Laboratory experiments of the MRI

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Outline

- Local stability analysis

- Experiments
  - Taylor-Couette flow
  - Standard MRI(SMRI)
  - Helical MRI (HMRI)
  - Non-magnetic flow
  - Magnetocoriolis waves

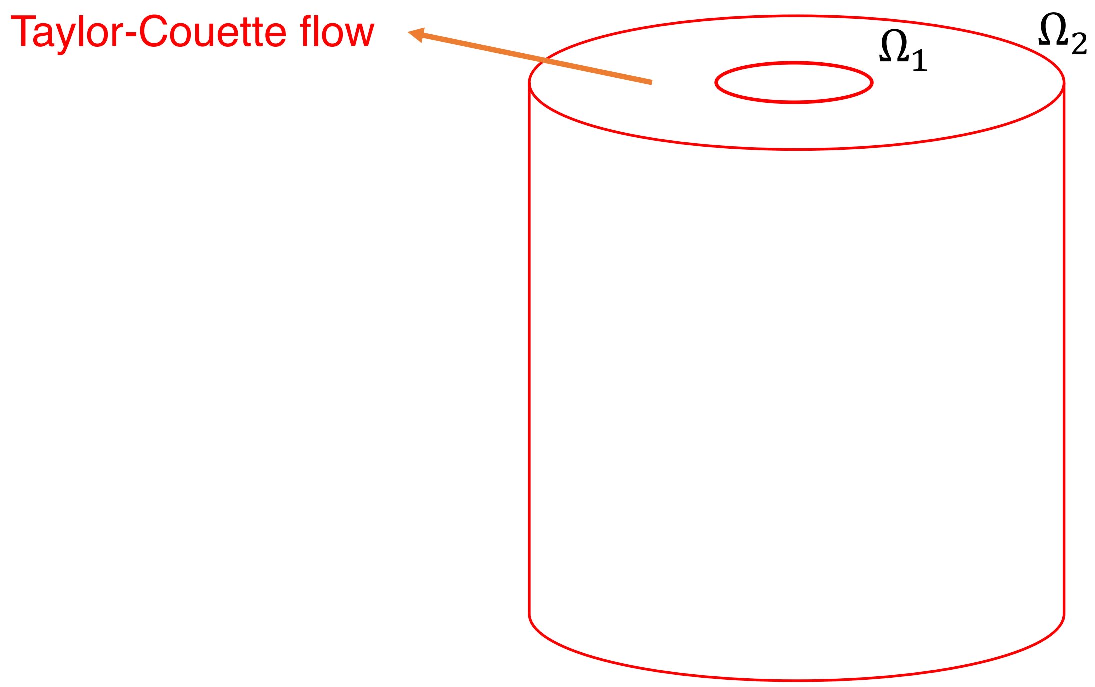
- Summary

Taylor-Couette flow

Why we need experiments?—To be a testbed for MHD codes

How? — flow: conductivity、temperature、price… **vessel, detection…**

Taylor-Couette flow:
consists of a viscous fluid (metal) confined in the gap between two rotating cylinders.



Local stability analysis

Dynamics of liquid metals—Incompressible and dissipative MHD equations,

$$
\begin{aligned}
0 &= \boldsymbol{\nabla}\cdot\boldsymbol{V} \\
0 &= \boldsymbol{\nabla}\cdot\boldsymbol{B} \\
\frac{\partial \boldsymbol{B}}{\partial t} &= \boldsymbol{\nabla}\times(\boldsymbol{V}\times\boldsymbol{B}) + \eta\boldsymbol{\nabla}^2\boldsymbol{B} \\
\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\cdot\boldsymbol{\nabla})\boldsymbol{V} &= \frac{(\boldsymbol{B}\cdot\boldsymbol{\nabla})\boldsymbol{B}}{\mu_0\rho} - \frac{1}{\rho}\boldsymbol{\nabla}\left(p + \frac{B^2}{2\mu_0}\right) \\
&\quad +\nu\boldsymbol{\nabla}^2\boldsymbol{V},
\end{aligned}
$$

The equilibrium quantities: $\boldsymbol{B}_0 = (0, 0, B)$ $\quad$ $\boldsymbol{V}_0 = (0, r\Omega, 0)$

The balance of forces: $\partial p_0/\partial z = 0$ $\quad$ $\partial p_0/\partial r = \rho r\Omega^2$

Local stability analysis

WKBJ methods (Goodman & Ji 2001)

The perturbations: $\boldsymbol{B}_1 = (B_r, B_\theta, B_z)$ $\boldsymbol{V}_1 = (V_r, V_\theta, V_z)$
All proportional to: $\exp(\gamma t - i k_z z - i k_r r)$ γ : growth rate

