Cosmic Ray : Propagation and Transport

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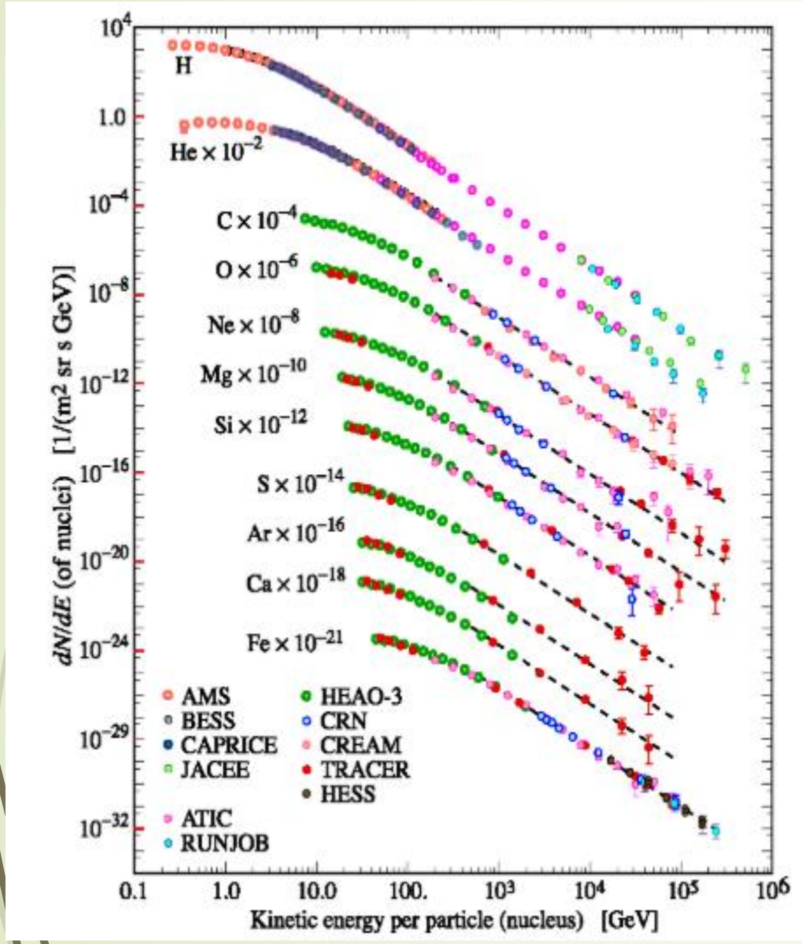
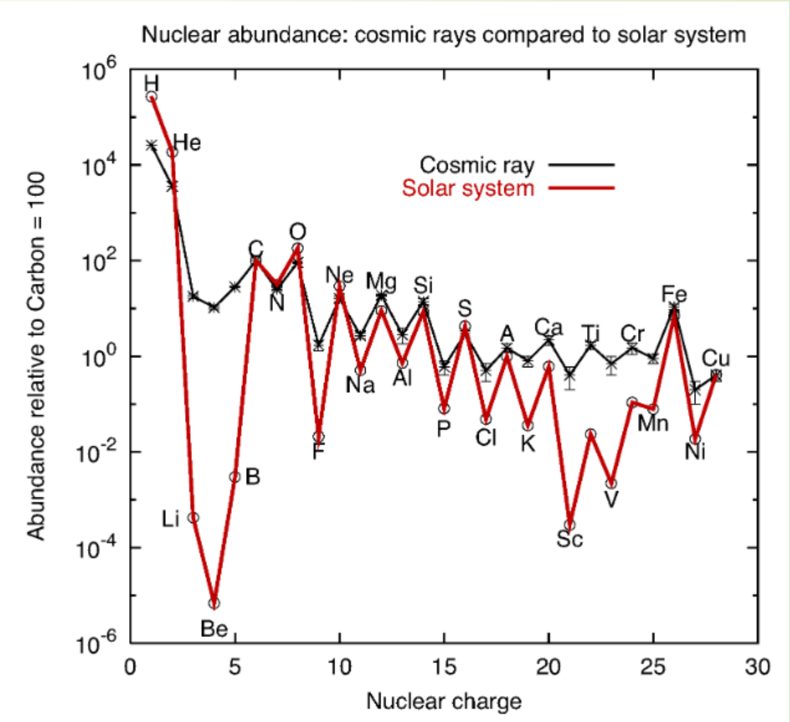
Outline

