Magnetic Reconnection
and
Plasmoid Instability

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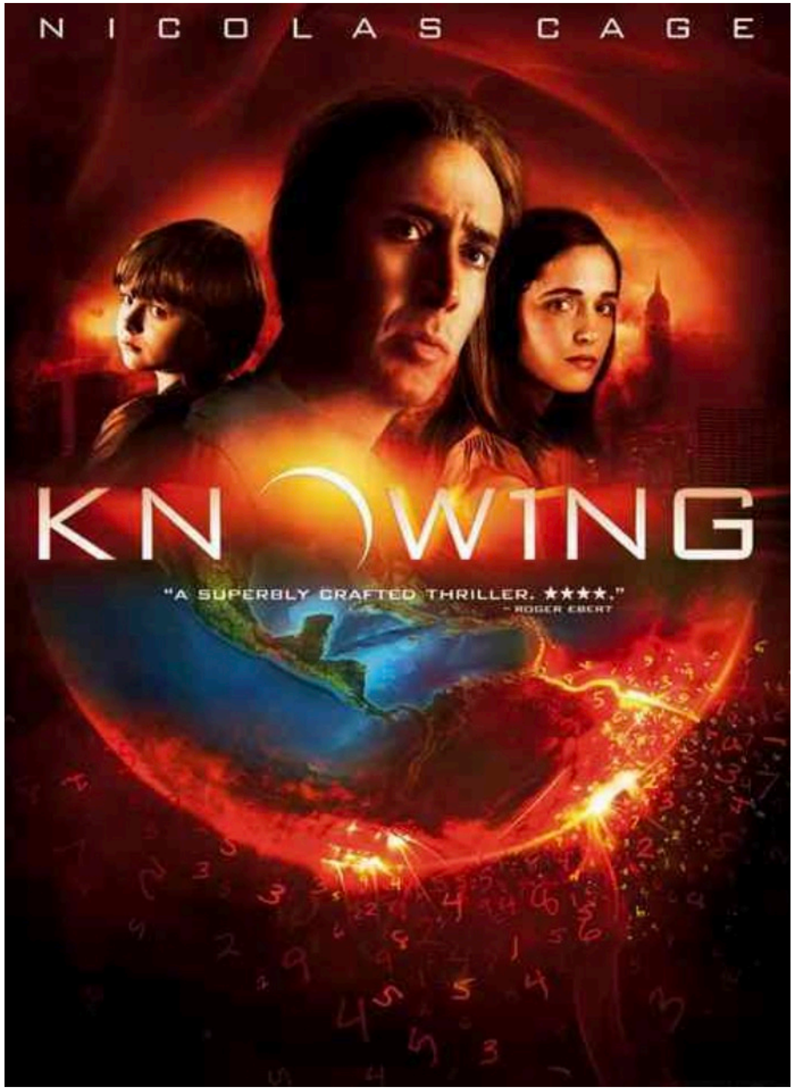
Supervised by Prof.Xuening Bai

Outline

- Introduction
- Therical model of plasmoid formation
- Simulation results of plasmoid chains and instability
- Summary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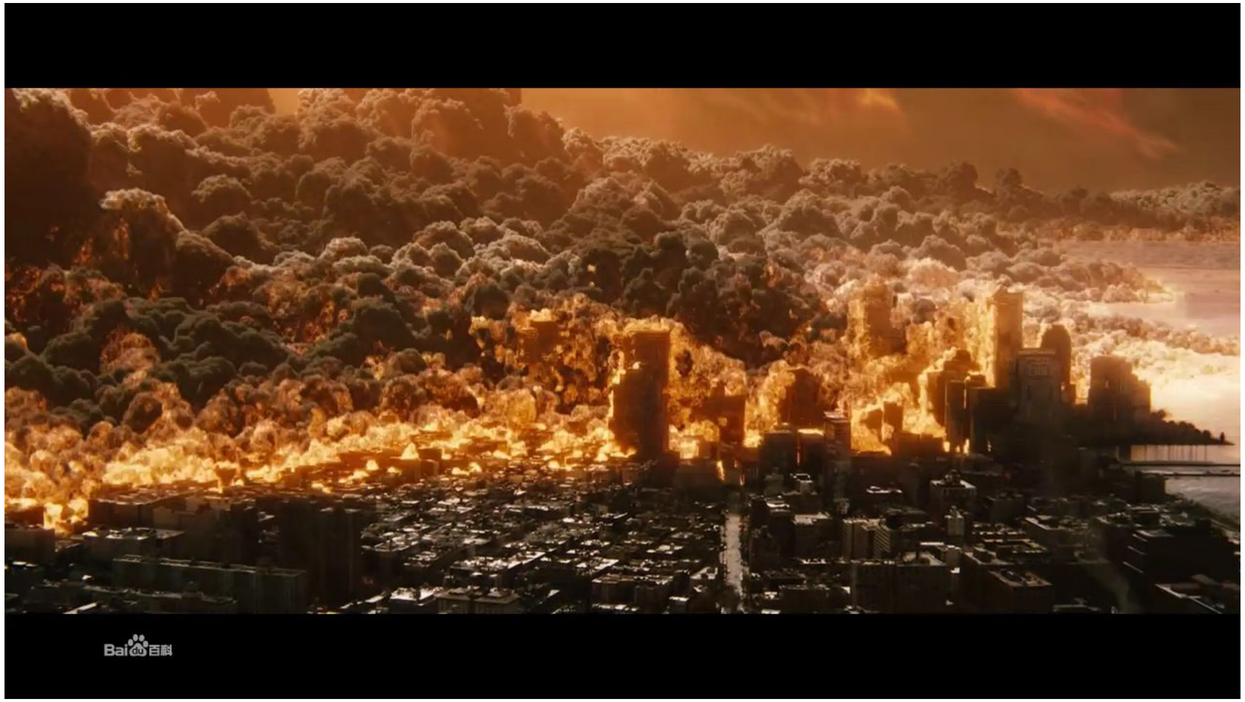
KNOWING
"A SUPERBLY CRAFTED THRILLER. ★★★★."
- ROGER EBERT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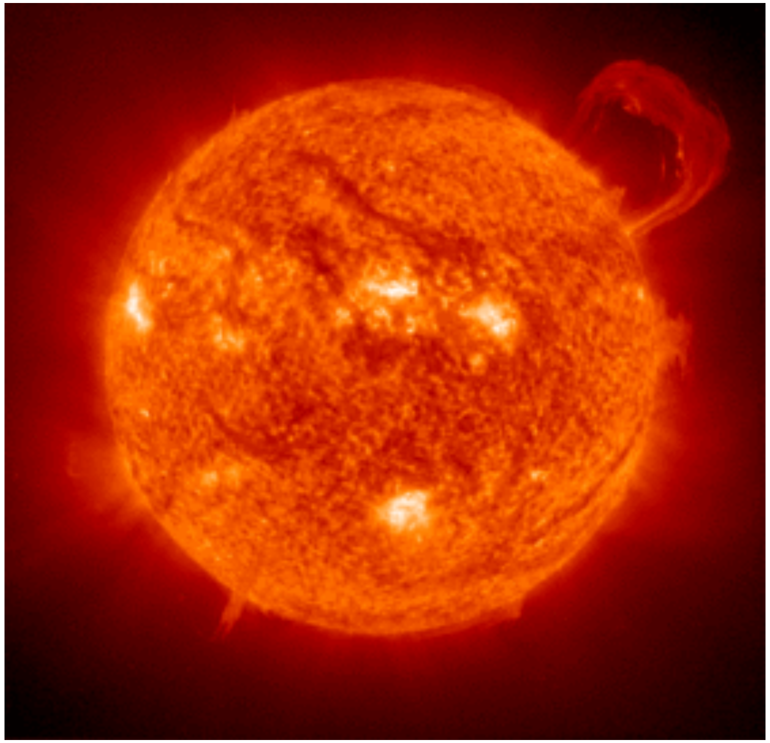
Solar flares and coronal mass ejection

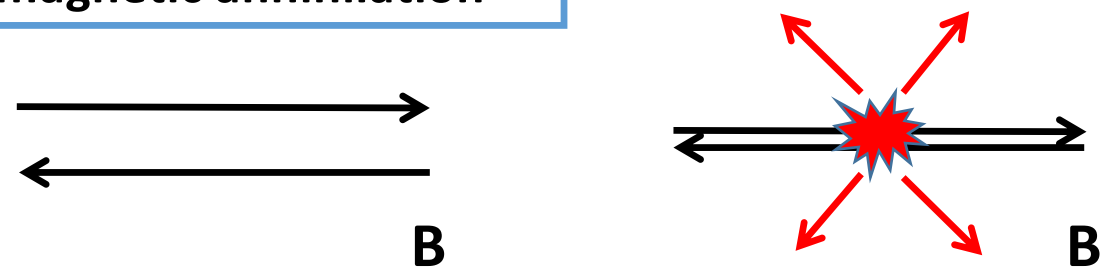


Solar flares are large eruptions of electromagnetic radiation from the Sun lasting from minutes to hours.

