Special Topics in Theoretical Astrophysics (2019 Spring)

Fermi acceleration of particles

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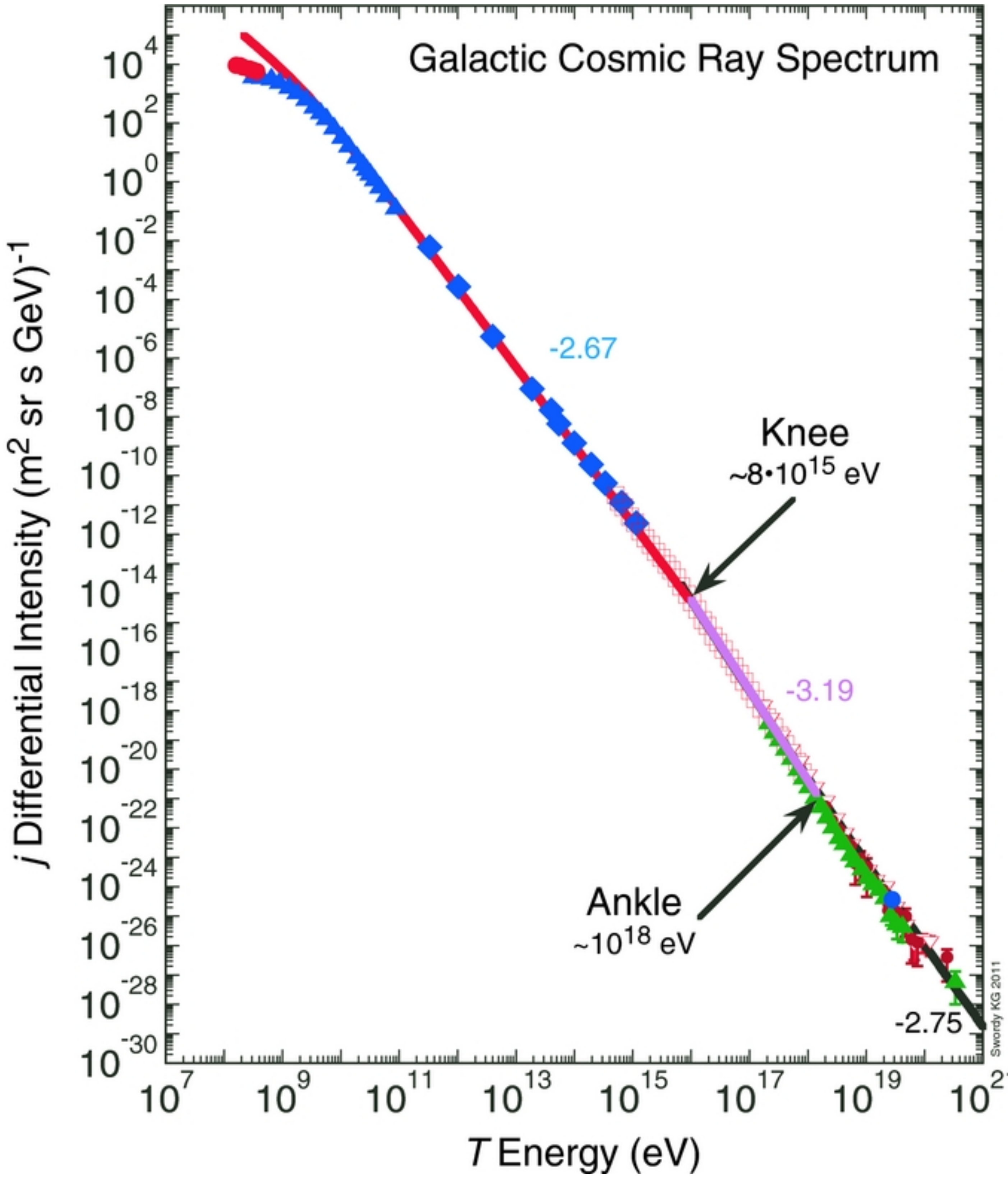
Advised by Prof. Xuening Bai

Why do we want to know how particles get accelerated?



Fisk L A, Gloeckler G. Acceleration of galactic cosmic rays in the interstellar medium[J]. The Astrophysical Journal, 2011, 744(2): 127.

1949

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

Second-order acceleration mechanism: not fast enough!

1954

THE ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS
VOLUME 119 JANUARY 1954 NUMBER 1

