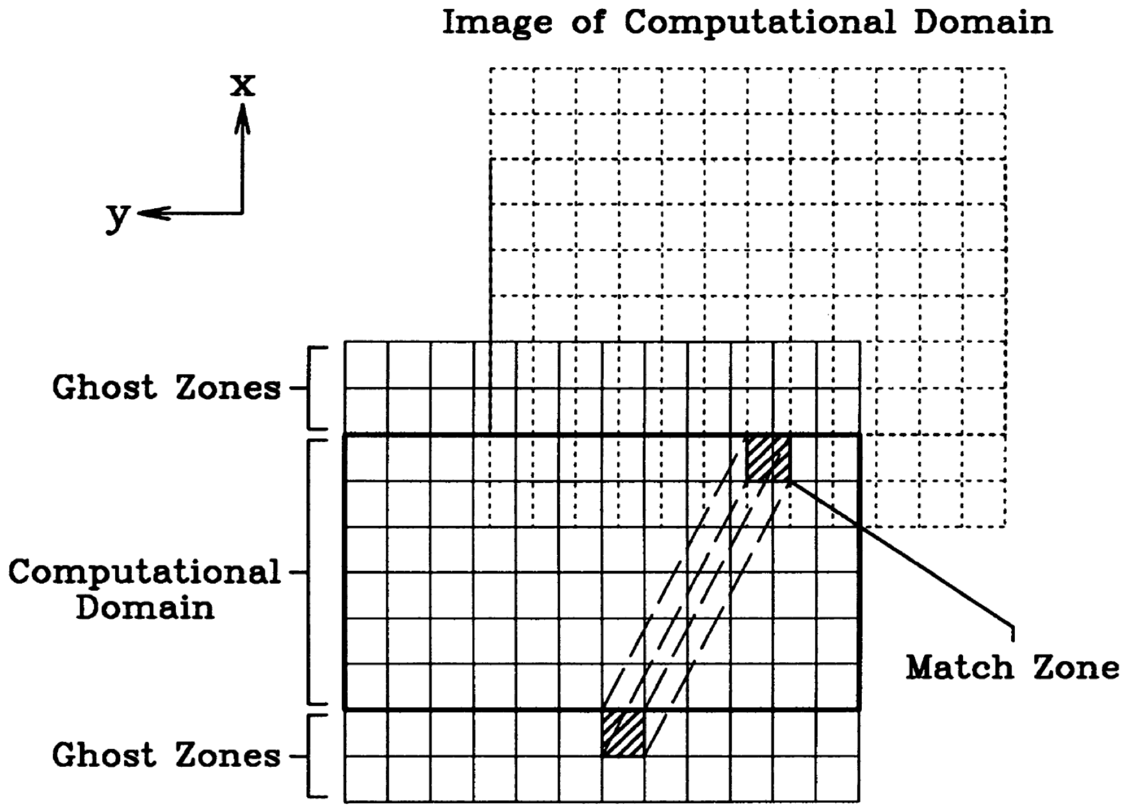
Simulations: shearing box system

- The computational domain is surrounded by identical domains moving with a fixed shear velocity



Simulations: shearing box system

- Boundary conditions are very important



$$f(x,y,z) = f(x + L_X, y - q\Omega L_X t, z)$$

$$f(x,y,z) = f(x, y + L_Y, z)$$

$$f(x,y,z) = f(x, y, z + L_Z)$$

Periodic points

$$t = \frac{nL_y}{q\Omega L_x}, \; n = 1, 2, 3...$$

Simulations: Numerical scheme

- Average physical quantities over volume or over time and volume to avoid random noise

- Fourier decomposition for the power spectrum

- Lagrangian wavenumber $k_x = \frac{2\pi n_x}{L_x} \quad k_y = \frac{2\pi n_y}{L_y} \quad k_z = \frac{2\pi n_z}{L_z}$