Solar flares usually take place in active regions, which are areas on the Sun marked by the presence of strong magnetic fields.

magnetic fields evolution → instability → release energy

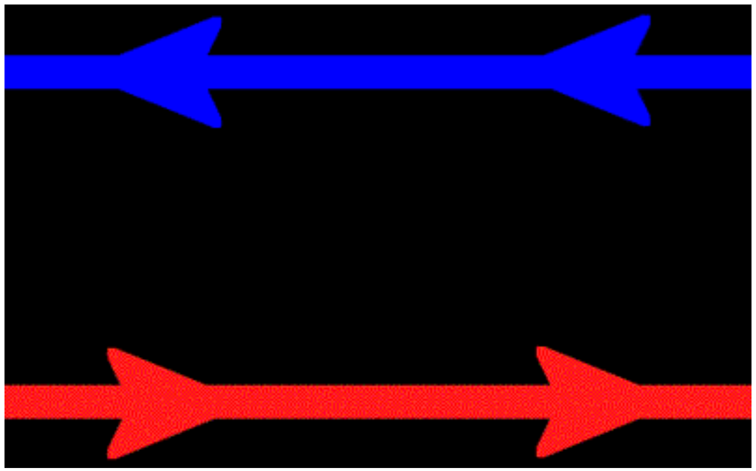
magnetic annihilation

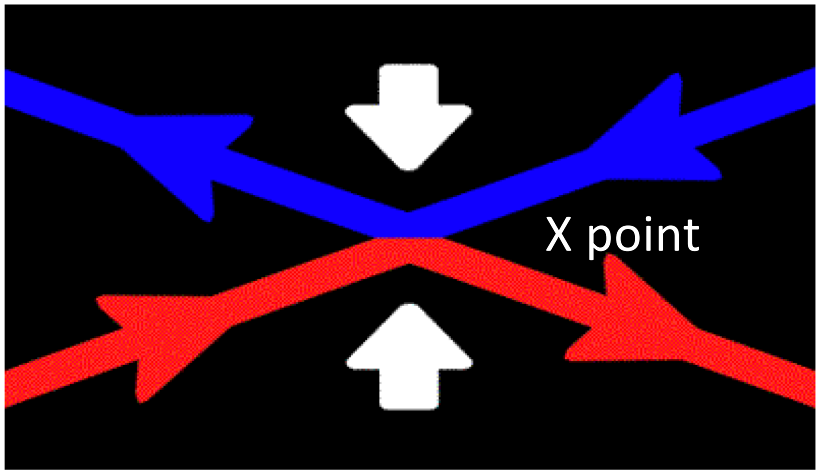


Giovanelli, R. G., 1946: Nature 158, 81

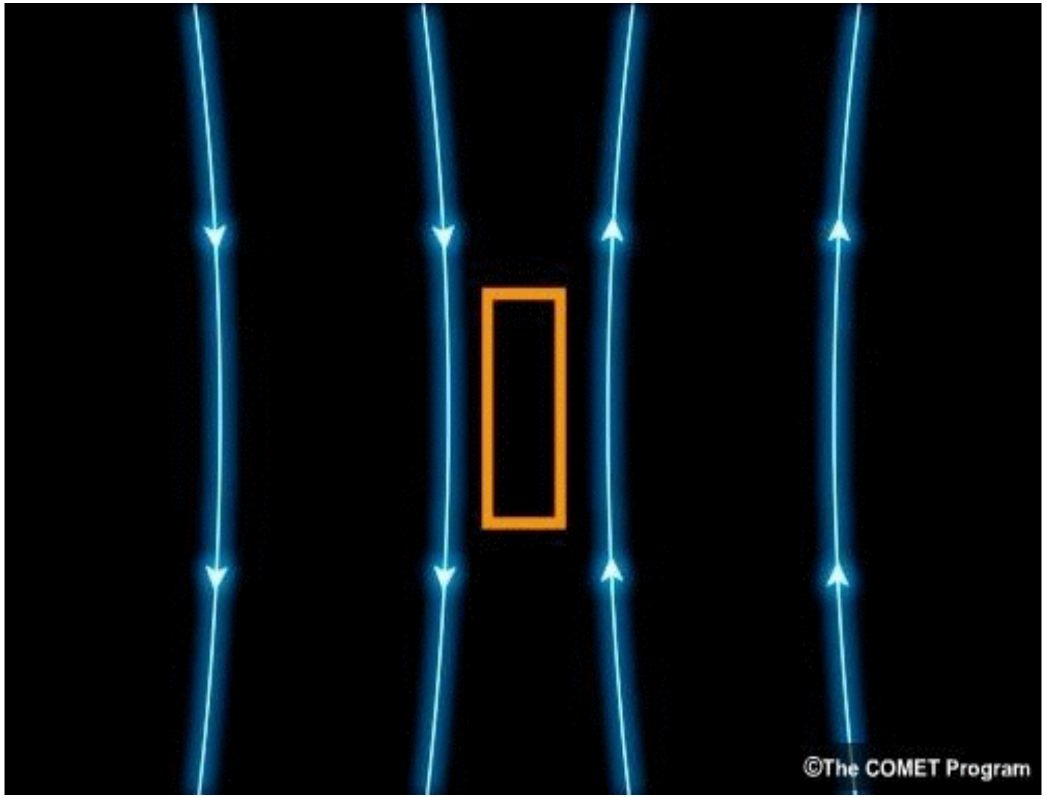
Magnetic Reconnection

- A rapid rearrangement of magnetic field topology plasma phenomenon in which oppositely directed magnetic field lines are driven together, break and rejoin.
- A violent release of magnetically-stored energy and its conversion into heat and into non-thermal particle energy.