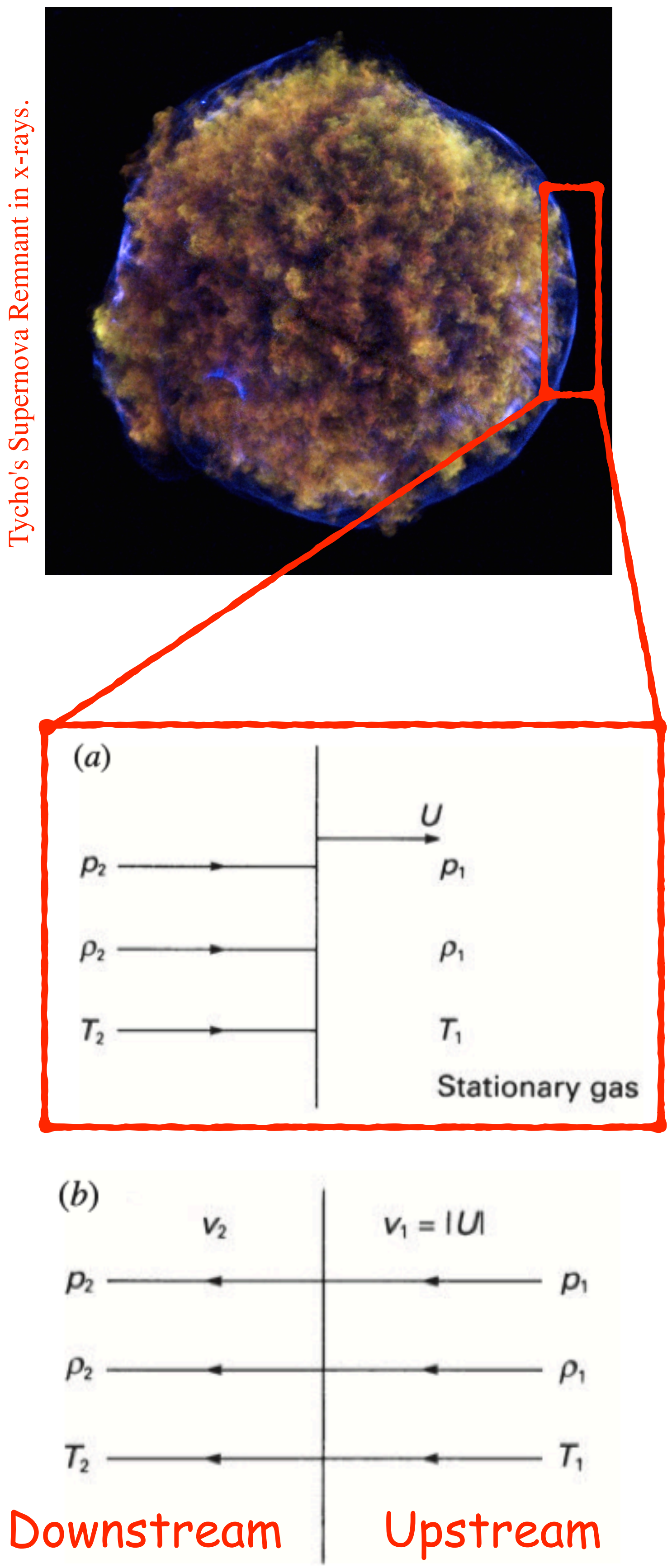
GALACTIC MAGNETIC FIELDS AND THE ORIGIN OF COSMIC RADIATION*

E. FERMI
Institute for Nuclear Studies, University of Chicago
Received September 11, 1953

apply to the shock

first-order acceleration mechanism
a.k.a. Diffusive Shock Acceleration(DSA)

What is shock?



Shock reference frame

perturbation velocity $\gg$ sound speed

↓

Mass conservation : $\rho_1 v_1 = \rho_2 v_2$

Energy conservation : $\rho_1 v_1 \left(\frac{1}{2} v_1^2 + w_1 \right) = \rho_2 v_2 \left(\frac{1}{2} v_2^2 + w_2 \right)$

Momentum conservation : $p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2$

↓

Mach number : $M_1 = U/c_1 = v_1/c_1$

Ratio of specific heat capacities : γ

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{(\gamma+1)}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)}{(\gamma-1)}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)M_1^2}{(\gamma+1)^2}$$