- Introduction about cosmic ray(CR)
- CR Propagation
  - The Unperturbed System
  - In a turbulence
    - Spatial diffusion
      - Pitch-Angle Scattering
    - Momentum Diffusion
- Propagation Equation
- Simulation result
- Summary

Cosmic Ray(CR)





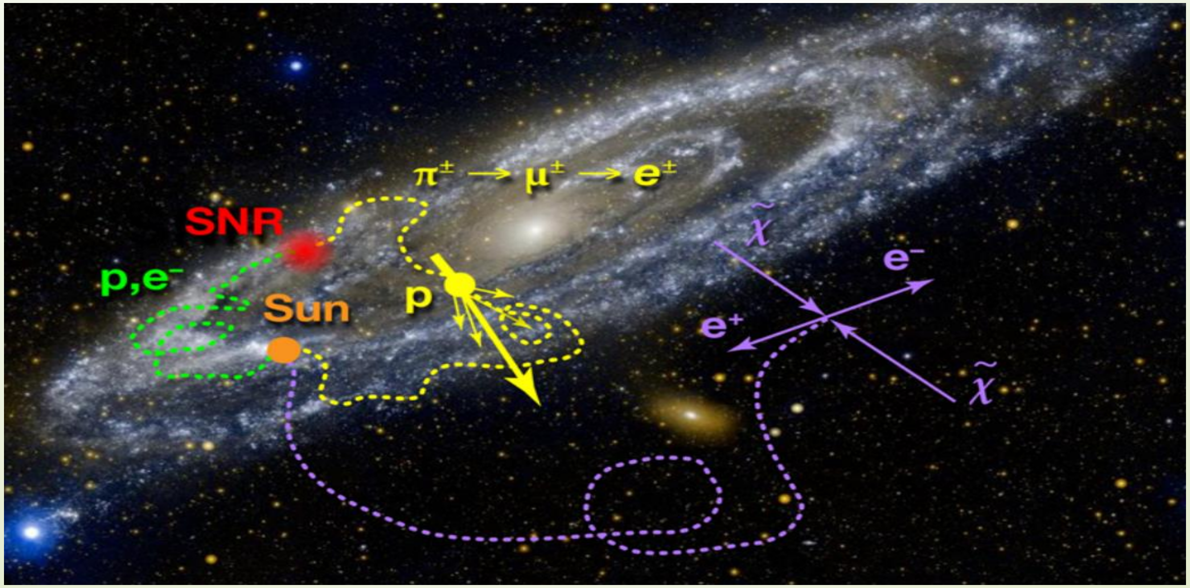
$$N(E) \propto E^{-2.7}$$

- Indicating an **stellar origin of cosmic rays**
- **Secondary cosmic rays:** Li, Be, B
- By measuring the **primary-to-secondary ratio** we can infer the propagation and diffusion processes of CR

Beringer et al. (Particle Data Group), 2012

CR propagation

- All our knowledge of CR propagation comes via secondary CRs, with additional information from γ-rays and synchrotron radiation.
- ways of approaching CR propagation:
  - Particle motion in a turbulence



The Unperturbed System

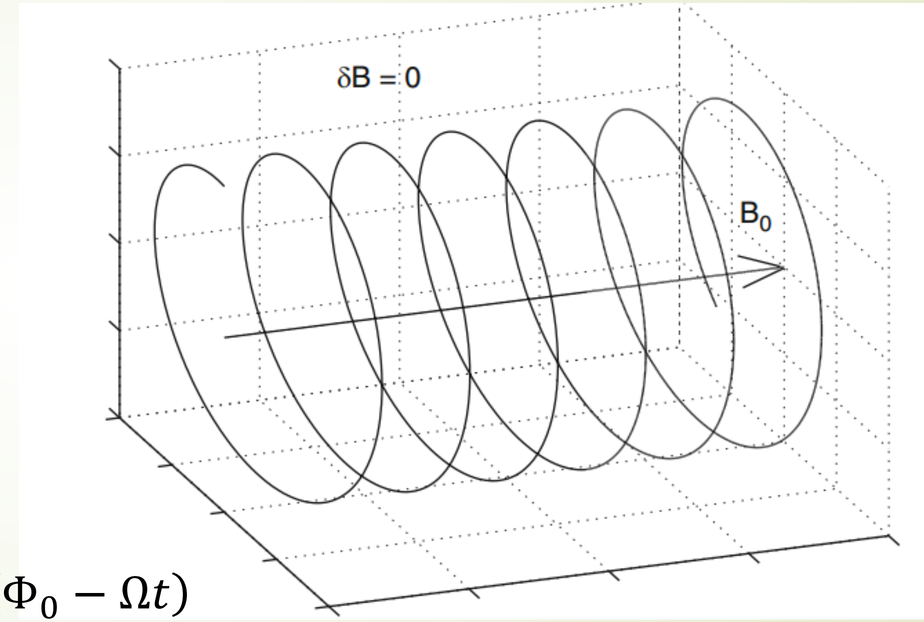
- Electric fields are less important for spatial diffusion

$$\frac{d}{dt}\boldsymbol{p} = q\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right)$$