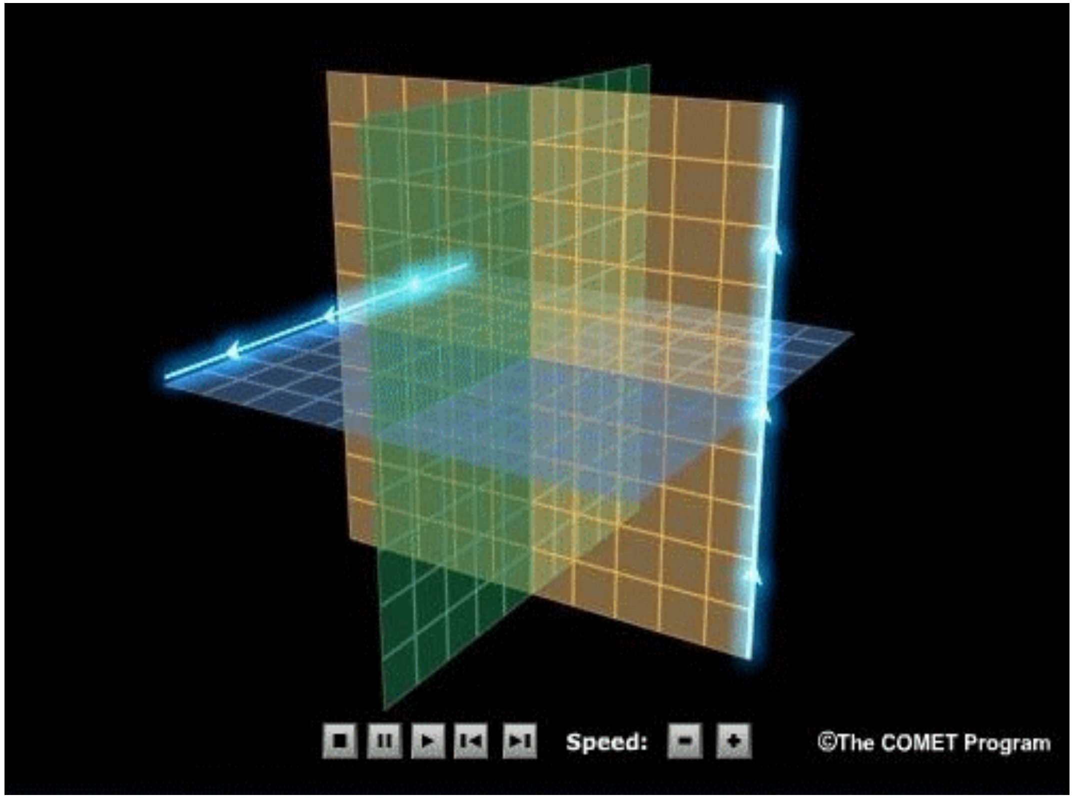




2D magnetic reconnection



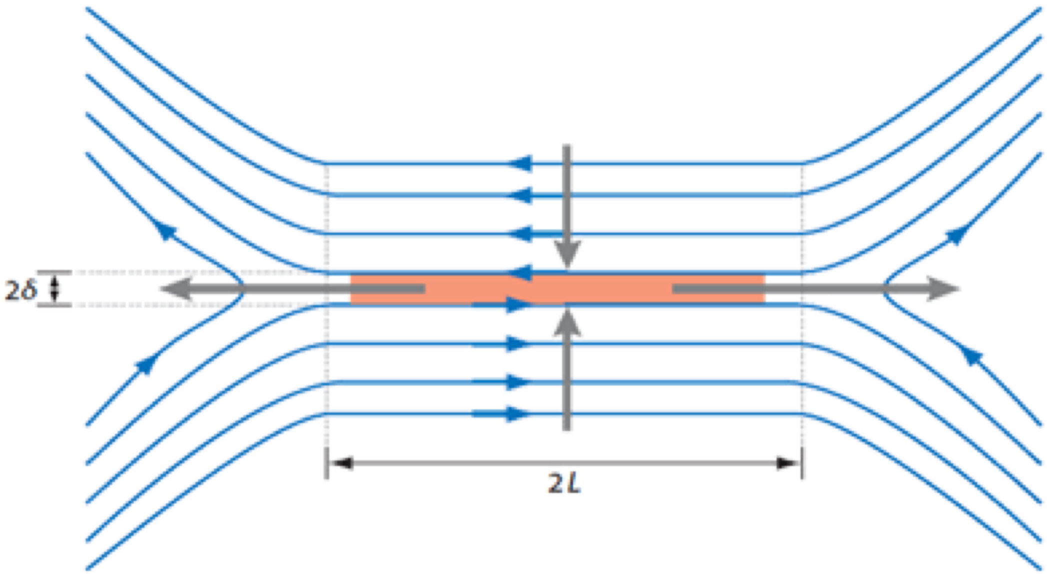
3D magnetic reconnection

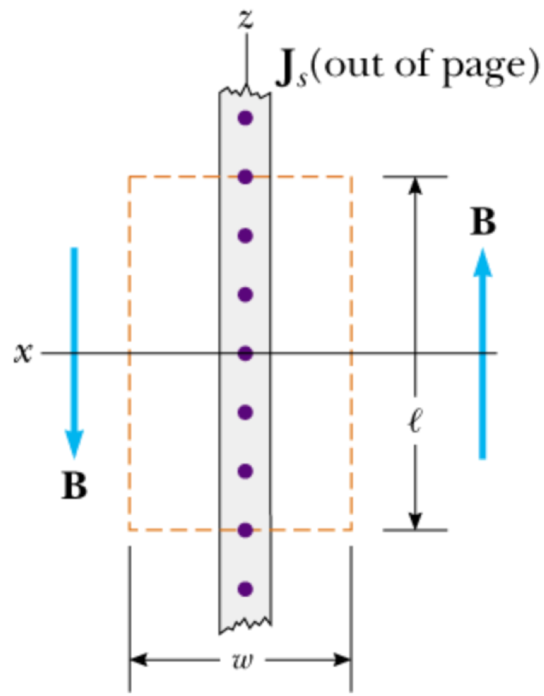


Therical models of magnetic reconnection

Sweet-Parker (SP) model

current sheet : a steady-state channel of length L and thickness δ_{SP}





Ampere's law

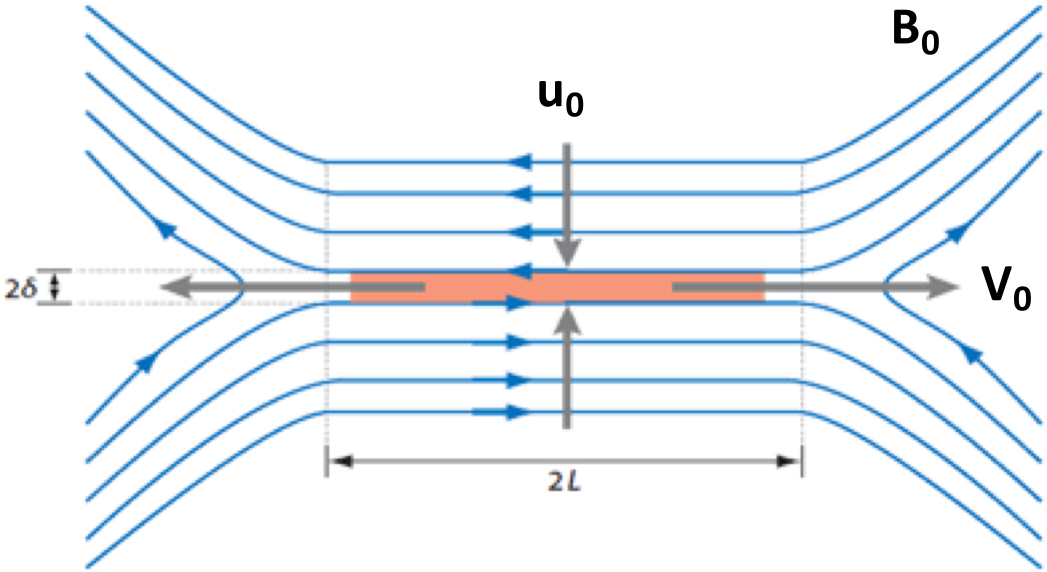
$$\oint_l B \bullet dl = \mu_0 \sum_{i=1}^{n} I_i$$

$$J_S = \frac{2B}{\mu_0}$$

Parker, E.N. 1957
Sweet, P.A. 1958
Zweibel & Yamada (2009)

Therical models of magnetic reconnection

Sweet-Parker (SP) model



2L: Current sheet's typical length

2δ: Current sheet's typical width

u_0: Inflow velocity

V_0: Outflow velocity

Lundquist number

$$S = \frac{LV_A}{\eta} \gg 1$$

V_A: Alfven speed

η: Resistivity

N F Loureiro, D A Uzdensky， 2015

From the resistive MHD equations :

① $V_0 = \frac{B_0}{\sqrt{\mu_0 \rho}} \equiv V_{A0}$

② reconnection rate $\frac{u_0}{V_0} \sim S^{-\frac{1}{2}}$

reconnection time $\tau_{rec} \sim \tau_A S^{\frac{1}{2}} \Leftarrow \tau_A = \frac{L}{V_A}$

③ aspect ratio $\frac{L}{\delta} \sim S^{\frac{1}{2}}$

Therical models of magnetic reconnection

Sweet-Parker (SP) model

$$S = \frac{LV_A}{\eta} \gg 1$$

Solar coronal conditions: $S \sim 10^{12} - 10^{14}$
Earth's magnetotail: $S \sim 10^{15} - 10^{16}$
modern tokamak such as JET: $S \sim 10^{6} - 10^{8}$

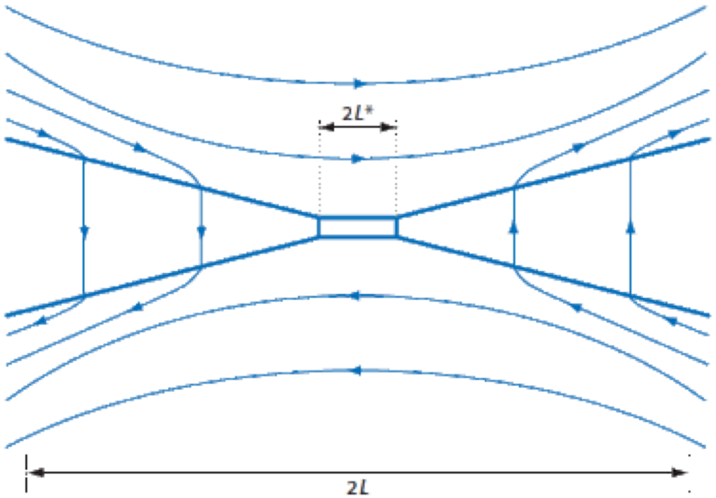
$$\tau_{rec} \sim \tau_A S^{\frac{1}{2}}$$