The linearized equations of motion:

$$
\begin{aligned}
0 &= k_r V_r + k_z V_z \\
0 &= k_r B_r + k_z B_z \\
\gamma B_r &= -i k_z B V_r - \eta k^2 B_r \\
\gamma B_\theta &= -i k_z B V_\theta + \frac{\partial \Omega}{\partial \ln r} B_r - \eta k^2 B_\theta \\
\gamma V_r - 2\Omega V_\theta &= -i \frac{k_z B}{\mu_0 \rho} B_r + i \frac{k_r}{\rho} p_1 + i \frac{k_r B}{\mu_0 \rho} B_z - \nu k^2 V_r \\
\gamma V_\theta + \frac{\kappa^2}{2\Omega} V_r &= -i \frac{k_z B}{\mu_0 \rho} B_\theta - \nu k^2 V_\theta \\
\gamma V_z &= i \frac{k_z}{\rho} p_1 - \nu k^2 V_z
\end{aligned}
$$

Local stability analysis

Dispersion relation:

$$[(\gamma+\nu k^2)(\gamma+\eta k^2)+(k_z V_A)^2]^2\frac{k^2}{k_z^2} + \kappa^2(\gamma+\eta k^2)^2 + \frac{\partial\Omega^2}{\partial\ln r}(k_z V_A)^2 = 0.$$

Alfvénic speed: $V_A \equiv B/\sqrt{\mu_0\rho}$

Three relevant frequencies:
- resistive $\omega_\eta \equiv \eta k^2$
- viscous $\omega_\nu \equiv \nu k^2$
- Alfvénic $\omega_A \equiv |k_z V_A|$

Four dimensionless parameters:
- Magnetic Prandtl number $P_{\rm m} \equiv \omega_\nu/\omega_\eta$
- Lundquist number $S \equiv \omega_A/\omega_\eta$ → Magnetic field
- Magnetic Reynolds number $R_{\rm m} \equiv \Omega/\omega_\eta$ → Free energy
- Vorticity parameter $\zeta \equiv (1/r\Omega)\partial(r^2\Omega)/\partial r = 2+\partial\ln\Omega/\partial\ln r$ → Free energy

Rayleigh stability criterion $\zeta \geq 0$

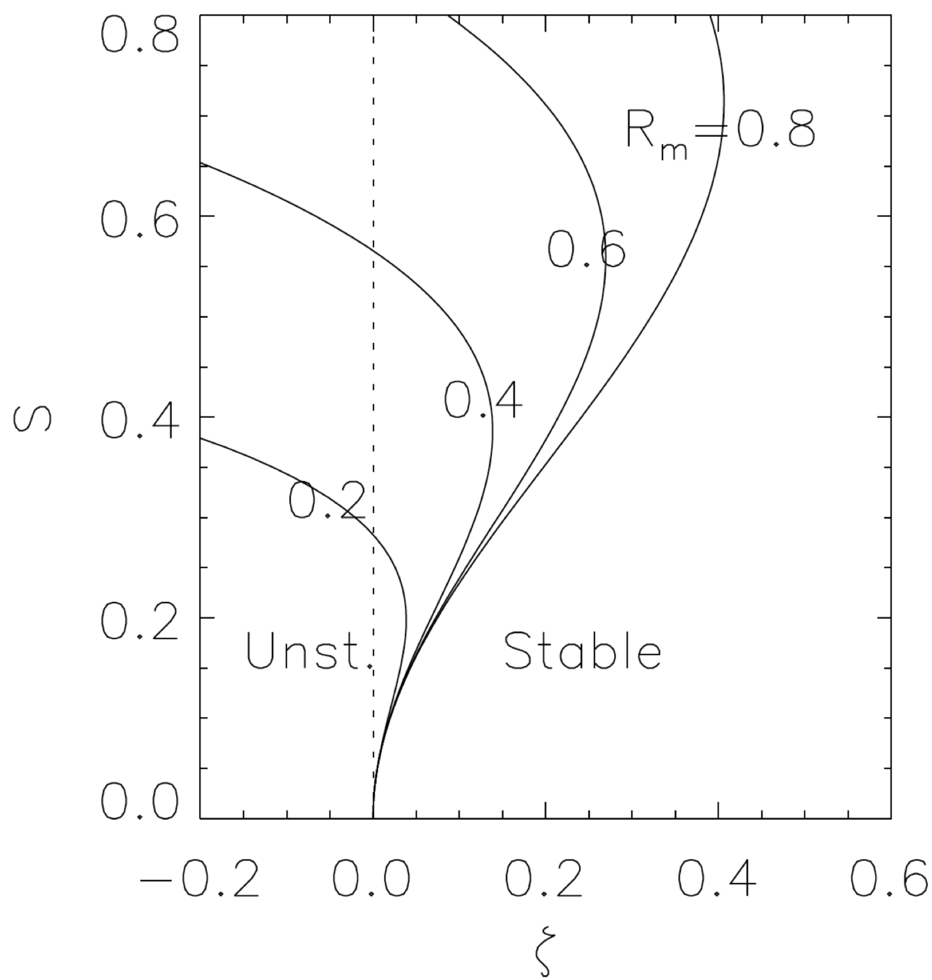
Condition for stability: $(P_{\rm m}+S^2)^2(1+\epsilon^2)+2\zeta R_{\rm m}^2-2(2-\zeta)R_{\rm m}^2 S^2 \geq 0,$ $\epsilon \equiv h/(r_2-r_1)$