- Eulerian wavenumber $k_x(t) = \frac{2\pi n_x}{L_x} + q\Omega k_y t$

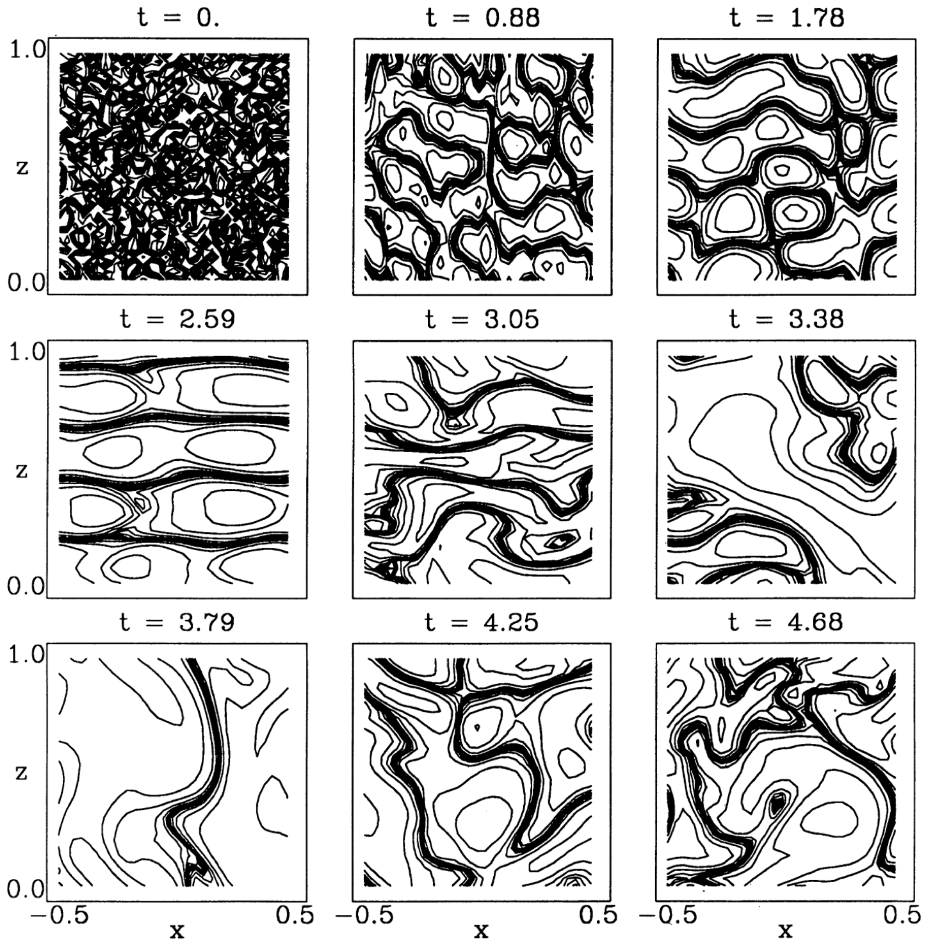
Simulations: numerical scheme

- Two important dimensionless ratios for initial configurations
- The plasma parameter

$$\beta = \frac{8\pi P_0}{B_0^2}$$

- Box size and wavelength ratio L_z/λ_c
- λ_c is the critical wave length of the fast mode
- $$\lambda_c = \frac{2\pi}{k_c} \qquad \lambda_c = 2\pi\sqrt{16/15}|\boldsymbol{v}_A|/\Omega$$ for Keplerian disk

Simulations: unstratified



Contour plots of perturbed velocity δv_y , which is the difference between actual velocity v_y and the background shearing flow (Hawley et al. 1995)

Simulations: basic results

- An axial magnetic field and toroidal magnetic field both lead to turbulence(with no assumption other than a weak magnetic field)
- It is magnetic field, rather than hydrodynamics, that sustains the angular momentum transport
- The resulting turbulence is nonlinear and nonisotropic