Define the parameter

$$\Omega := \frac{qB_0}{mc}\sqrt{1 - v^2/c^2}$$

μ: the pitch-angle



The trajectory can be solved

$$x(t) = x(0) + \frac{v_\perp}{\Omega}sin(\Phi_0) - \frac{v_\perp}{\Omega}sin(\Phi_0 - \Omega t)$$

$$y(t) = y(0) - \frac{v_\perp}{\Omega}cos(\Phi_0) + \frac{v_\perp}{\Omega}cos(\Phi_0 - \Omega t)$$

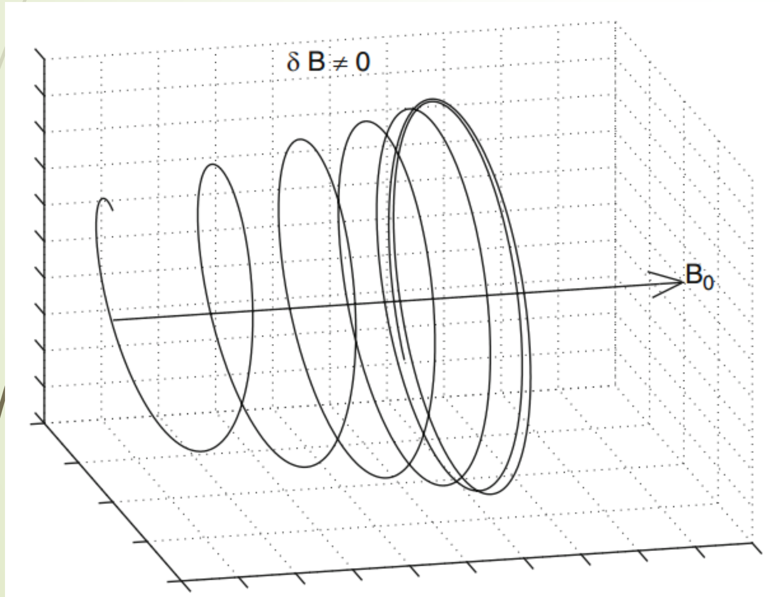
$$z(t) = z(0) + v_\parallel \mathrm{t}$$

gyromotion

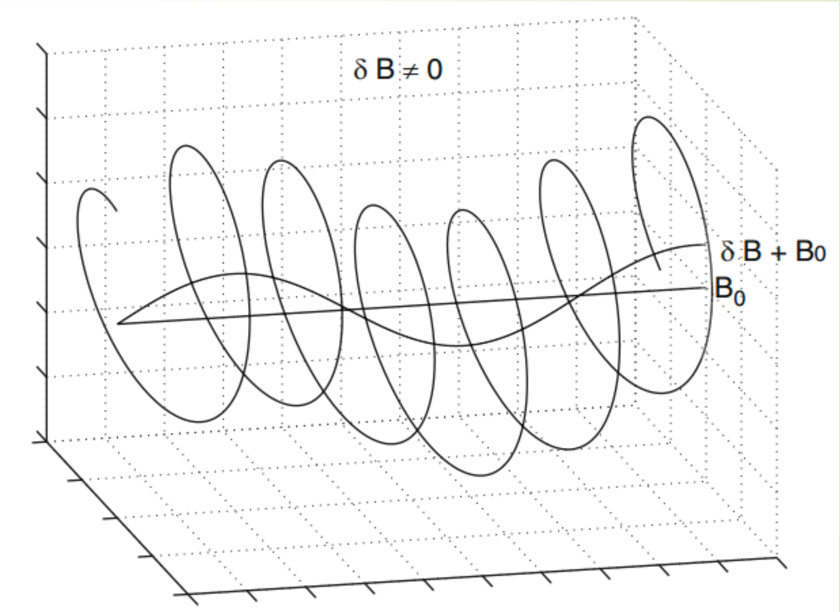
Shalchi (2009 book), chapter 1

In a turbulence…

- Affect the trajectory of the particles in two ways:
  - Spatial diffusion due to the turbulent magnetic fields
  - Momentum diffusion due to the turbulent electric fields