Small P_m limit:

$$\zeta \geq \frac{2S^2}{S^2+1}-\frac{S^4(1+\epsilon^2)}{2R_{\rm m}^2(S^2+1)}.$$

Local stability analysis

Stability diagram



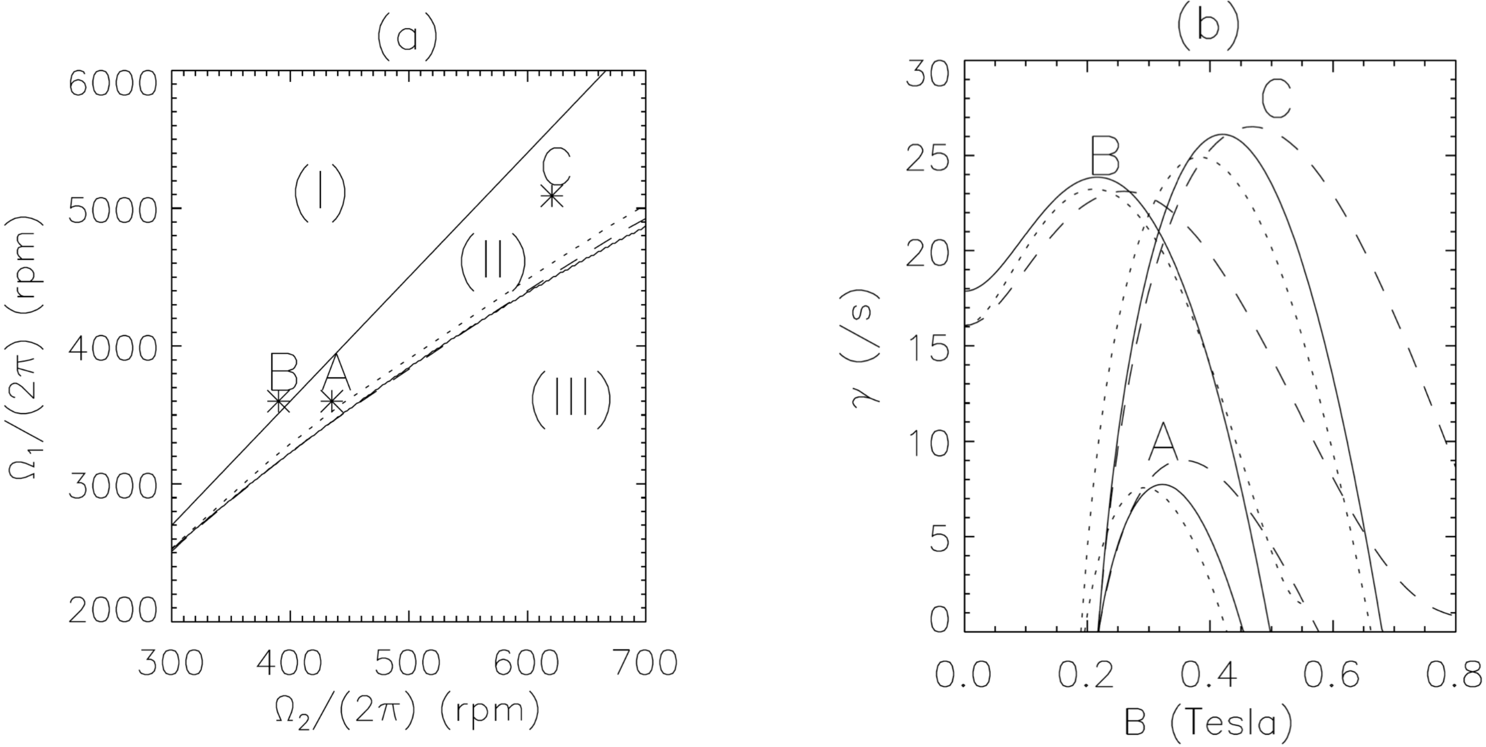


Table 1. Parameters for a gallium annulus with $r_1 = 0.05$m, $r_2 = 0.15$m, and $h = 0.1$m.