In solar-corona conditions: $\tau_A \approx 0.5$s

$\tau_{rec} \approx 2$ months

observed duration: 15mins ~ several hrs

Petschek model



$$L \gg L_*$$

$$\tau'_A = \frac{L_*}{V_A}$$

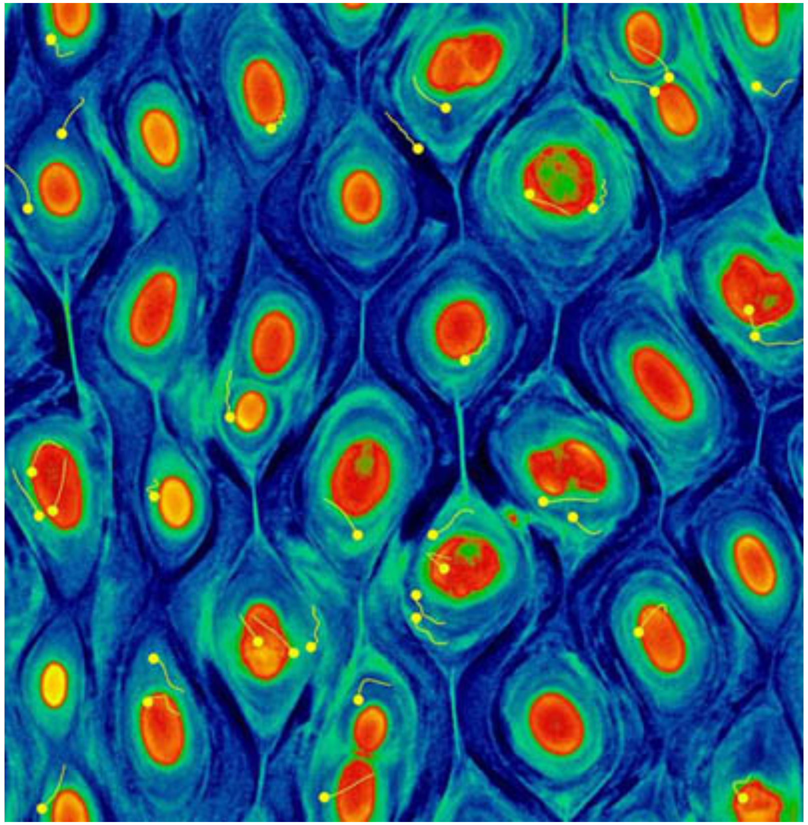
$$\tau'_{rec} \sim \tau'_A S^{\frac{1}{2}}$$

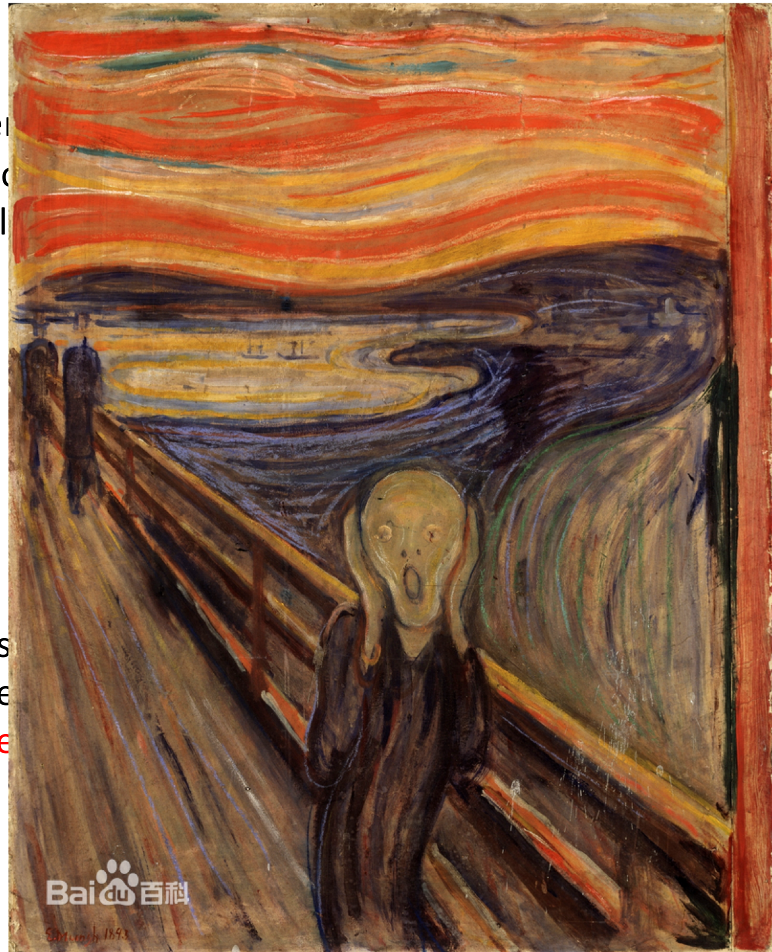
$$\frac{\tau'_{rec}}{\tau_{rec}} \sim \frac{L_*}{L}$$

Petschek, H.E., 1964

Plasmoid instability: another way to explain the reconnection rate

plasmoid





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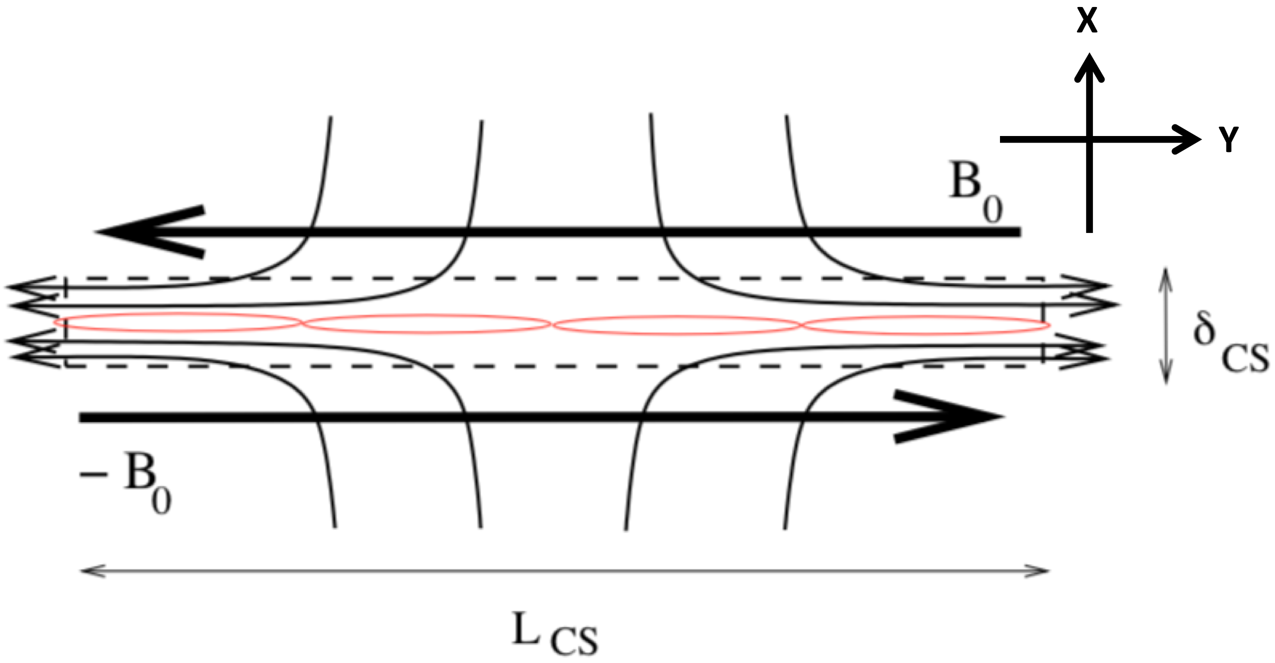
Lorenzo Sironi & Anatoly Spitkovsky

Therical model of plasmoid chains formation



$$\delta_{CS} \sim \frac{L_{CS}}{\sqrt{S}}$$

$$\tau_{SP} = \frac{L_{CS}}{V_A}\sqrt{S}$$

Induction equation

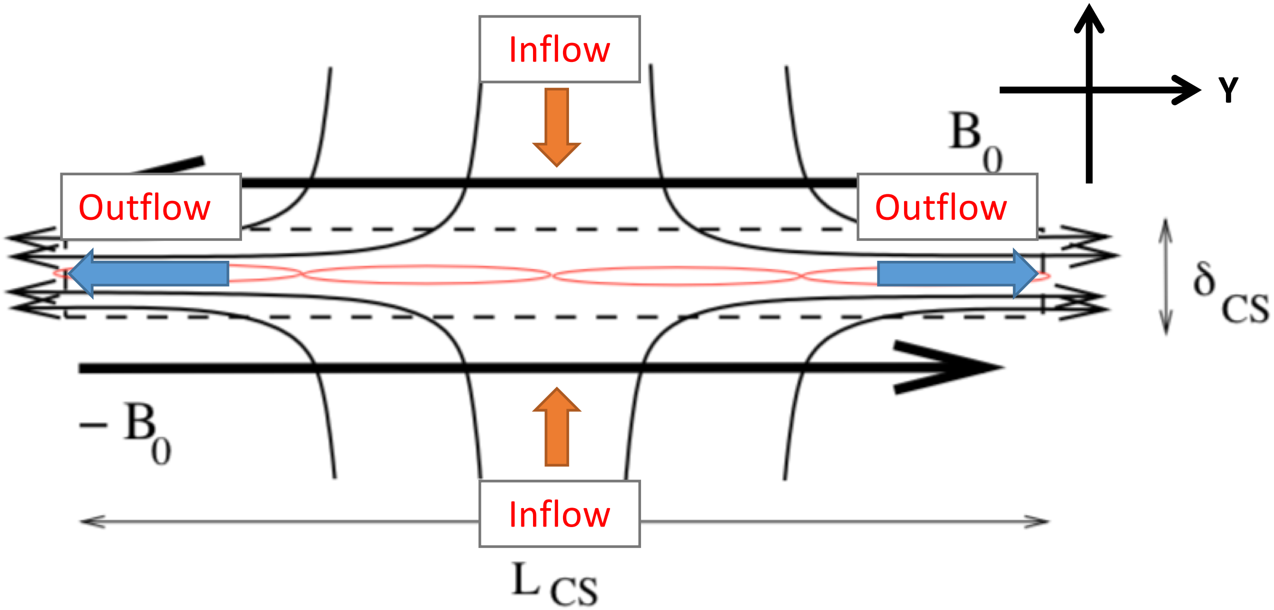
$$\partial_t \boldsymbol{B} + \boldsymbol{u} \cdot \boldsymbol{\nabla B} = \boldsymbol{B} \cdot \boldsymbol{\nabla u} - \boldsymbol{B \nabla} \cdot \boldsymbol{u} + \eta \nabla^2 \boldsymbol{B}$$