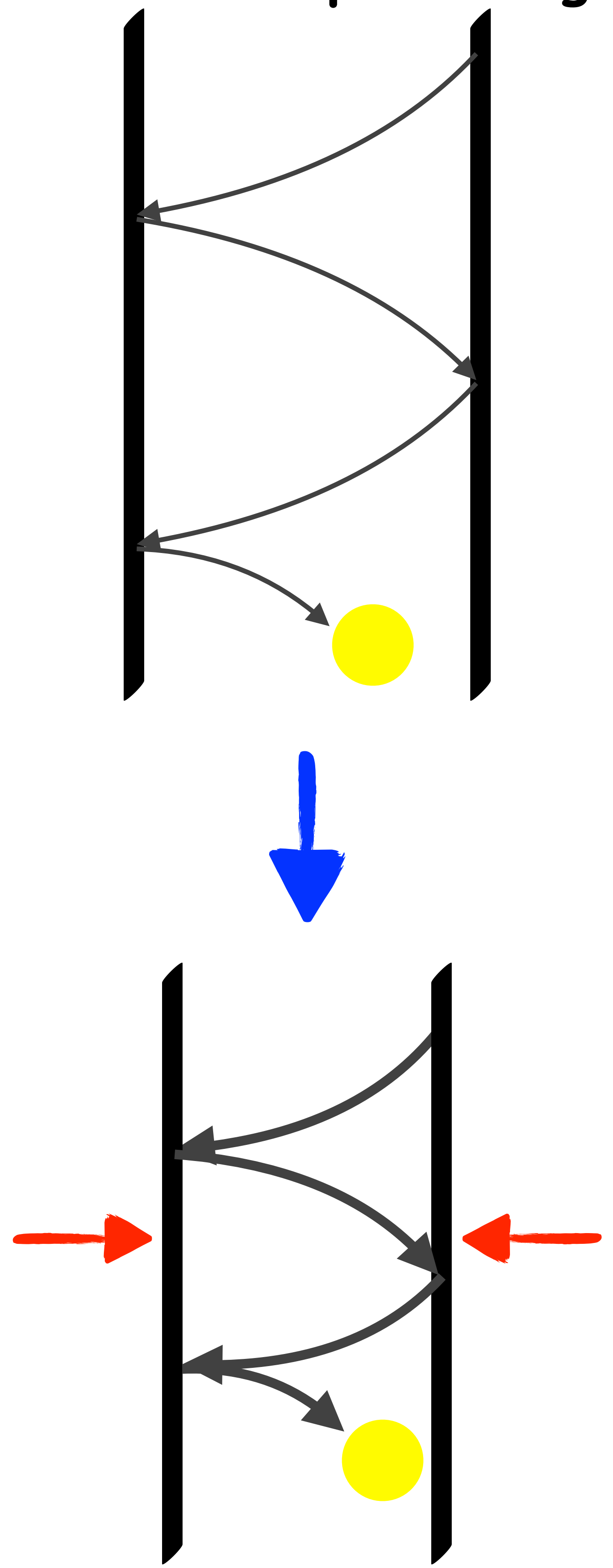
$\gamma = \frac{5}{3}, \quad M_1 \gg 1$

→

$$\rho_2/\rho_1 = 4$$

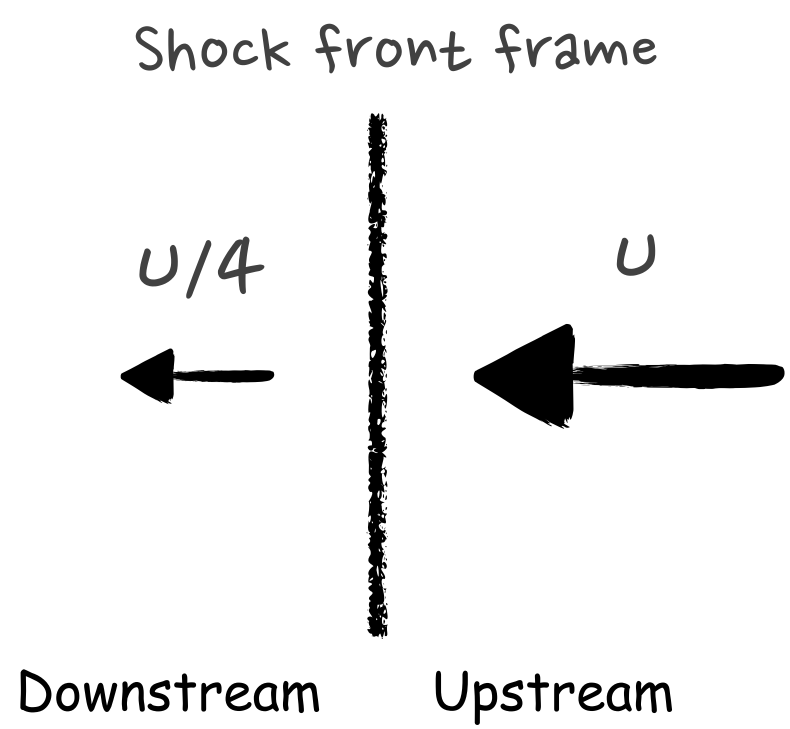
$$v_1/v_2 = 4$$

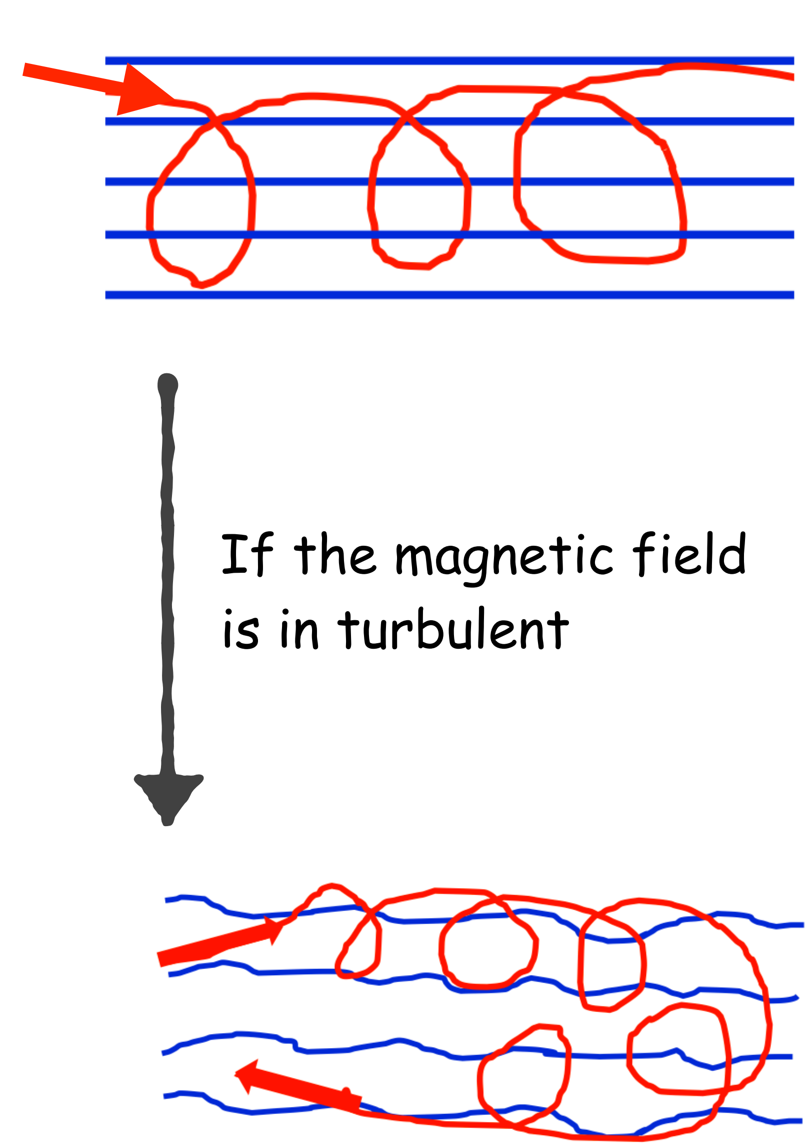
How does a particle get accelerated?