Parallel Scattering
(pitch-angle scattering)

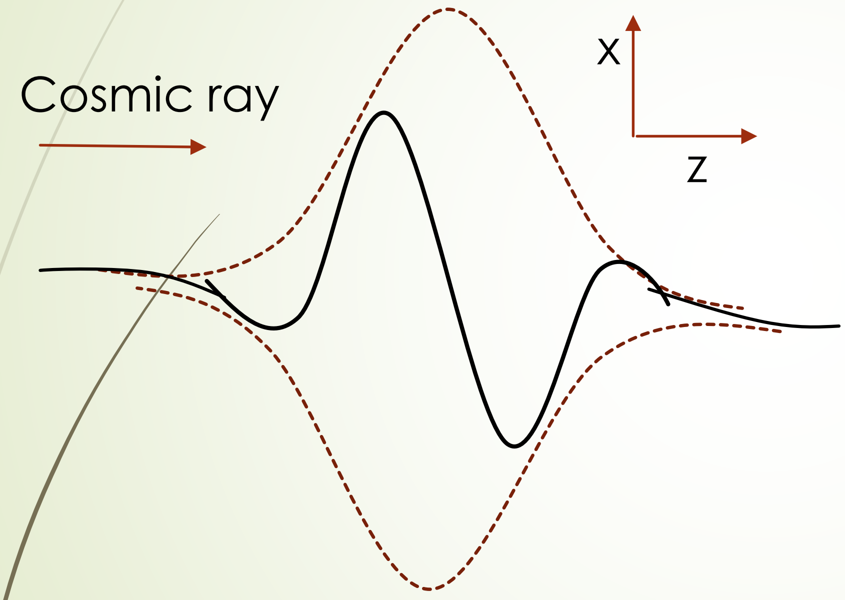


Perpendicular Scattering
(field line wandering)

Shalchi (2009 book), chapter 1

Pitch-Angle Scattering

- Consider how a cosmic ray interacts with Alfven packet



$\lambda \gg r_L$

$\lambda \ll r_L$

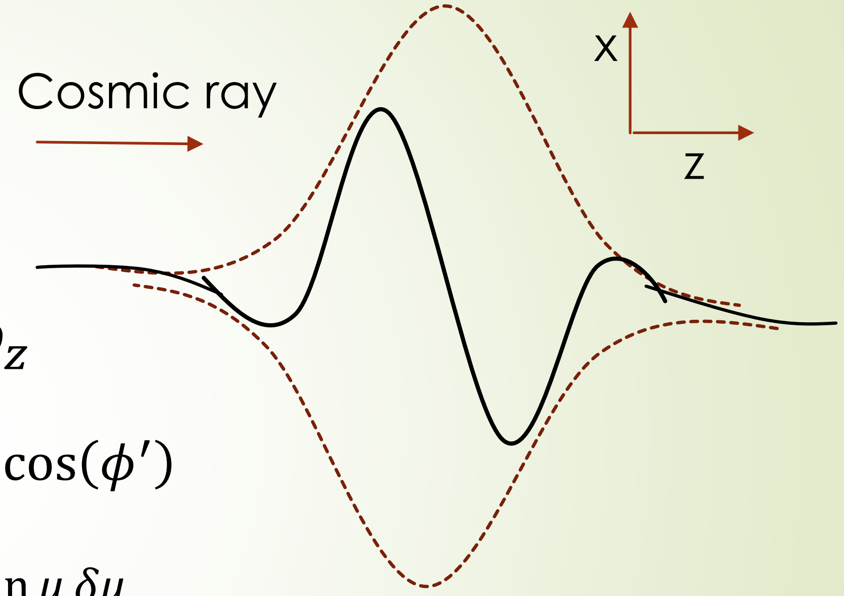
- For $\lambda \gg r_L$ or $\lambda \ll r_L$, the average change of pitch-angle during one period will be nearly zero.
- Pitch-Angle Scattering happens only when CR gyration is in resonance with the wave. $\lambda \approx r_L$

Pitch-Angle Scattering

$$\delta B_\perp = \hat{x}\delta B \sin(kz - wt)$$

$$v_y = v_\perp \sin(\Omega t + \phi)$$

$$z = z_0 + v_z t$$



Then we can get the change in p_z

$$\Delta p_z = e \int dt \left(\frac{v \times B}{c}\right)_z = \pi p \sin\mu \frac{\delta B}{B} \cos(\phi')$$

Where $\phi' = kz_0 - \phi$ $\Delta p_z = -p \sin\mu\, \delta\mu$

$$\Rightarrow \delta\mu = -\pi \frac{\delta B}{B}\cos(\phi') \Rightarrow D_{\mu\mu} = \frac{\langle(\Delta\mu)^2\rangle}{2t} = \frac{\pi}{8}\Omega\left\langle\left(\frac{\delta B}{B}\right)^2\right\rangle$$

$$\kappa_{zz} = \frac{v^2}{8}\int_{-1}^{+1} \mathrm{d}\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}}$$

it is demonstrated that pitch-angle diffusion in phase-space leads to parallel spatial diffusion in real space.