u: Velocity field
B: magnetic field
η: Resistivity

Loureiro et al. , 2007

Therical model of plasmoid chains formation

Description of velocity field inside the CS



Assumation

① Incompressibility and 2D current sheet

② Ignoring the flow vorticity

③ Only incorporating the key feature: linearly increasing Alfvenic outflow.

$$\Gamma_0 = 2V_A/L_{CS}$$

$$u_x = -\Gamma_0 x$$
$$u_y = \Gamma_0 y$$

Outflow is Alfvenic

Loureiro et al. , 2007

Therical model of plasmoid chains formation

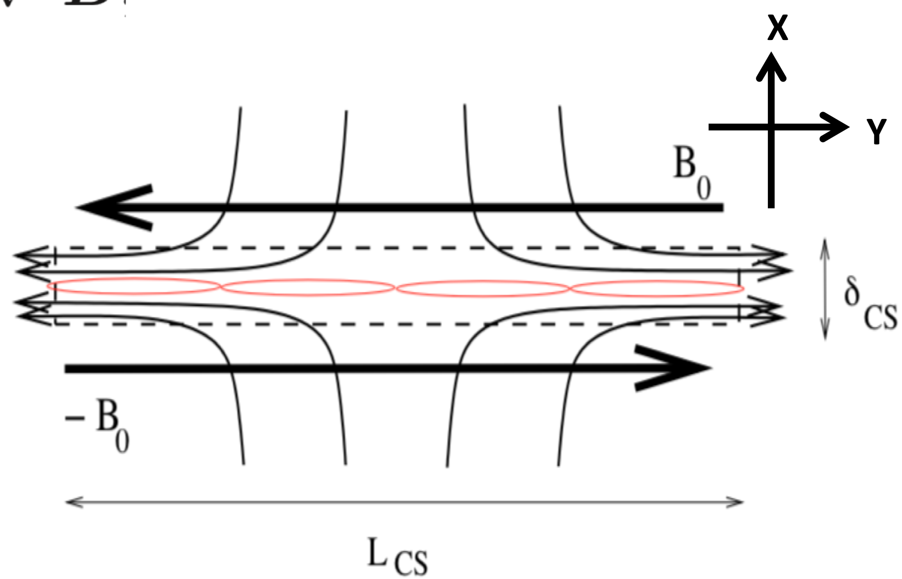
Induction equation

$$\partial_t \boldsymbol{B} + \boldsymbol{u} \cdot \boldsymbol{\nabla B} = \boldsymbol{B} \cdot \boldsymbol{\nabla u} - \boldsymbol{B\nabla} \cdot \boldsymbol{u} + \eta \nabla^2 \boldsymbol{B}$$

Simple equilibrium solution

$$B_{0x} = 0 \qquad B_{0y} = B_{0y}(x)$$

$$u_x = -\Gamma_0 x \qquad u_y = \Gamma_0 y$$



$$B_{0y} = V_A f(\xi) \quad {}^2_{CS}\partial_x^2 B_{0y} + \partial_x$$

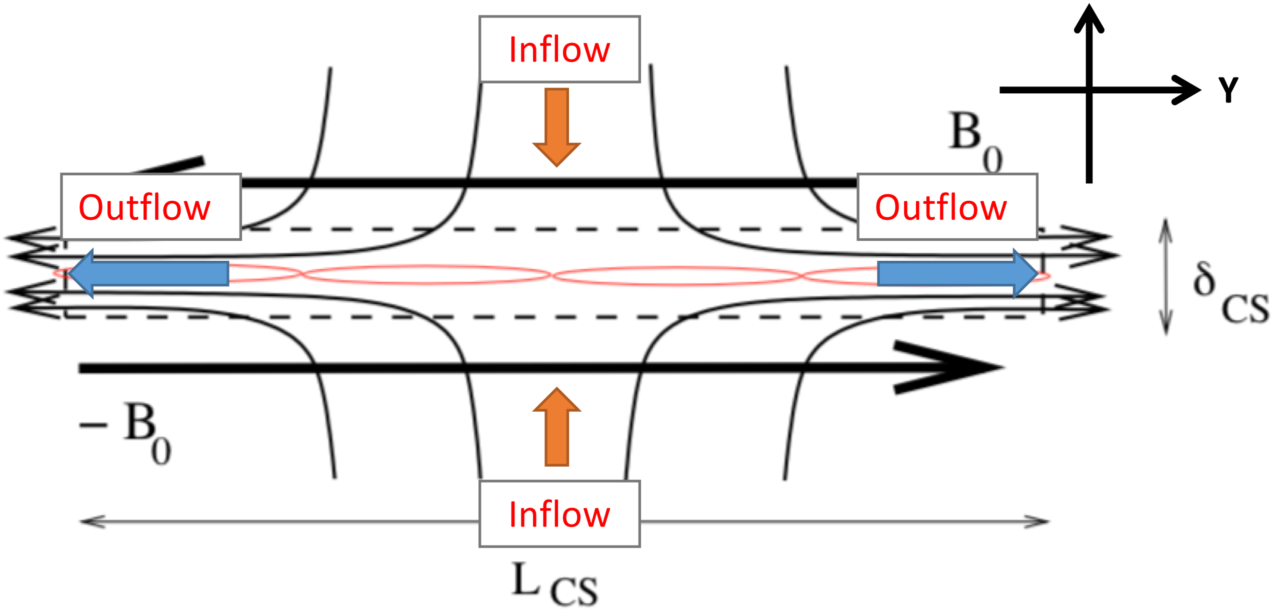
$$f(\xi) = \alpha e^{-\xi^2/2} \int_0^{\xi} dz\, e^{z^2/2}$$

$$\xi = x/\delta_{CS}$$

Loureiro et al. , 2007

Therical model of plasmoid chains formation

Description of Inflow



$$\vec{u} = \begin{cases} (u_0\ ,\ 0)\ for\ x < -x_0 \\ (-\Gamma_0 x\ ,\ \Gamma_0 y)\ for\ \ -x_0 < x < x_0 \\ (-u_0\ ,\ 0)\ for\ x > x_0 \end{cases}$$

Assumation

① Constant inflows outside the sheet

② Inside and outside solution matched point: $x=x_0$

Alfven Speed

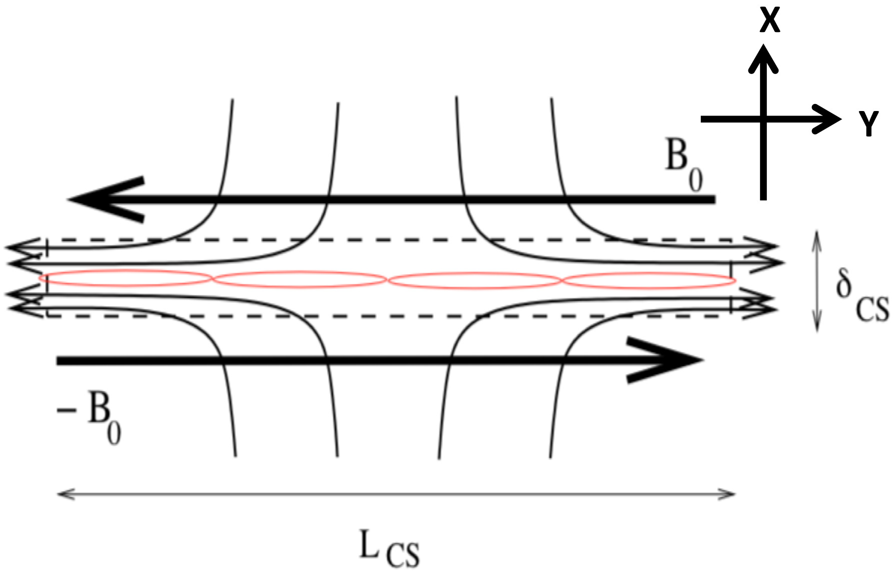
$$V_0 = \frac{B_0}{\sqrt{\mu_0 \rho}} \equiv V_{A0}$$