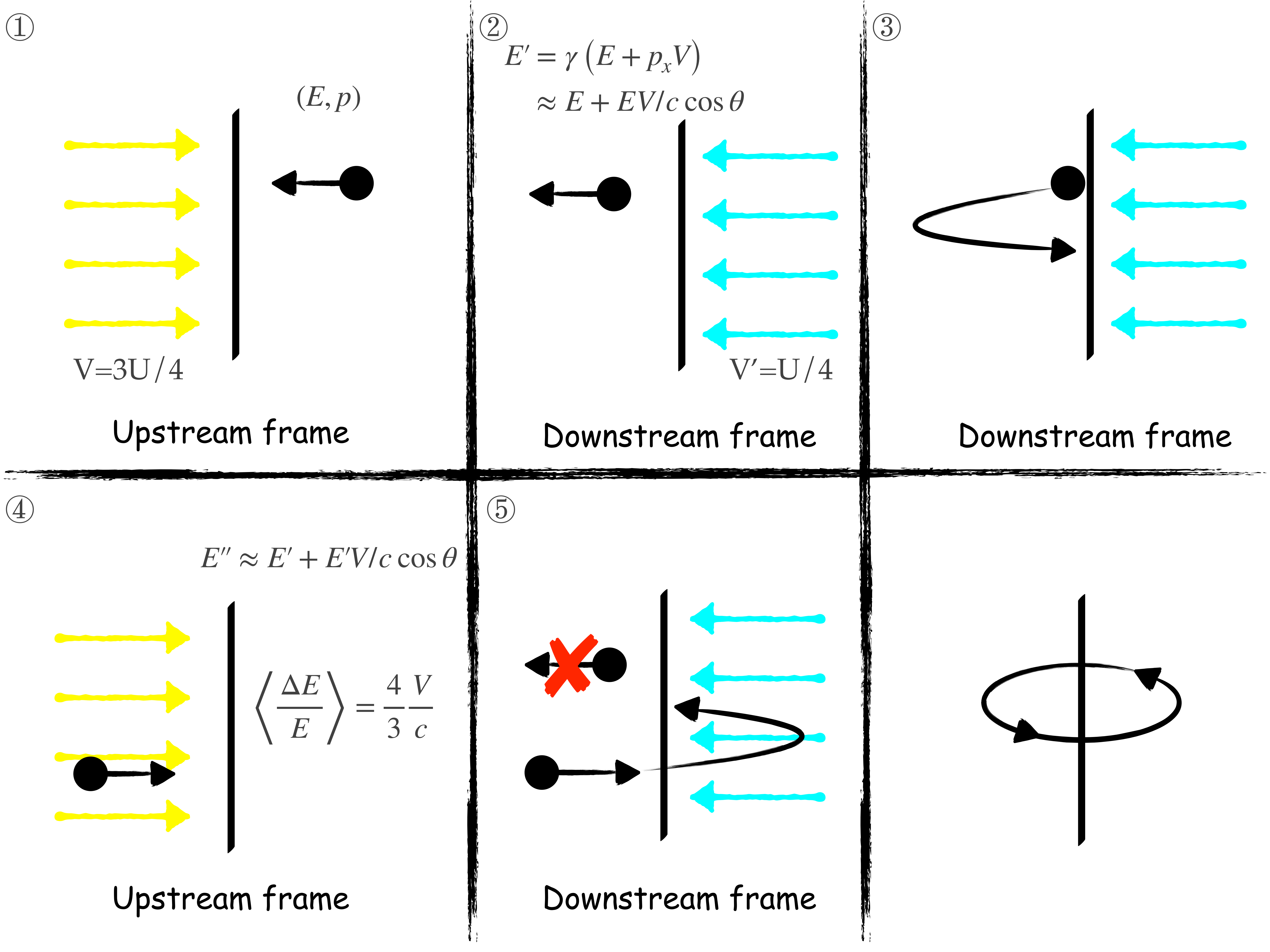


Conditions for the Pingpong to accelerate:

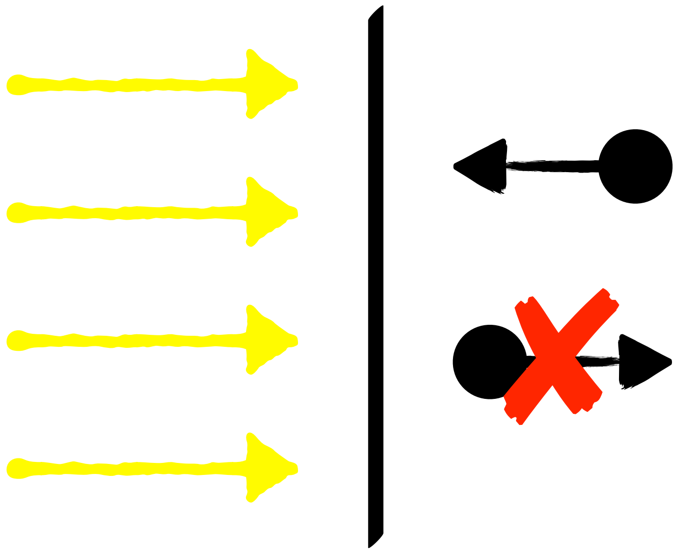
- ☑ Two walls are getting closer
- ☑ It is better to be elastic collision