Momentum Diffusion

In addition to spatial diffusion, the scattering of CR particles on randomly moving MHD waves leads to stochastic acceleration, which is described in the transport equation as diffusion in momentum space with some diffusion coefficient D_{pp}

$$D_{pp} = \frac{p^2 V_a^2}{9 D_{xx}}$$

Where V_a is the Alfven velocity, D_{xx} is the spatial diffusion coefficient.

Other factors…

- Cosmic-ray interactions

Cosmic-ray particles are expected to interact during their travel.

1.Coulomb collisions: the collision rate

$$n\sigma v \sim 10^{-20} s^{-1}$$

Where for a 1 GeV particle propagating in the ISM

($n \sim 1 cm^{-3}, \sigma \sim 10^{-30} cm^2$)

Coulomb collisions can be neglected.

Note: collisions hardly affect the bulk CR population, but they totally determine the secondaries. And it's also important for the diffuse gamma ray emission.

2. Spallations processes: It occurs when C, N, O, Fe nuclei impact on interstellar hydrogen and the large nuclei is broken up into smaller nuclei

Propagation Equation

$\psi(p) = 4\pi p^2 f(\vec{p})dp,$

f is the CR density in phase space

Source term

Spatial diffusion

$$\frac{\partial \psi(\vec{r}, p, t)}{\partial t} = q(\vec{r}, p, t) + \vec{\nabla} \cdot (D_{xx}\vec{\nabla}\psi - \vec{V}\psi) + \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2}\psi - \frac{\partial}{\partial p}\left[\dot{p}\psi - \frac{p}{3}(\vec{\nabla} \cdot \vec{V})\psi\right] - \frac{1}{\tau_f}\psi - \frac{1}{\tau_r}\psi$$