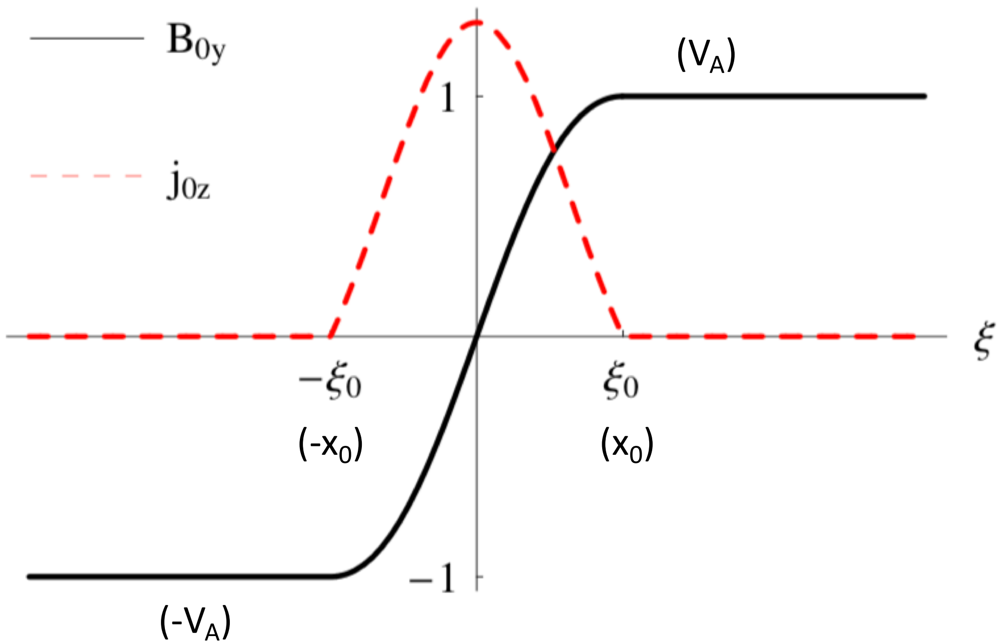
$$B_{0y}(\pm x_0) \sim \pm V_A$$

$$B_{0y} = V_A f(\xi)$$

Loureiro et al. , 2007

Therical model of plasmoid chains formation





$$j_{0z} = \partial_x B_{0y}$$

$$\xi = x/\delta_{CS}$$

Loureiro et al. , 2007

Therical model of plasmoid chains formation

Reduced MHD equation

$$\partial_t \nabla_\perp^2 \phi + \{\phi, \nabla_\perp^2 \phi\} = \{\psi, \nabla_\perp^2 \psi\},$$
$$\partial_t \psi + \{\phi, \psi\} = \eta \nabla_\perp^2 \psi + E_0.$$

Where

$$\{\phi, \psi\} \equiv \partial_x \phi \, \partial_y \psi - \partial_y \phi \, \partial_x \psi$$

ϕ : Stream function of the inplane velocity

ψ : Flux function of the magnetic field

$$u = (-\partial_y \phi \ , \ \partial_x \phi)$$

$$B = (-\partial_y \psi \ , \ \partial_x \psi)$$

$$\phi_0 = \begin{cases} \Gamma_0 xy \ for\,|x| < x_0 \\ \pm \Gamma_0 x_0 y \ for\,|x| > x_0 \end{cases}$$

$$\partial_x \psi_0 = V_A f(\xi)$$

Small perturbations to the equilibrium

$< -x_0$

$$\phi = \phi_0 + \delta\phi$$

$-x_0 < x < x_0$

$$\psi = \psi_0 + \delta\psi$$

$> x_0$

Solutions

$$\delta\phi(x, y, t) = \phi_1(x, t) exp[ik(t)y]$$

$$\delta\psi(x, y, t) = \psi_1(x, t) exp[ik(t)y]$$

$$\mathrm{k(t) = k_0 exp(-\Gamma_0 t)}$$

Loureiro et al. , 2007

Therical model of plasmoid chains formation

$$\delta\phi(x,y,t) = \phi_1(x,t)exp[ik(t)y]$$

$$\delta\psi(x,y,t) = \psi_1(x,t)exp[ik(t)y]$$

$$\mathrm{k(t) = k_0 exp(-\Gamma_0 t)}$$

Exponentially solutions

$$\phi_1(x,t) = -\mathrm{i}\Phi(\mathrm{x})\mathrm{exp}(\gamma \mathrm{t})$$

$$\psi_1(x,t) = \Psi(\mathrm{x})\mathrm{exp}(\gamma \mathrm{t})$$

Reduced MHD equation

$$\partial_t \nabla_\perp^2 \phi + \{\phi, \nabla_\perp^2 \phi\} = \{\psi, \nabla_\perp^2 \psi\},$$
$$\partial_t \psi + \{\phi, \psi\} = \eta \nabla_\perp^2 \psi + E_0.$$

$$\lambda(\Phi'' - \kappa^2\epsilon^2\Phi) = -f(\xi)(\Psi'' - \kappa^2\epsilon^2\Psi) + f''(\xi)\Psi,$$
$$\lambda\Psi - f(\xi)\Phi = \frac{1}{\kappa}(\Psi'' - \kappa^2\epsilon^2\Psi),$$

$$\kappa = k_0 V_A/\Gamma_0 = k_0 L_{CS}/2$$

$$\epsilon = (\eta\Gamma_0)^{\frac{1}{2}}/V_A = 2\delta_{CS}/L_{CS}$$

$$\lambda = \gamma/\Gamma_0\kappa$$

Loureiro et al. , 2007

Therical model of plasmoid chains formation

$$\lambda\Phi'' = -\alpha\xi\Psi'',$$
$$\lambda\Psi - \alpha\xi\Phi = \frac{1}{\kappa}\Psi''.$$

$$\kappa = k_0 V_A/\Gamma_0 = k_0 L_{CS}/2$$
$$\epsilon = (\eta\Gamma_0)^{\frac{1}{2}}/V_A = 2\delta_{CS}/L_{CS}$$
$$\lambda = \gamma/\Gamma_0\kappa$$

Conclusion

① Instability growth rate $\gamma_{max}/\Gamma_0 \sim \epsilon^{-\frac{1}{2}}$

Super-Alfvenically fast

Stable current sheets with aspect ratios above some critical value can not exist. $S_c \sim 10^4$

② Plasmoids formed along the sheet $\kappa_{max} \sim \epsilon^{-\frac{3}{4}}$

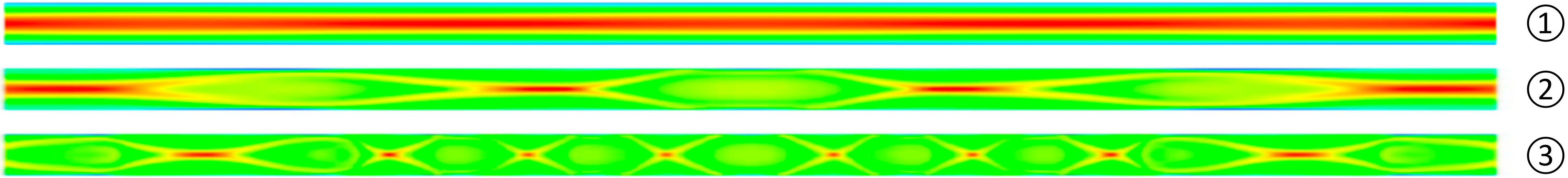
③ Inner layer width $\delta/\delta_{CS} \sim \epsilon^{\frac{1}{4}}$

Loureiro et al. , 2007

Simulation results of plasmoid chains and instability

2D MHD simulations of an SP reconnection layer
uniform resistivity
large S: $10^4<S<10^8$

