Solar dynamo

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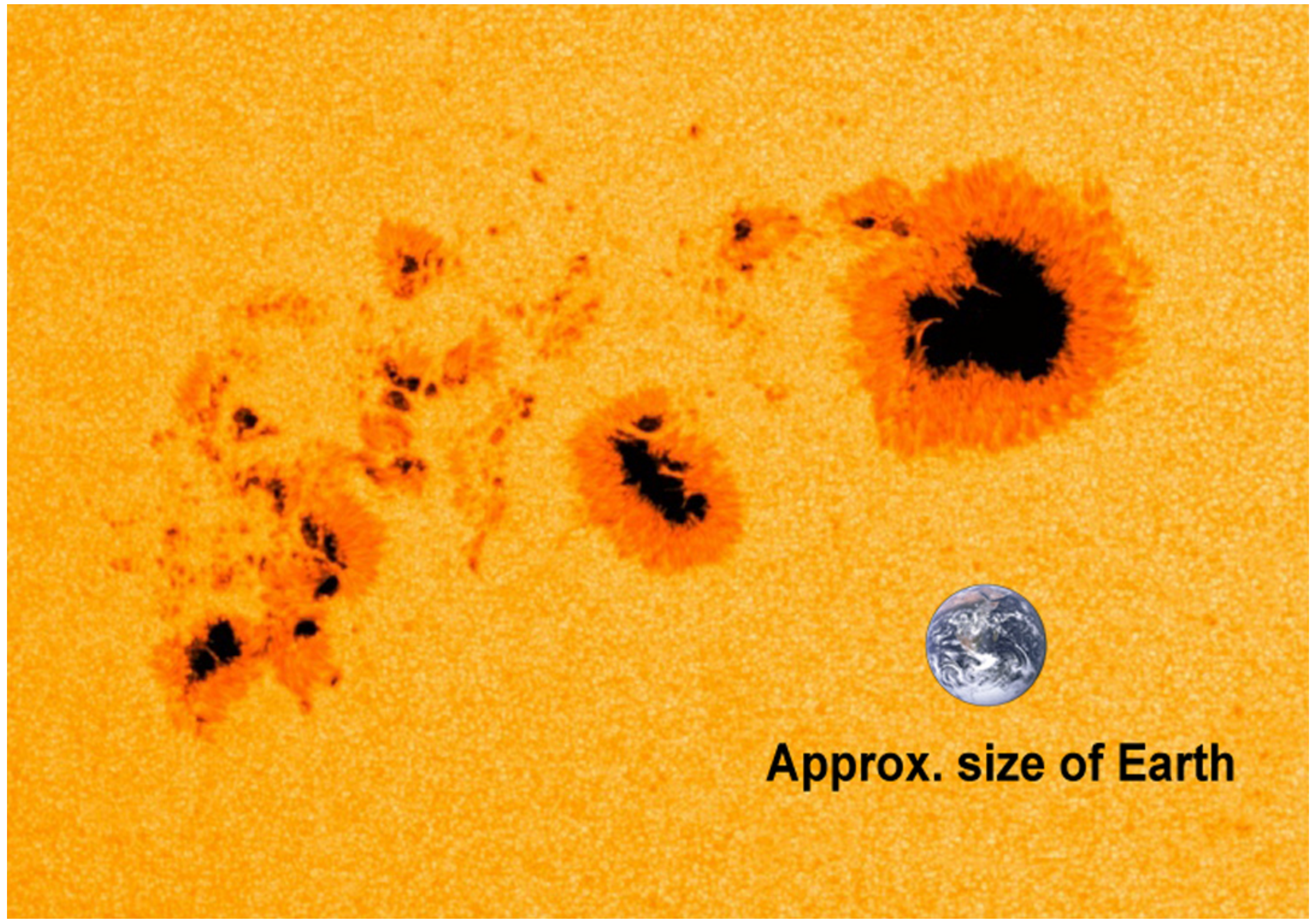
OUTLINE