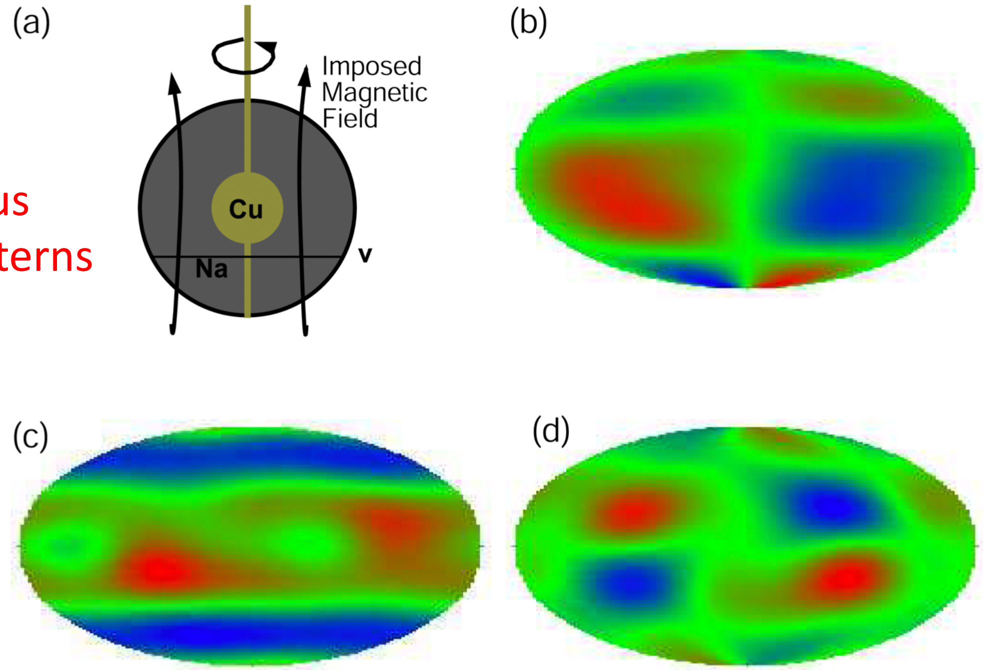
point	Ω_1(rpm)	Ω_2(rpm)	R_m	ζ
A	3600.00	435.00	0.3319	0.06293
B	3600.00	390.00	0.3143	-0.01899
C	5089.77	620.70	0.4715	0.06984

Local analysis → global analysis (Goodman & Ji 2001)

Practical issues: geometric optimization
periodic vertical boundary conditions
nonlinear hydrodynamical instability (Goodman & Ji 2001)

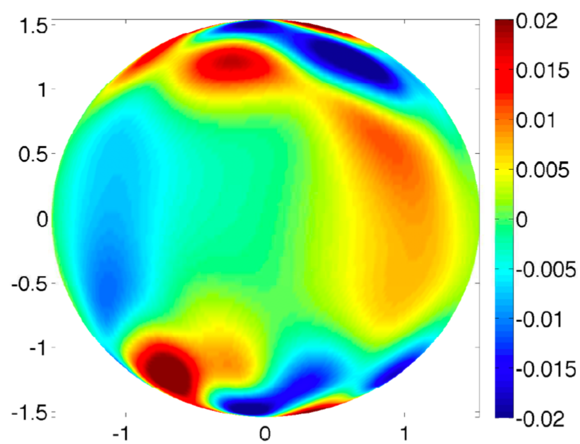
Standard MRI(SMRI)