Time evolution of the instability



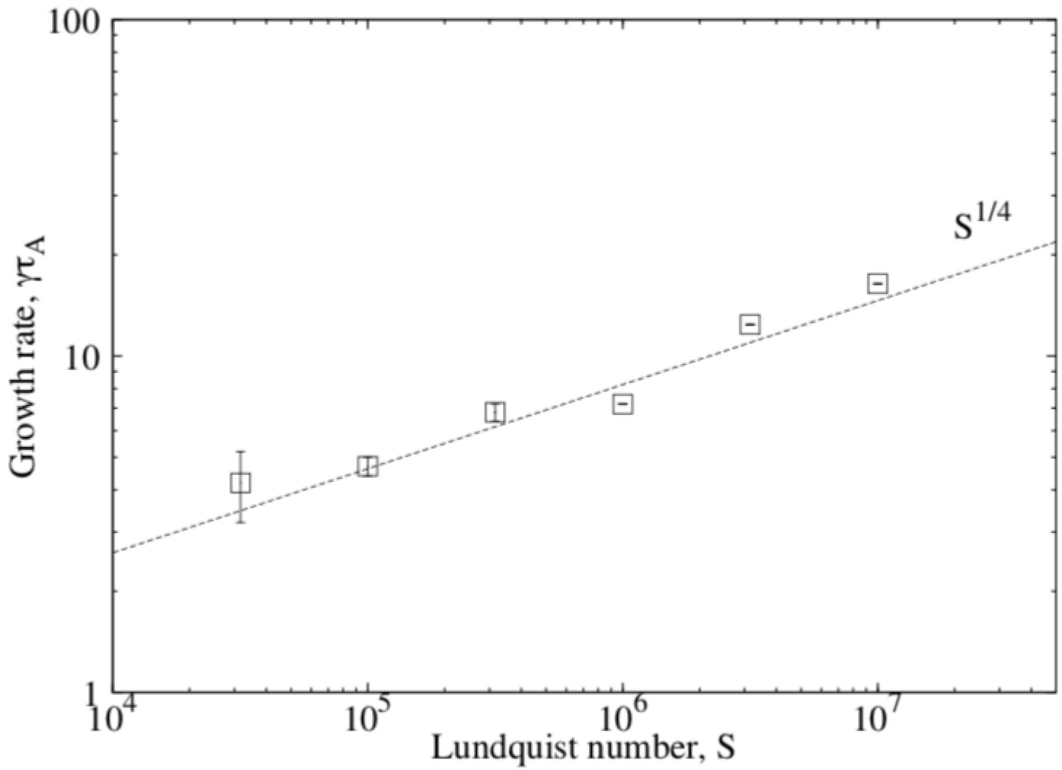
①:$S=10^4$, $t=4.5t_A$
②:$S=10^5$, $t=2.9t_A$
③:$S=10^6$, $t=2.6t_A$

For $S<=10^4$, no plasmoids are observed.

For $S>10^4$, layer becomes unstable and plasmoids form with reconnection occurring at multiple X-points.

Samtaney et al. , 2009

Simulation results of plasmoid chains and instability



Instability growth rate

$$\gamma_{max}/\Gamma_0 \sim \epsilon^{-\frac{1}{2}}$$

Super-Alfvenically fast

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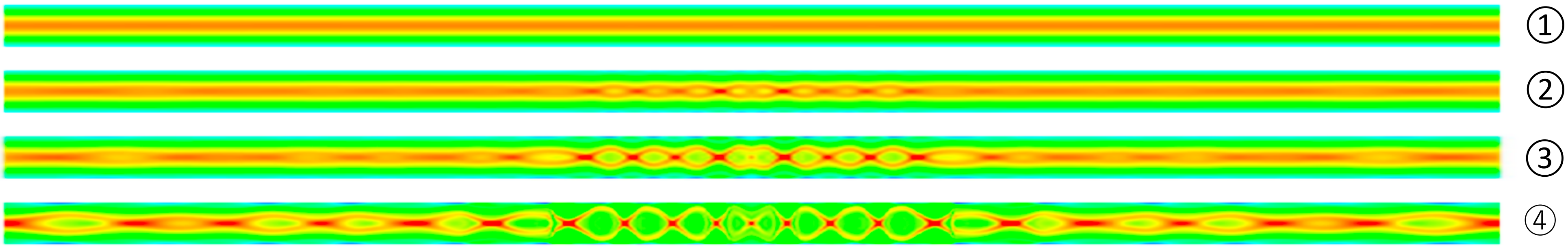
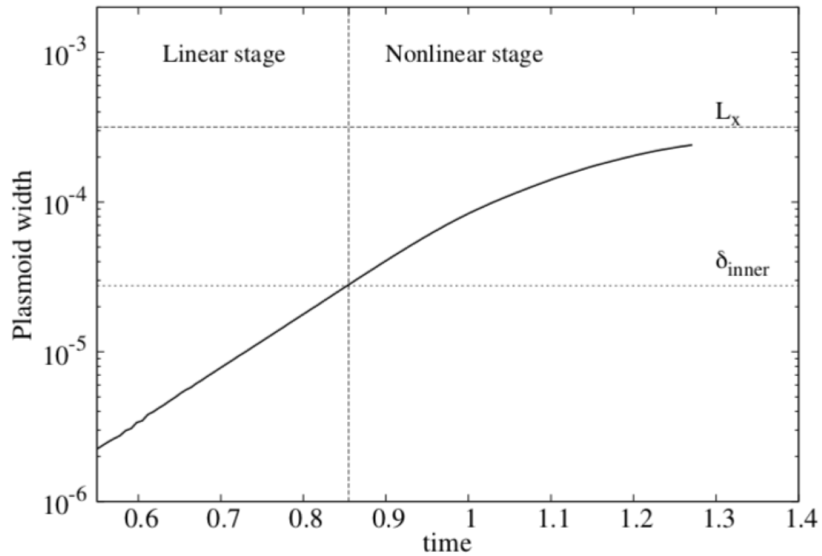
Simulation results of plasmoid chains and instability

Time evolution of the instability



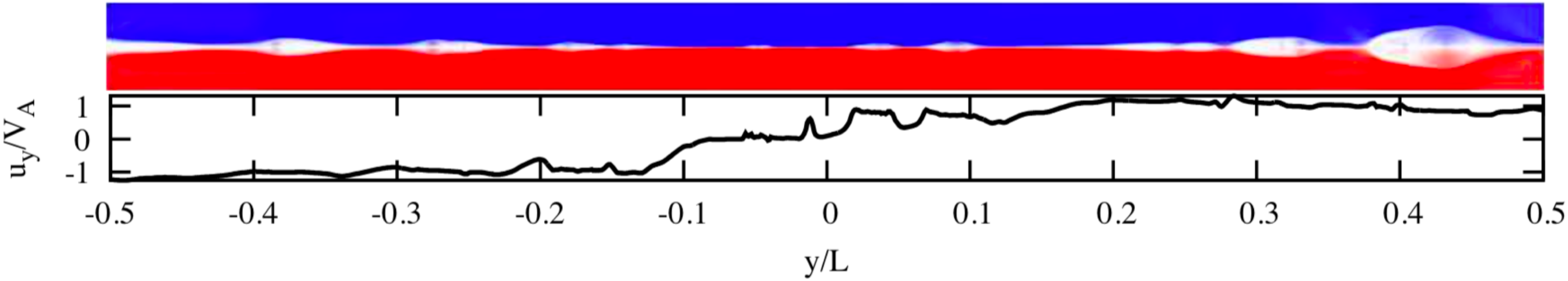
①:S=10^7, t=0.63t_A , ②:S=10^7, t=0.96t_A , ③:S=10^7, t=1.09t_A , ④:S=10^7, t=1.27t_A