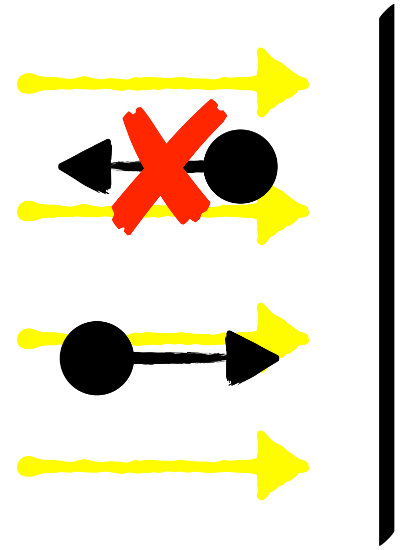






①
(E, p)
V=3U/4
Upstream frame
②
$E' = \gamma\left(E + p_x V\right)$
$\approx E + EV/c\cos\theta$
V′=U/4
Downstream frame
③
Downstream frame
④
$E'' \approx E' + E'V/c\cos\theta$
$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}\frac{V}{c}$
Upstream frame
⑤
Downstream frame





$$P_{\text{escape}} = \frac{N_{\text{escape}}}{N_{\text{left}}} = \frac{U}{c}$$

$$P = 1 - P_{\text{escappe}} = 1 - \frac{U}{c}$$

$$\frac{dN_{\text{left}}}{dSdt} = \int_{\pi/2}^{\pi} \frac{1}{2} Nc \cos\theta \sin\theta d\theta = \frac{Nc}{4}$$

$$\frac{dN_{\text{escape}}}{dSdt} = \frac{NU}{4}$$

Energy of the particle after one collision: $E = E_0(1 + \frac{U}{c})$

Number of particles staying in the acceleration region after one collision: $N = N_0(1 - \frac{U}{c})$

After k collisions: $E = E_0\left(1 + \frac{U}{c}\right)^k$ $\quad N = N_0\left(1 - \frac{U}{c}\right)^k$

Eliminating the k and using $U \ll c$

$$N(E)\mathrm{d}E \propto E^{-2}\mathrm{d}E$$

Strong shock
Relativistic particles

$$f(p)dp \propto p^{-4}dp$$

More general version

$$N(E)\mathrm{d}E \propto E^{-1.5}\mathrm{d}E$$

Strong shock
Non-relativistic particles

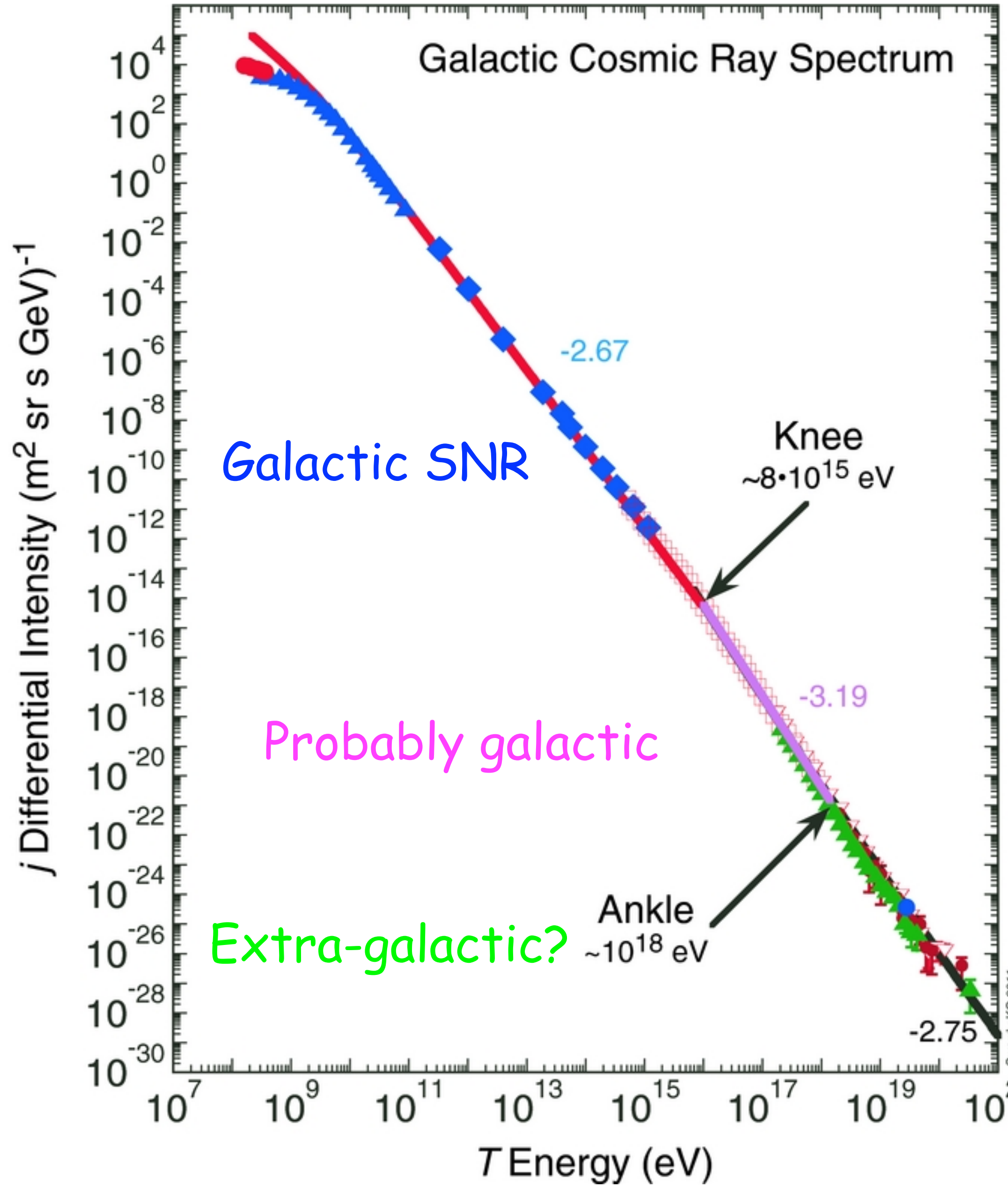
Power-law slope:

Simple leaky-box model: $$\frac{\partial N}{\partial t} = Q(E) - \frac{N(E)}{t_{\rm esc}(E)}$$

Steady-state: $$\frac{\partial N}{\partial t} = 0 \quad \Rightarrow \quad N(E) = Q(E)t_{\rm esc}(E)$$