Spherical apparatus
and instability patterns



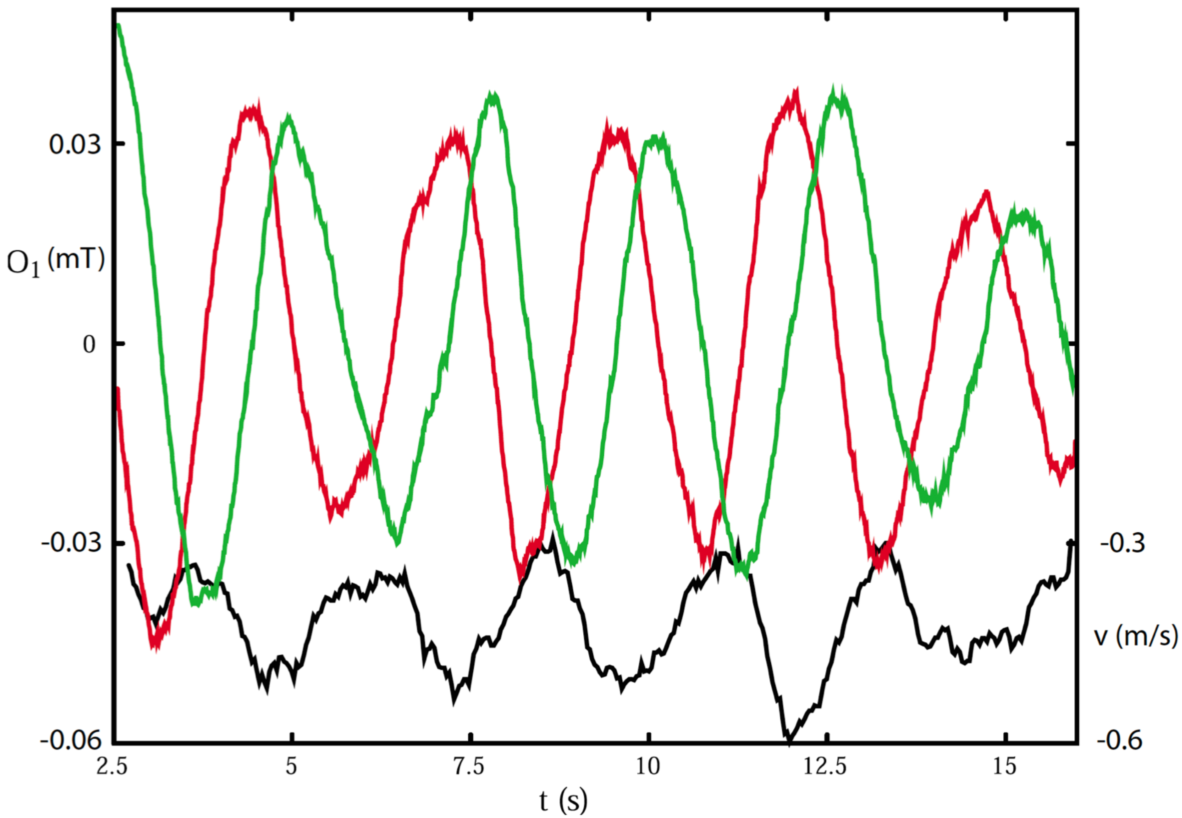
(Daniel R. Sisan, et al. 2004)

Simulation



(Christophe Gissinger,et al. 2011)

Standard MRI(SMRI)

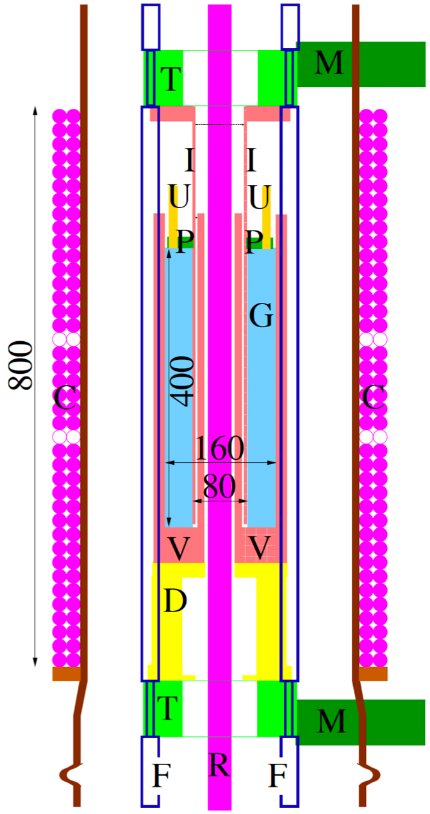


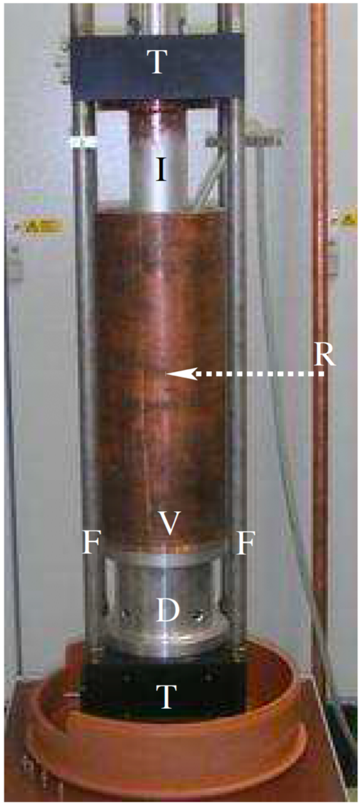
(Daniel R. Sisan, et al. 2004)

Coupled fluctuations in the velocity and induced magnetic fields.