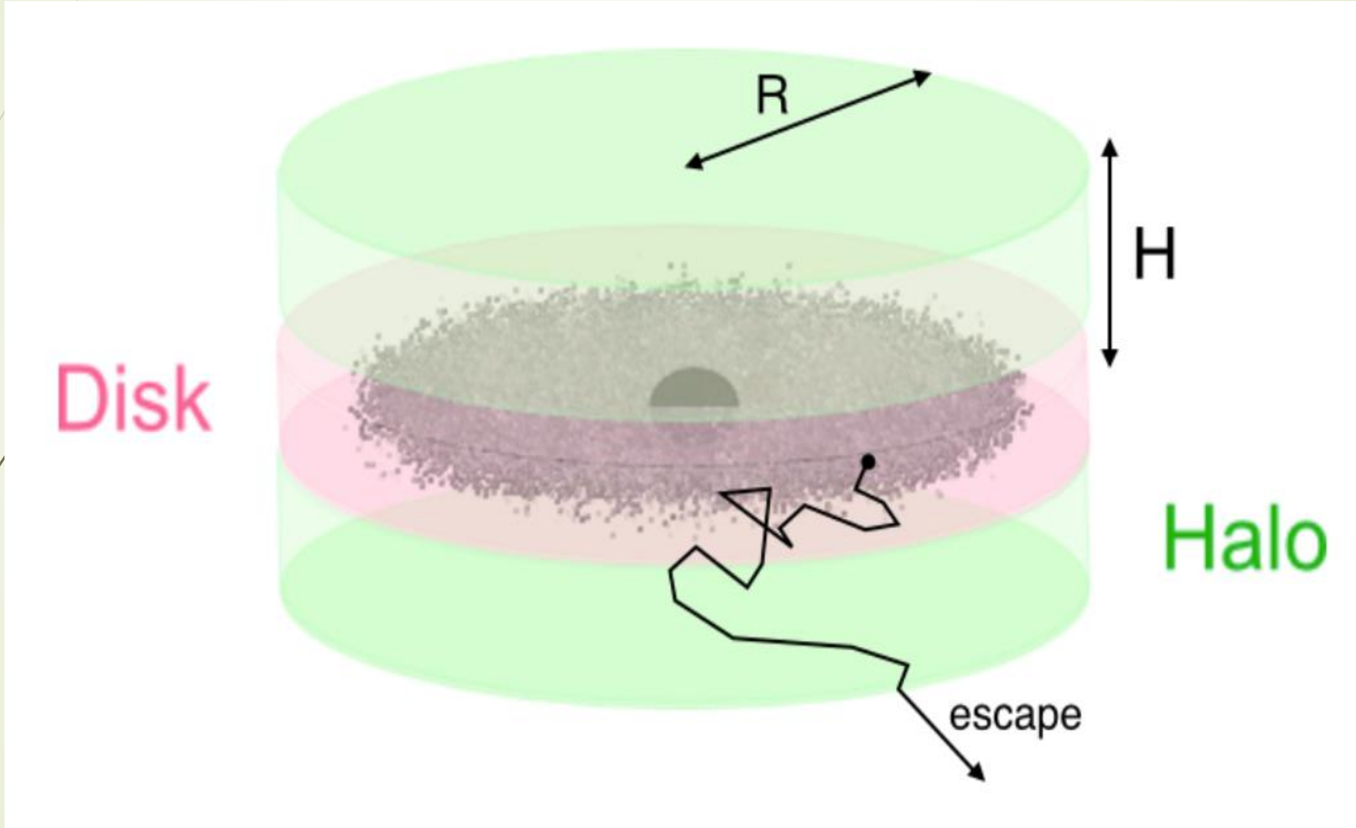
Momentum diffusion

Spallation and decay

Simulation

- A powerful tool: **GALPROP**
  - Aim to enable simultaneous predictions of all relevant observations
- Here I just introduce a very simple model called **Leaky Boxes model**, in which we don't need to consider many details in Propagation Equation.

Leaky Boxes model



CRs are assumed be accelerated in the galactic plane and to propagate freely within a cylindrical box of size H and radius R and reflected at the boundaries. And CRs have a non-zero probability to escape from the box.

Leaky Boxes model

$$\frac{\partial N_i}{\partial t} = D\Delta N_i = -\frac{N_i}{T_{esc}}$$

↓

$$N_i = n_0 exp(-t/T_{esc}) = n_0 exp(-z/H)$$

Here we can get $D \propto T_{esc}^{-1}$

How to estimate the escape timescale T_{esc}?

Escape Timescale T_{esc}

Consider the Secondary-to-Primary ratios: B/C

(Boron is chiefly produced by Carbon)

The Boron source production rate

the cross-section of Carbon into Boron

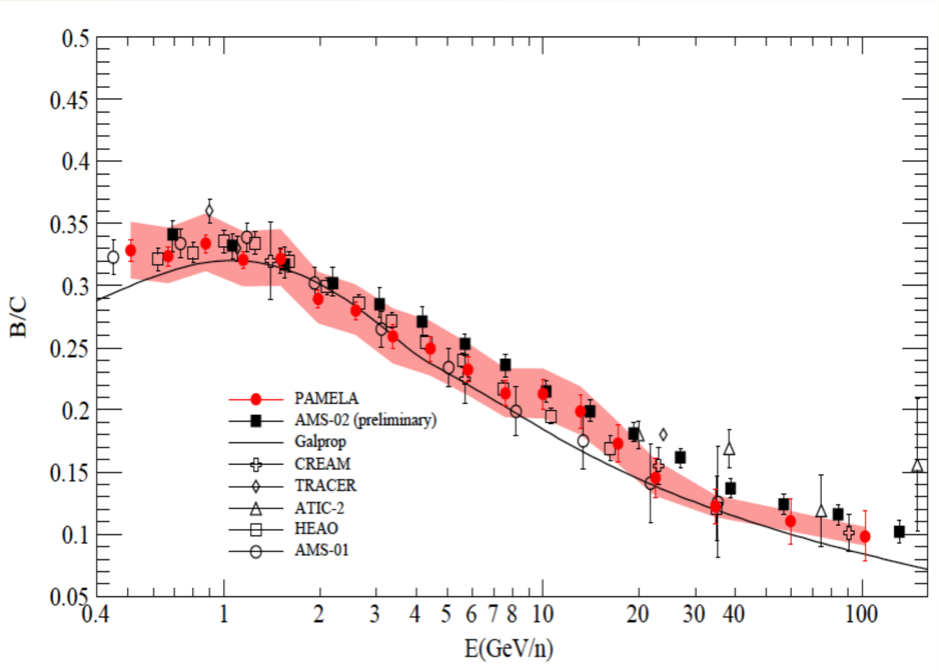
$$Q_B(E) \simeq n_H \beta c \sigma_{\to B} N_C$$