Using $t_{\rm esc} \propto E^{-0.6}$, $Q(E) \propto E^{-2}$

$$N(E) \propto E^{-2.6}$$



Maximum energy:

Order of magnitude analysis: $$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\Rightarrow E \sim BU$$

$$\Rightarrow E_{\rm max} = \int zeE{\rm d}x = zeBUL$$

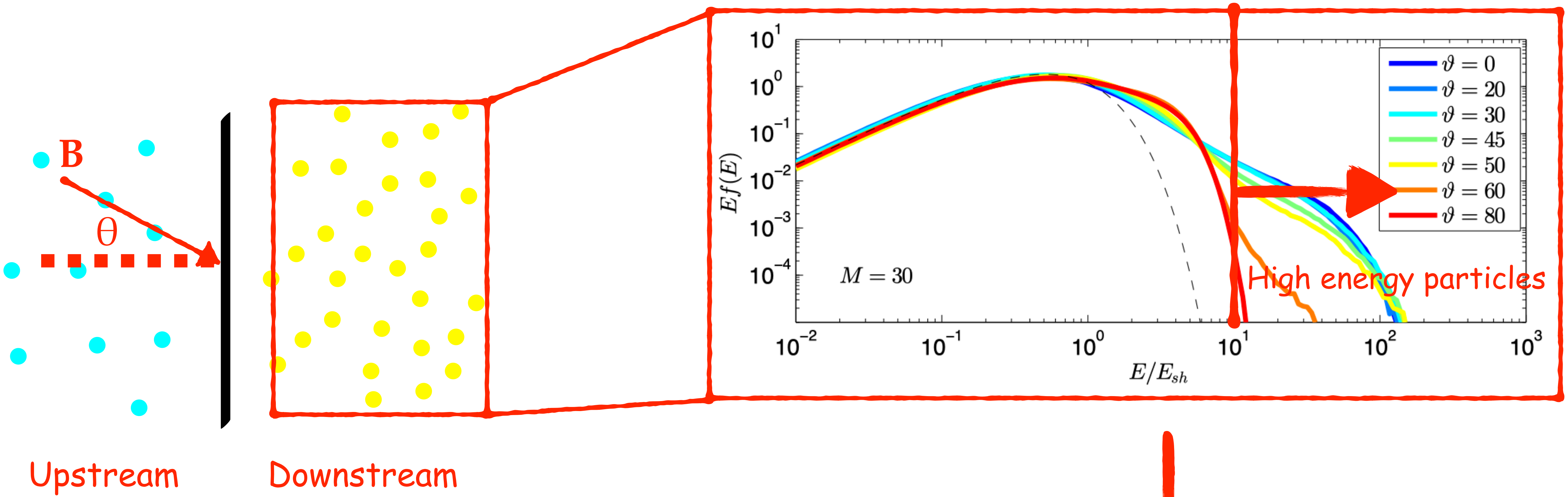
$$B = 10^{-10}{\rm T},\ U = 10^4{\rm kms}^{-1},\ t \approx 10^3 \text{ years}$$

$$E_{\rm max} \sim 10^{14}{\rm eV}$$

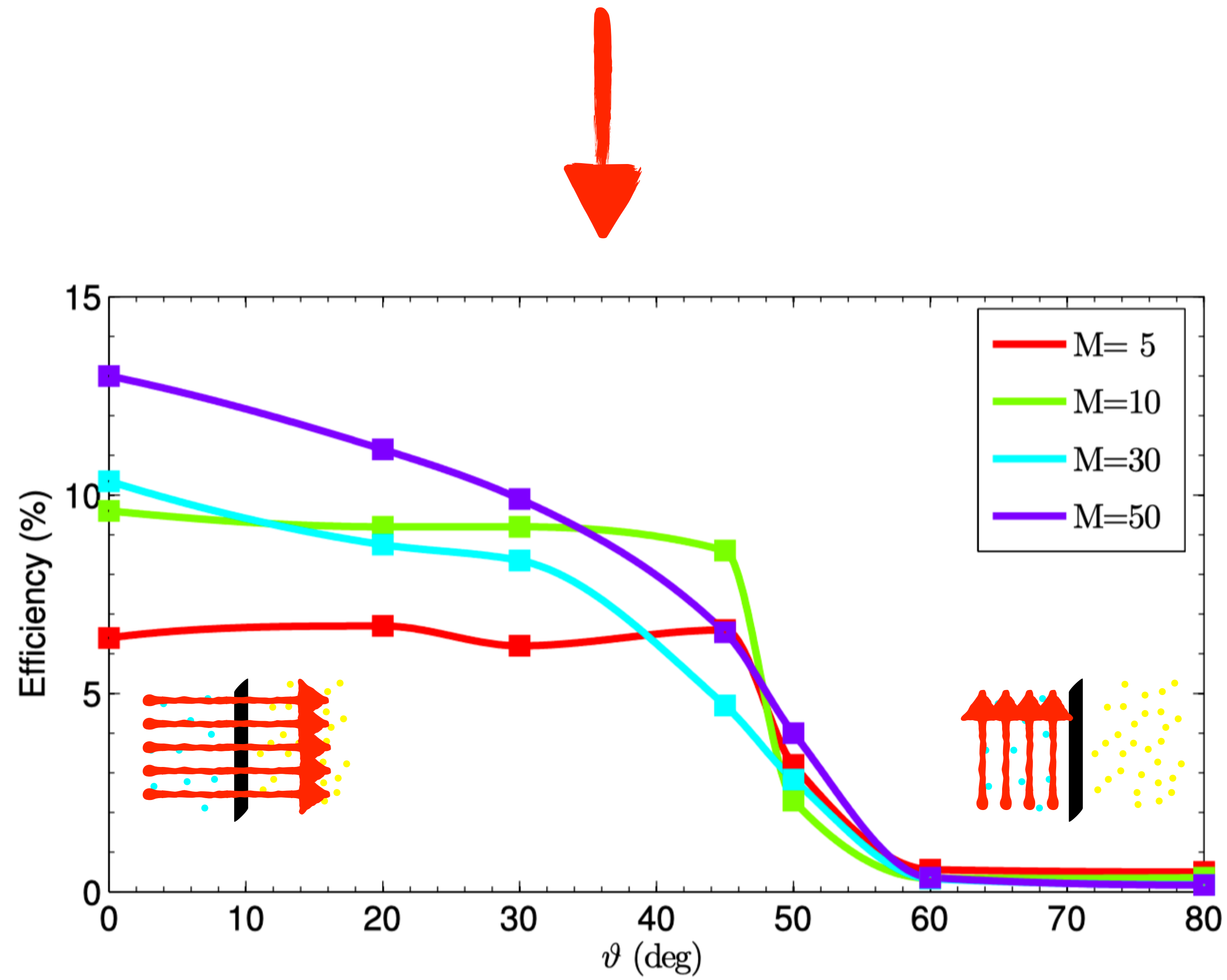
Limitation of Fermi acceleration mechanism