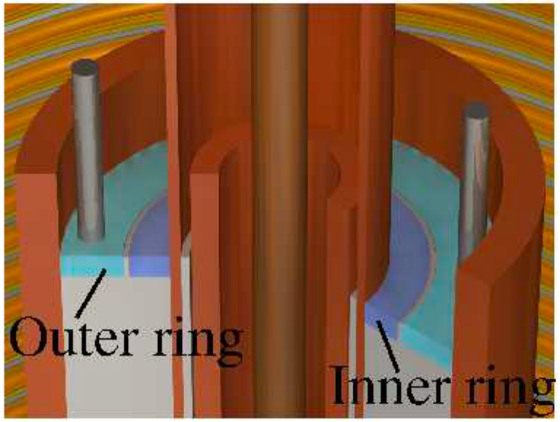
Background hydrodynamic turbulence?

Helical MRI (HMRI)





(Frank Stefani, et al. 2006)

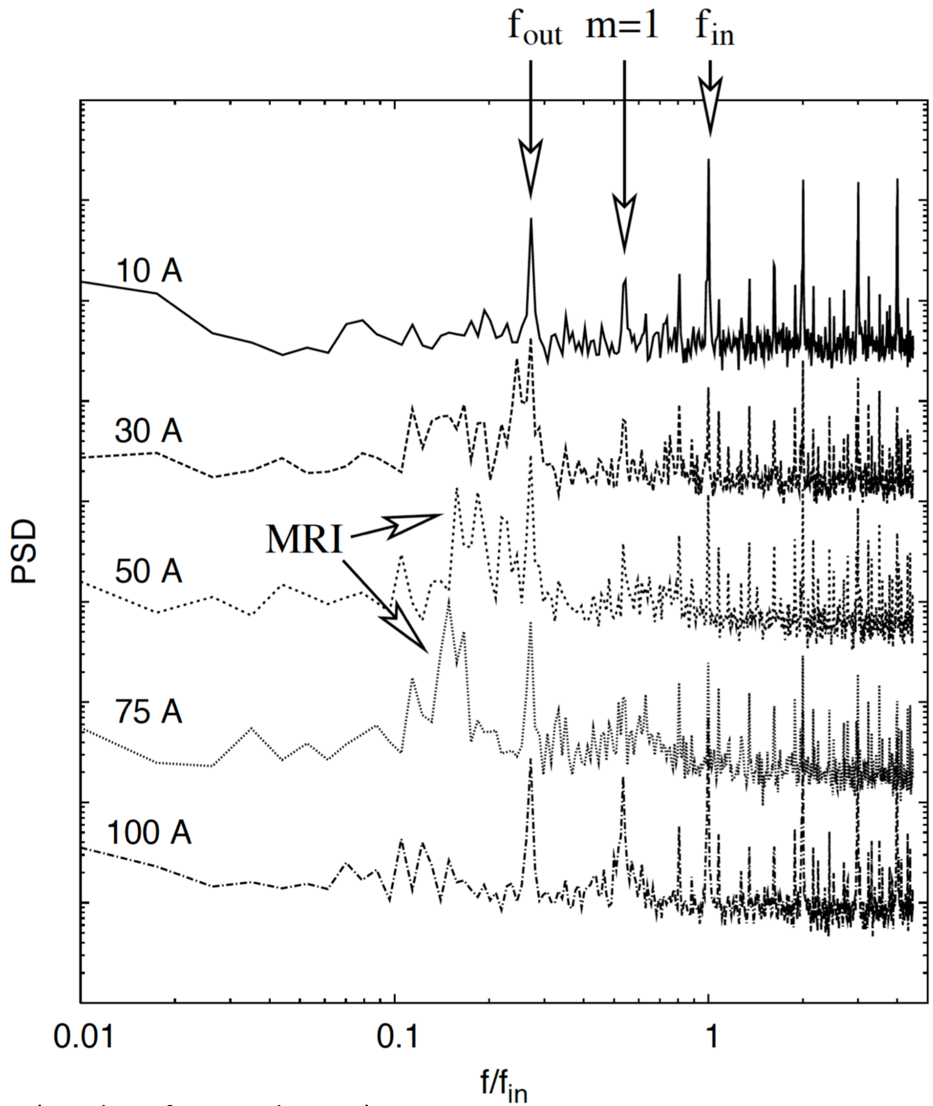


(Frank Stefani, et al. 2009)