And the production rate is also related to lifetime of Boron τ

$$Q_B = \dot{N}_B = \frac{N_B}{\tau}$$

$$\frac{N_B}{N_C} \simeq n_H \beta c \sigma_{\to B} \tau$$

Escape Timescale T_{esc}



Observations show that

$$\frac{N_B}{N_C} = n_H \beta c \sigma_{\to B} \tau = 0.4 \beta E^{-0.3}$$

$$n_H \tau \simeq 10^{14} \left(\frac{E}{\text{GeV}}\right)^{-0.3} \text{ s cm}^{-3}$$

$$n_H \tau_{f,B} \approx 1.4 \times 10^{14} \; s \; cm^{-3}$$

$$n_H T_{esc} = \frac{1}{\frac{1}{n_H \tau} - \frac{1}{n_H \tau_{f,B}}} \approx 10^{14} \left(\frac{E}{GeV}\right)^{-0.55} s \; cm^{-3}$$

The lifetime τ includes:

1. $\tau_{f,B}$:the **catastrophic loss** time due to the partial fragmentation of Boron
2. T_{esc}: the escape timescale

$T_{esc} \sim 3Myr$ and $T_{esc}(E) \propto E^{-0.55}$

$D(E) \propto E^{0.55}$

Back to Leaky Boxes model

Add the source term: $\frac{\partial N_i(E)}{\partial t} = Q_i(E) - \frac{N_i(E)}{T_{esc}(E)}$

Solve for the stationary state: $N_i(E) = Q_i(E)T_{esc}(E)$

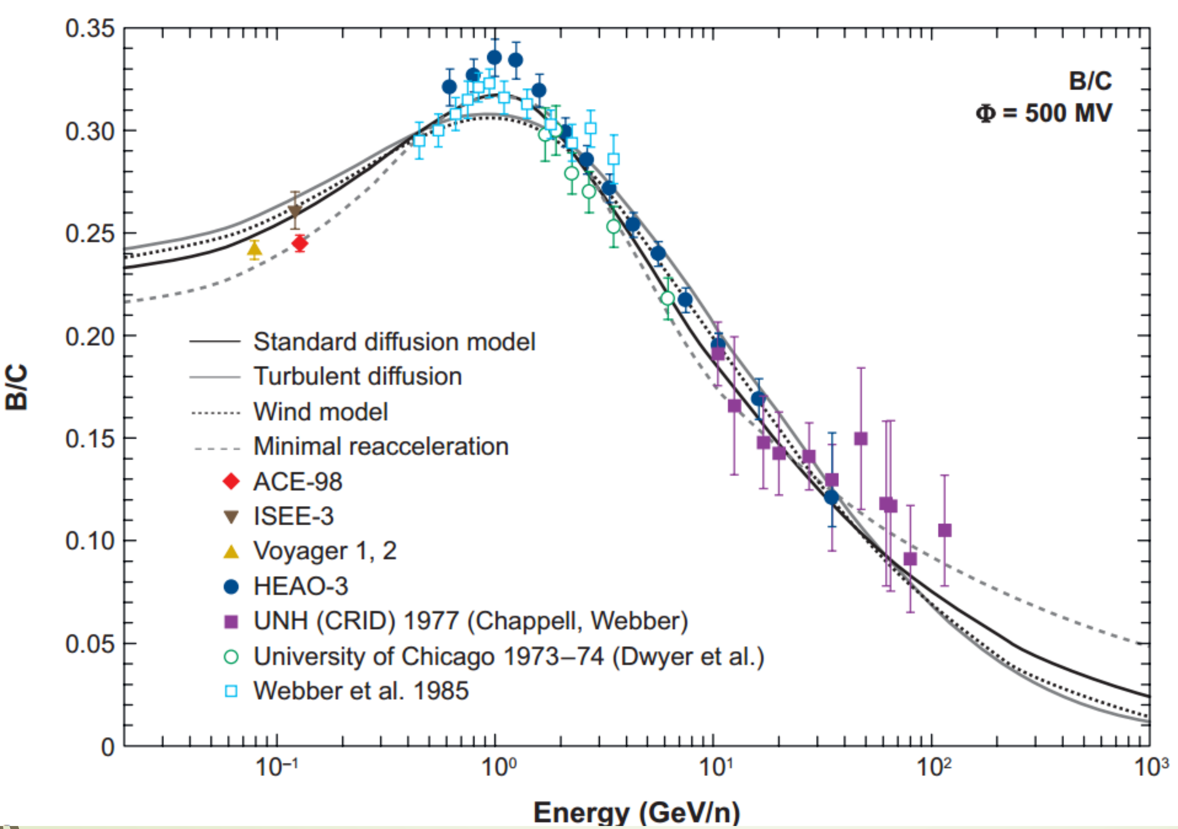
$T_{esc}(E) \propto E^{-0.55}$ $\quad$ $N(E) \propto E^{-2.7}$

$\Rightarrow$ $Q(E) \propto E^{-2.1}$

So the difference of Power-law slope between observation and Fermi acceleration mechanism can be explained by the propagation progress.