1. Cannot give us the efficiency of the energy conversion
2. Cannot explain the origin of turbulent magnetic field
3. Cannot explain the extreme high energy particles
4. Cannot explain how the particle get their energy initially
5. ……

1. Efficiency of the acceleration



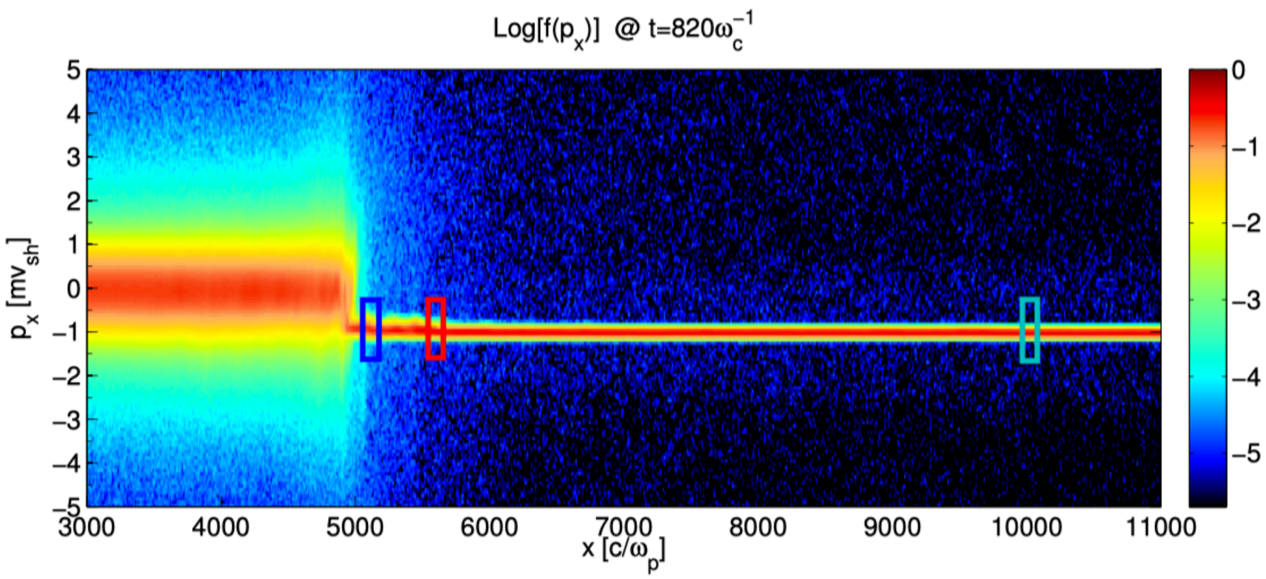
- Significant energy transferred to high energy particles
- Orientation of background magnetic field is critical
- Strong shock can accelerate particles effectively