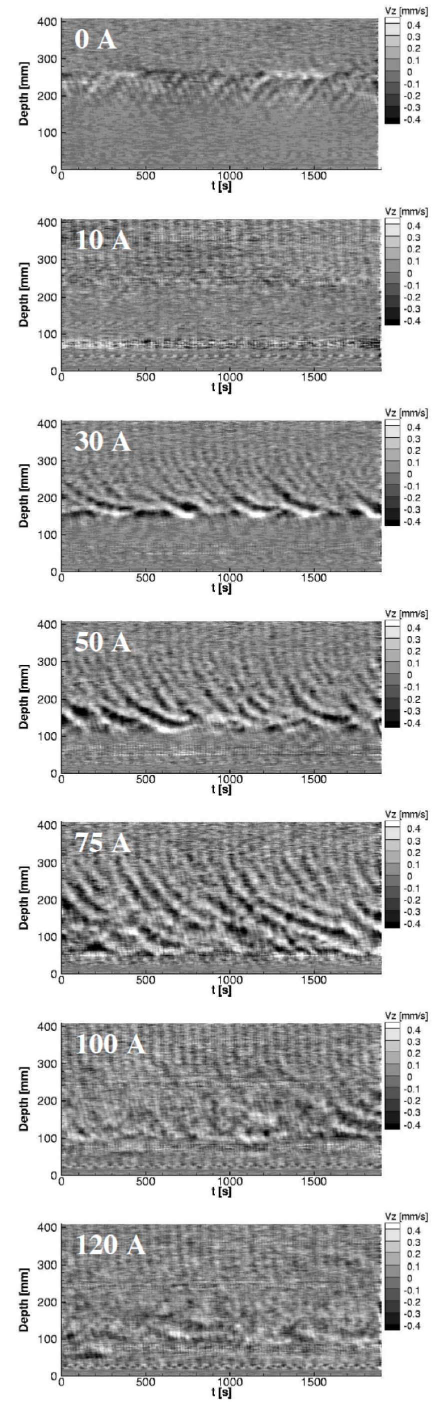
Alloy GaInSn

Azimuthal magnetic field, MRI is then possible with far smaller Reynolds (Re)

Helical MRI (HMRI)

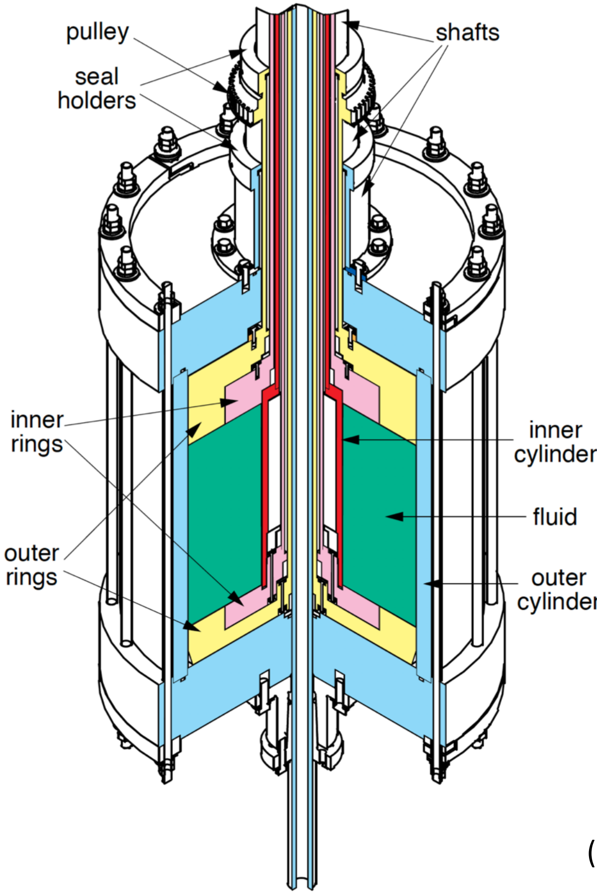


(Frank Stefani, et al. 2006)



Travelling waves

Non-magnetic flow



(Hantao Ji, et al. 2006)

The MRI (other than hydrodynamic instabilities) appears to be the only plausible source of accretion disk turbulence.