Other simulated Result

- Secondary-to-Primary Ratios



The models cannot be distinguished on the basis of these types of data alone, and they all provide an adequate fit.

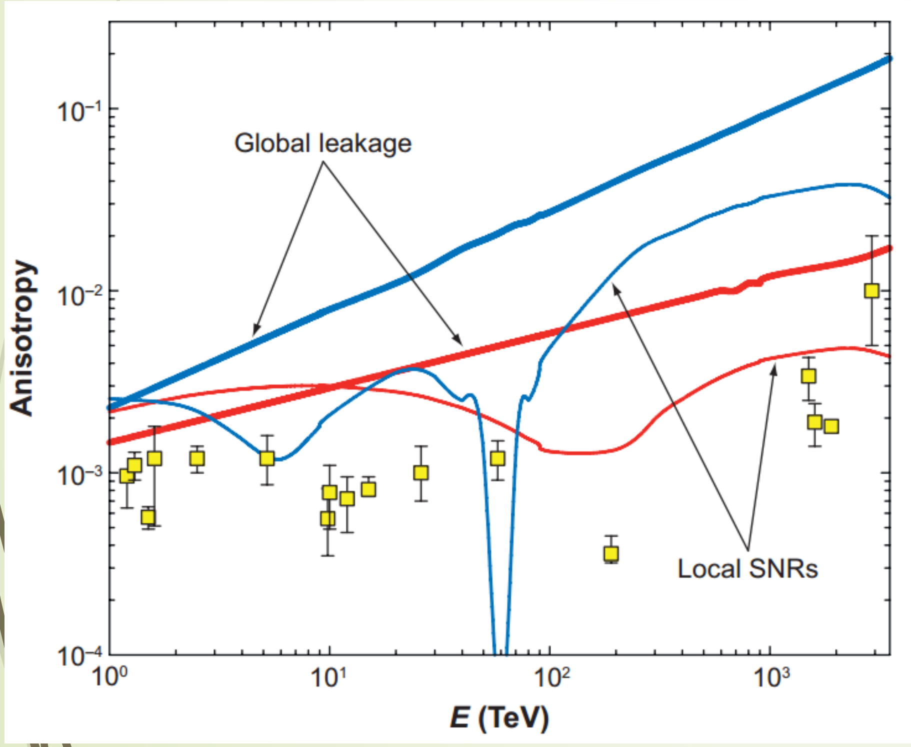
Jones FC, Lukasiak A, Ptuskin V, Webber W. Astrophys. J. 547:264 (2001)

Other simulated Result

- Anisotropy

High isotropy is a distinctive quality of CRs observed on Earth

The first angular harmonic of anisotropy is at the level of $\delta{\sim}10^{-3}$



Red line: reacceleration model

Blue line: plain diffusion model

thick lines: the effects of the global leakage from the Galaxy

Thin lines: the contribution from local supernova remnants

Ambrosio M, et al. Phys. Rev. D 67:042002 (2003)

Summary

- The Propagation Equation is the basic theory for us to study the CR propagation

$$\frac{\partial \psi(\vec{r},p,t)}{\partial t} = q(\vec{r},p,t) + \vec{\nabla}\cdot(D_{xx}\vec{\nabla}\psi - \vec{V}\psi) + \frac{\partial}{\partial p}p^2 D_{pp}\frac{\partial}{\partial p}\frac{1}{p^2}\psi - \frac{\partial}{\partial p}\left[\dot{p}\psi - \frac{p}{3}(\vec{\nabla}\cdot\vec{V})\psi\right] - \frac{1}{\tau_f}\psi - \frac{1}{\tau_r}\psi$$

- Based on Leaky Boxes model, we can use the Secondary-to-Primary ratios B/C to estimate the CR's residence time in galaxy is about 3Myr.
- The difference of Power-law slope between observation and Fermi acceleration mechanism can be explained by the propagation progress. $Q(E) \propto E^{-2.1}$ $N(E) \propto E^{-2.7}$
- The anisotropy of CR is at the level of $\delta \sim 10^{-3}$.