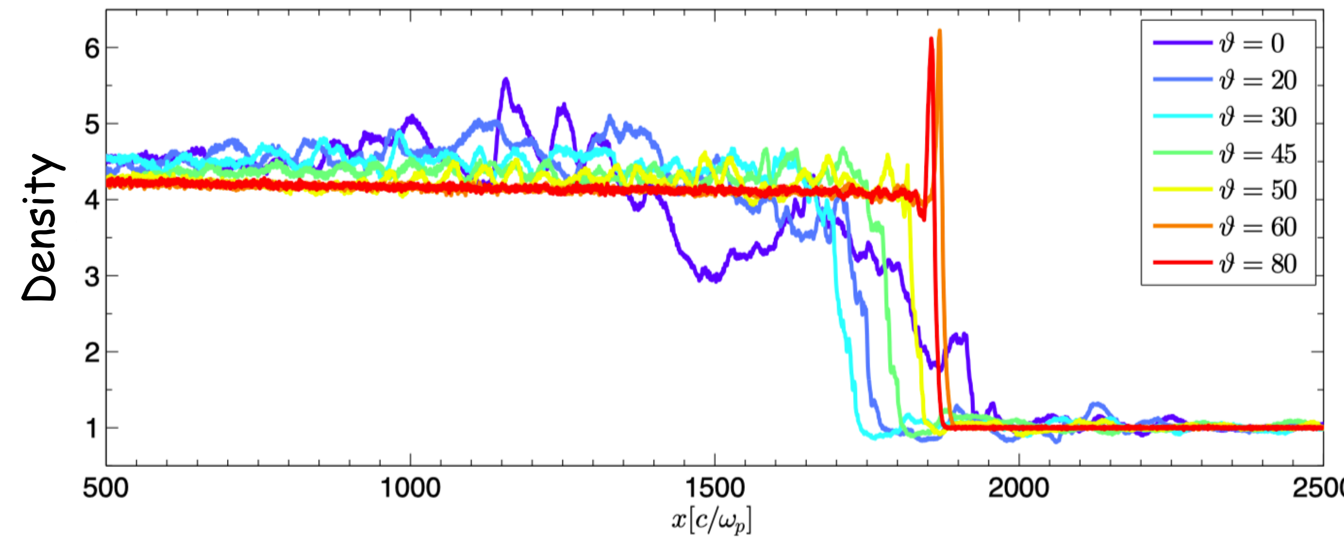


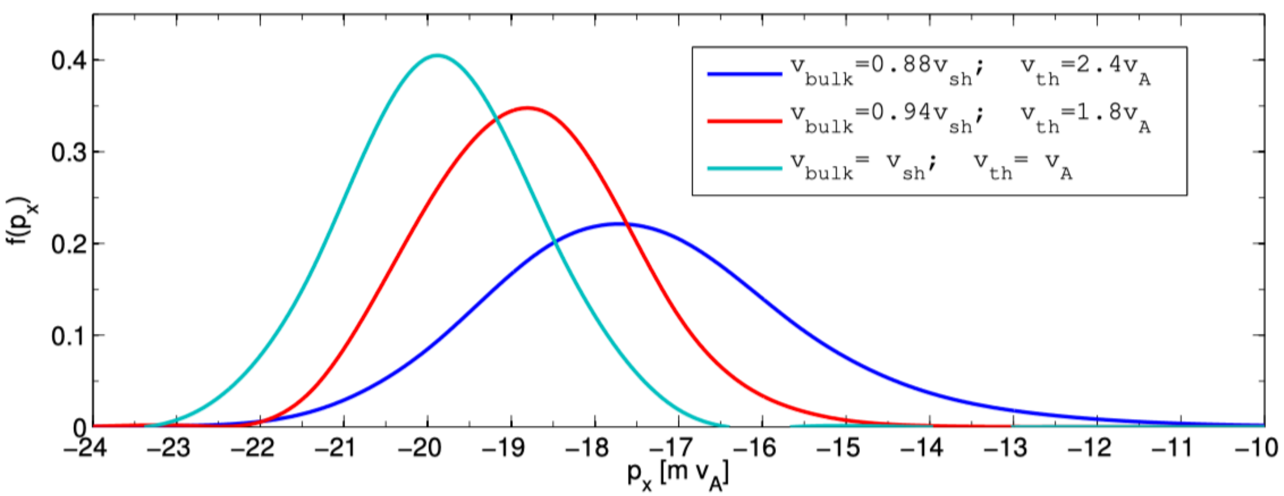
Caprioli D, Spitkovsky A. Simulations of ion acceleration at non-relativistic shocks. I. Acceleration efficiency[J]. The Astrophysical Journal, 2014, 783(2): 91.

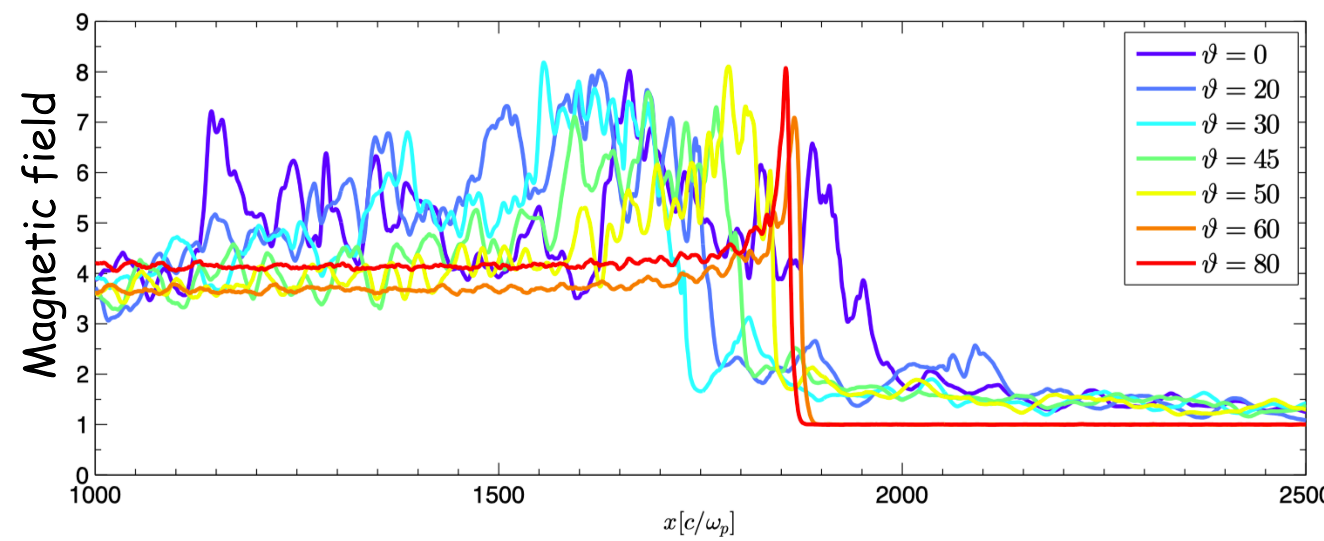
2. Back-reaction from the high energy particles

Since there are noticeable energy transferred to high energy particles, does this modify the shock itself?









- High energy particles can modify the phase space property and magnetic field
- The turbulent magnetic field could be much greater than the background

Summary

- DSA provides us an efficient way to accelerate particles
- DSA predicts a universal power-law energy distribution up to 1e14 eV
- DSA can convert the energy to high energy particles efficiently
- High energy particles can induce the turbulent magnetic field to accelerate themselves