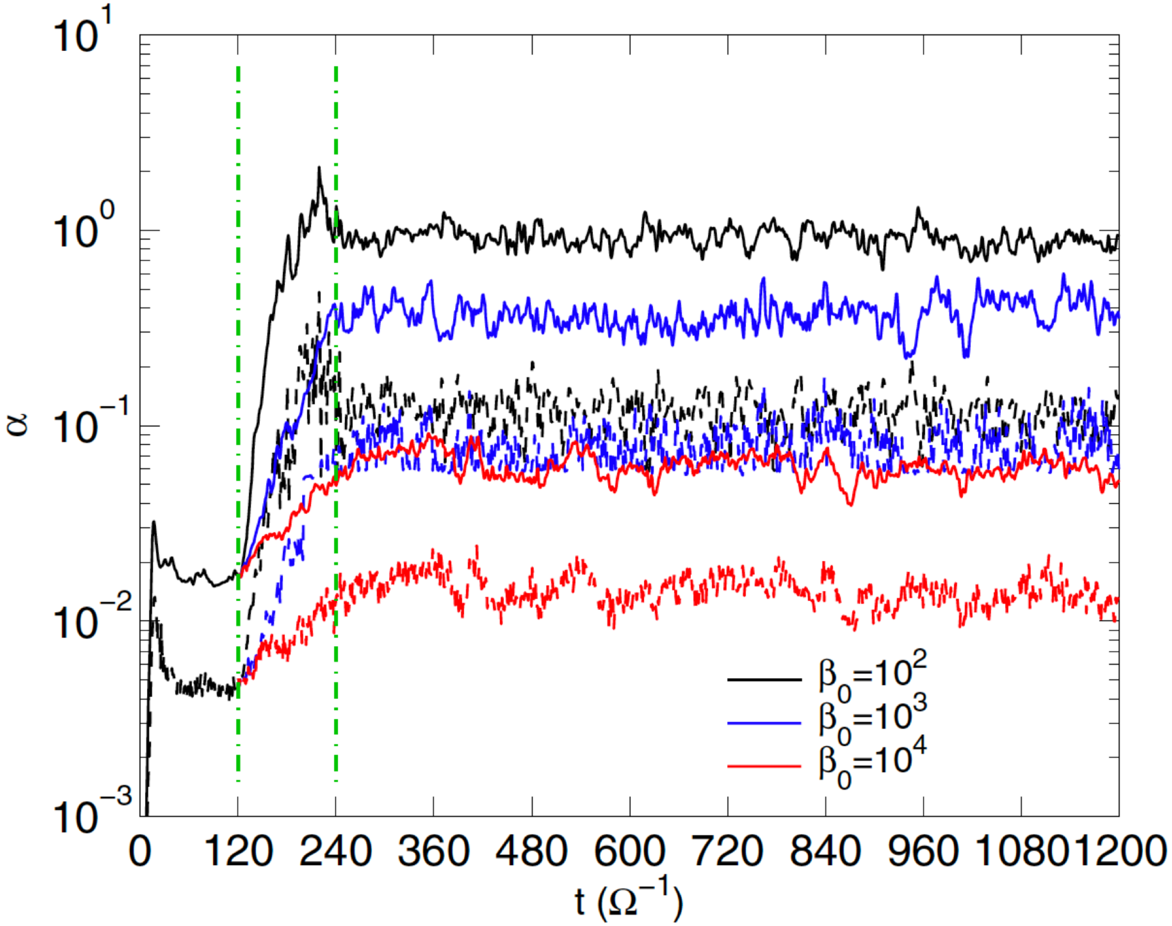
Simulations: basic results

- Vertical field: $\alpha = 0.2 - 0.7$
- Toroidal field: $\alpha = 0.02 - 0.08$
- Simulation results converge when there is a net magnetic flux
- Channel modes break up in 3D simulation
- The total stress is mostly correlated with the magnetic pressure, rather than the gas pressure

Simluations:zero net vertical magnetic flux

- Prandtl number is $Pm = \frac{Rm}{Re}$ $R_m = \frac{ul}{\eta}$ $R_e = \frac{ul}{\nu}$
- Turbulent activity is an increasing function of the magnetic Prandtl number Pm (Fromang et al. 2007II)
- Turbulence disappears when the Prandtl number falls below a critical value Pm_c

Simluations:stratified



- Bai & Stone (2013)
- Time history of mass weighted Maxwell (solid) and Reynolds (dashed) stresses from simulations.
- The behavior of the MRI turbulence strongly depends on β_0

Summary

- MRI is a significant mechanism to transport angular momentum in accretion disks
- When the dissipation coefficients are small enough not to affect its linear stage, the nonlinear development of the MRI leads to MHD turbulence
- Angular momentum transport can be initiated and sustained by the presence of a weak magnetic field

Summary

- The turbulence is subsonic and the Maxwell stress dominates the Reynolds stress by a factor of a few

- The value of α ranges from 10^{-3} to a few times 10^{-1} depending on the magnetic field strength

Thank you