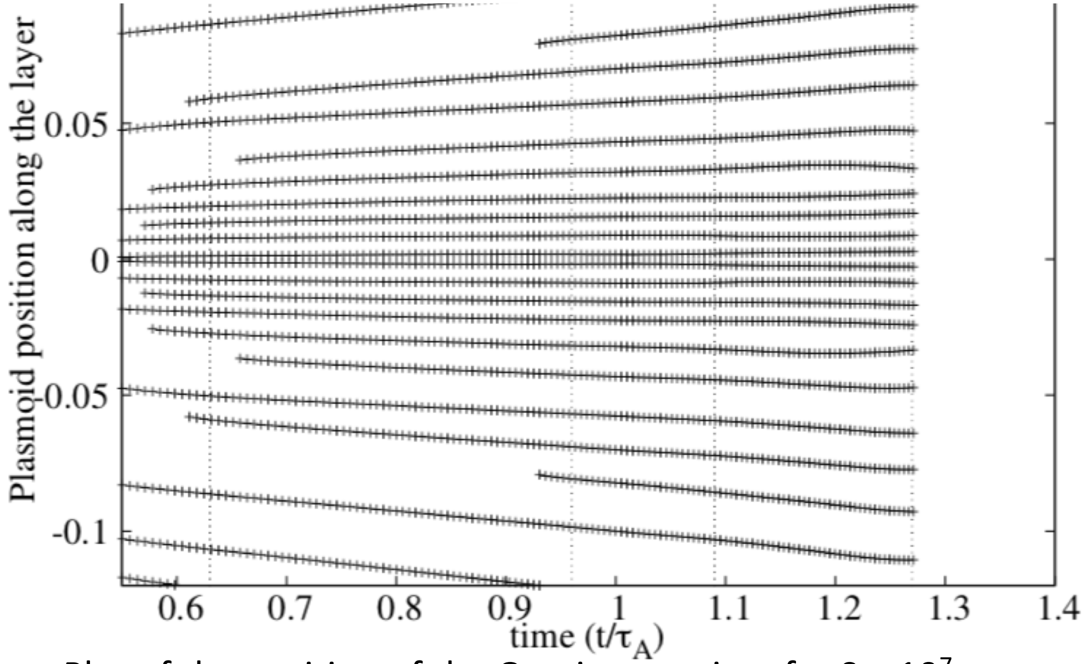


$$\delta/\delta_{CS} \sim \epsilon^{\frac{1}{4}}$$

Samtaney et al. , 2009

Simulation results of plasmoid chains and instability





Plot of the position of the O points vs. time for $S = 10^7$

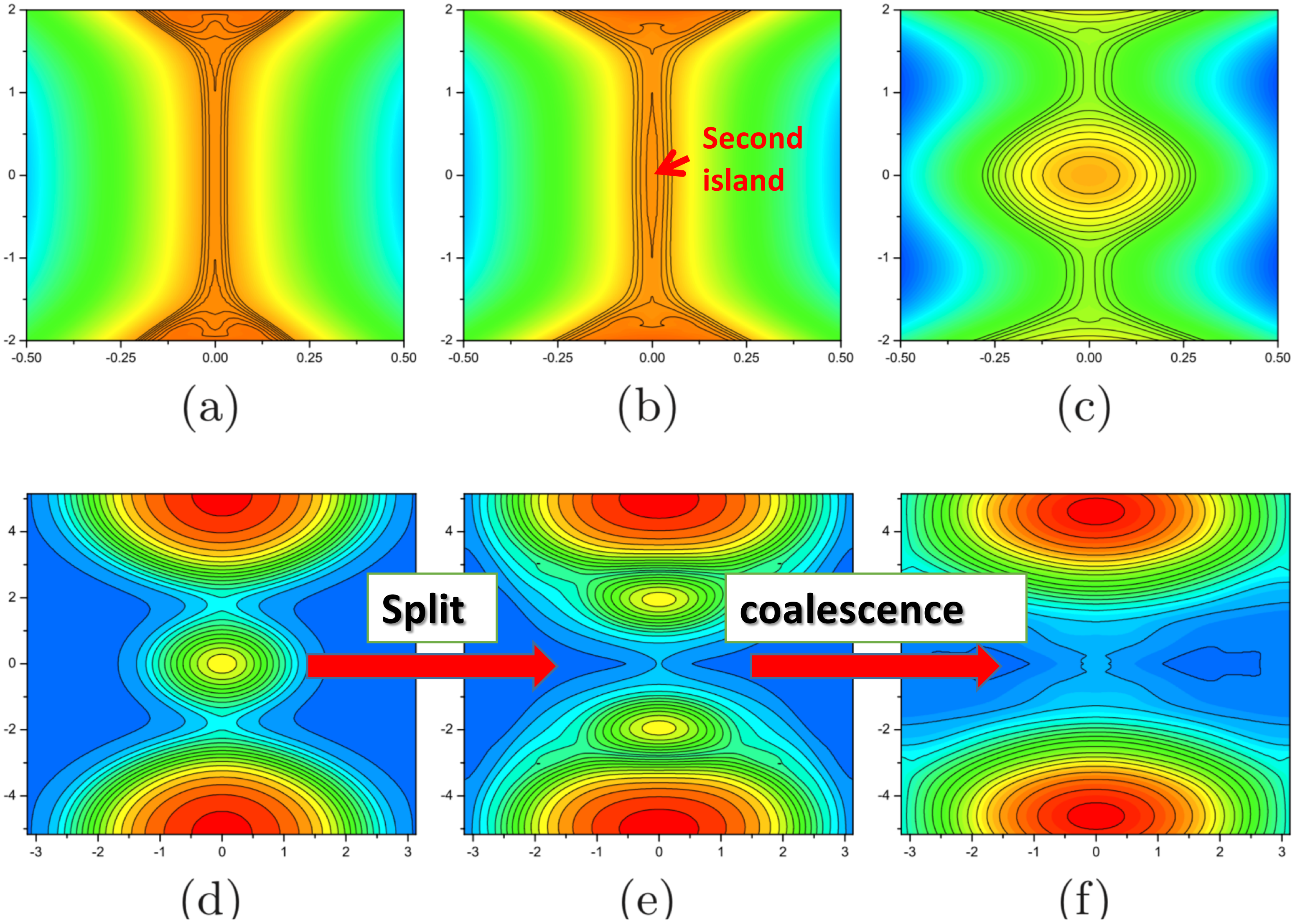
Description of outflow

$$u_x = -\Gamma_0 x$$

$$u_y = \Gamma_0 y$$

Samtaney et al. , 2009
Loureiro et al. , 2011

Simulation results of plasmoid chains and instability



Secondary plasmoid

(a-c): current sheet instability

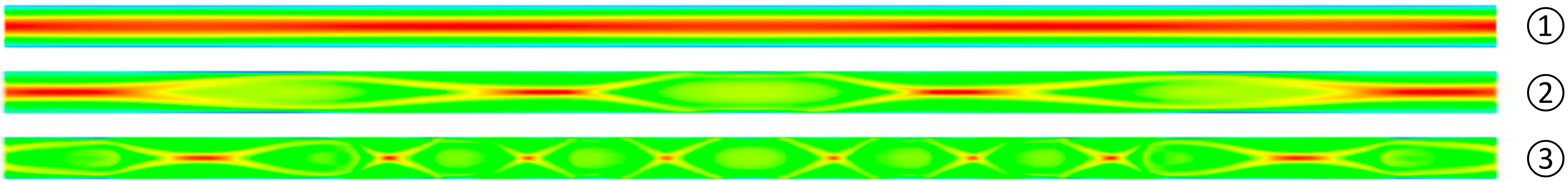
(d-f): subsequent nonlinear evolution of the secondary island

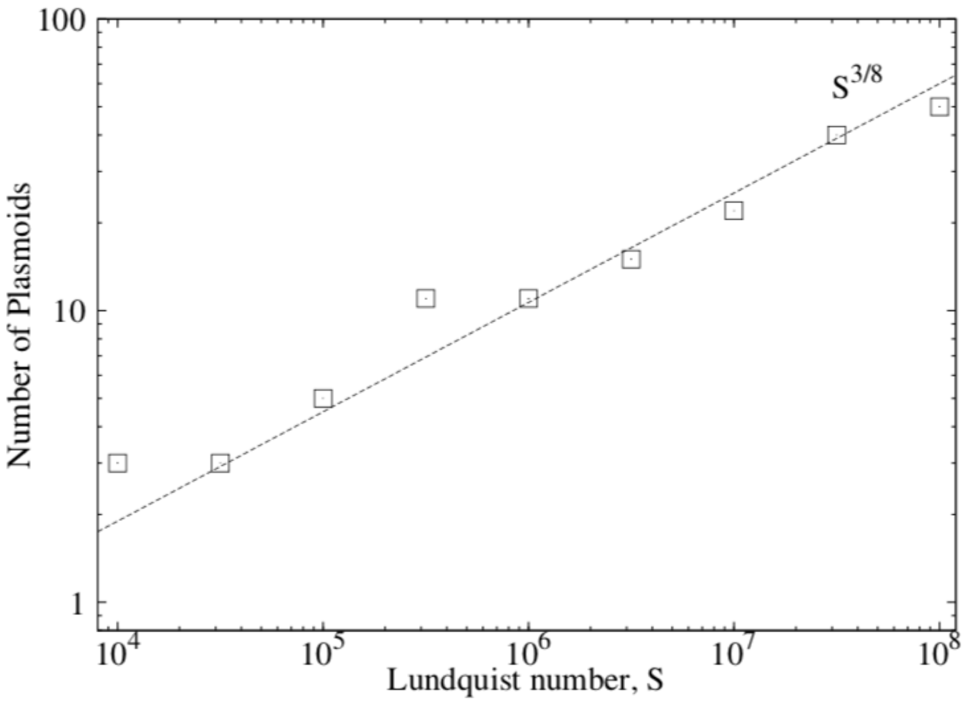
A linear perturbation to the current sheet, with two X points forming at the ends of the sheet.

Loureiro et al. , 2005

Simulation results of plasmoid chains and instability

Spatial Structure of the Plasmoid Chain





Plasmoids' maximum number

$$\kappa_{max} \sim \epsilon^{-\frac{3}{4}}$$

Samtaney et al. , 2009

Summary

- Magnetic reconnection: oppositely directed magnetic field lines are driven together, break and rejoin, resulting in a conversion of magnetically-stored energy into plasma energy.
- Layer will become unstable and plasmoids will form if S is larger than $S_c(10^4)$.
- Instability growth rate $\gamma_{max}/\Gamma_0 \sim \epsilon^{-\frac{1}{2}}$
- Inner layer width $\delta/\delta_{CS} \sim \epsilon^{\frac{1}{4}}$
- Plasmoids formed along the sheet $\kappa_{max} \sim \epsilon^{-\frac{3}{4}}$
- Plasmoids formation and instability can explain fast connection of high Lundquist number events.