Magnetocoriolis waves

MHD equations in a rotating frame:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + 2\mathbf{\Omega}\times\mathbf{v} = -\nabla P + \frac{1}{\mu_o\rho}(\mathbf{B}\cdot\nabla)\mathbf{B} + \nu\nabla^2\mathbf{v}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \eta\nabla^2\mathbf{B}$$

$$P = \frac{p}{\rho} + \frac{1}{2}\frac{B^2}{\mu_0\rho} - \frac{1}{2}|\mathbf{\Omega}\times\mathbf{r}|^2$$

→ Dispersion relation

↓ limitation

Magnetocoriolis waves

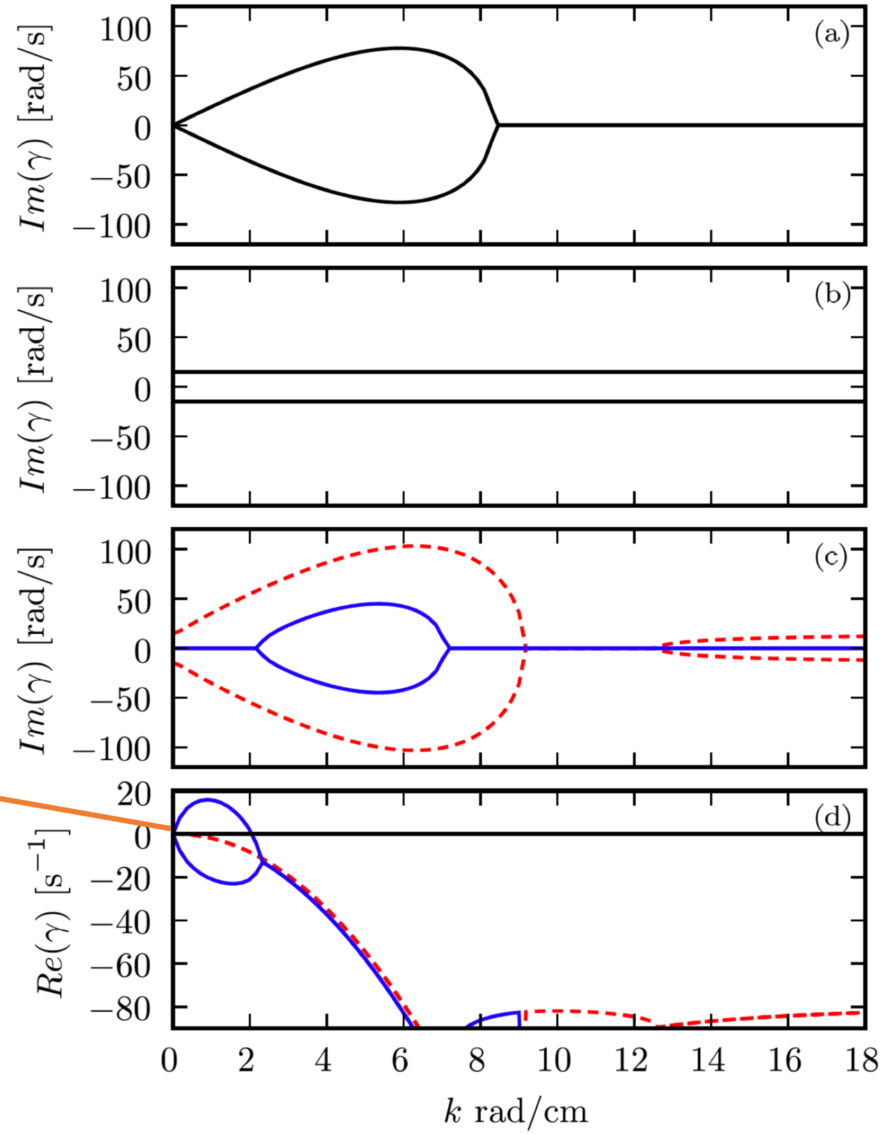
- Alfvénic wave(Lorentz forces) $(\omega - i\gamma_\nu)(\omega - i\gamma_\eta) - \omega_A^2 = 0$
- Inertial waves(Coriolis forces) $(\omega - i\gamma_\nu)^2 + (2\Omega k_z/k)^2 = 0$

→

- higher frequency fast wave
- lower frequency slow wave

Magnetocoriolis waves

Unstable—MRI



(M.D. Nornberg, et al. 2010)

Summary

- Preliminary experimental design can be done through Local (global) stability analysis.
- Standard MRI(SMRI) shows that Lorentz forces are key to the instability, but is affected by the hydrodynamical turbulence background.
- Helical MRI (HMRI) can produce characteristic travelling wave which is a main feature of MRI.
- It is magnetic field, rather than hydrodynamics, that sustains the angular momentum transport .
- Great debate: effects of the no-slip axial boundaries found in Taylor-Couette experiments which do not match the open stratified boundaries of accretion disks (Freja Nordsiek1,et al. 2014).