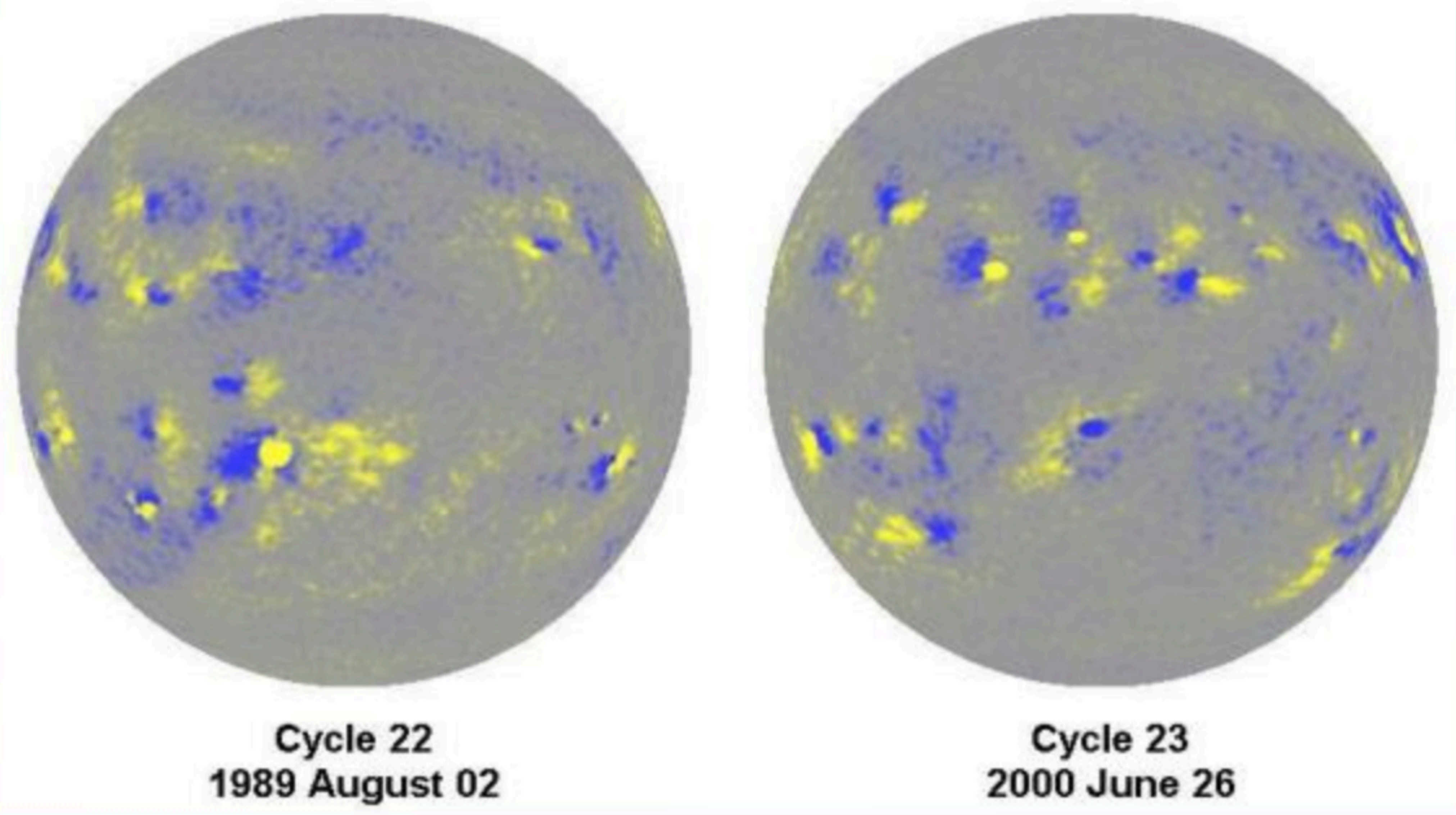
- Introduction: sunspots and solar cycle
- Solar dynamo model
  - $\alpha\Omega$ dynamo
  - Interface dynamo (Babcock-Leighton mechanism)
  - Flux transport dynamo
- Summary

Observation: sunspots

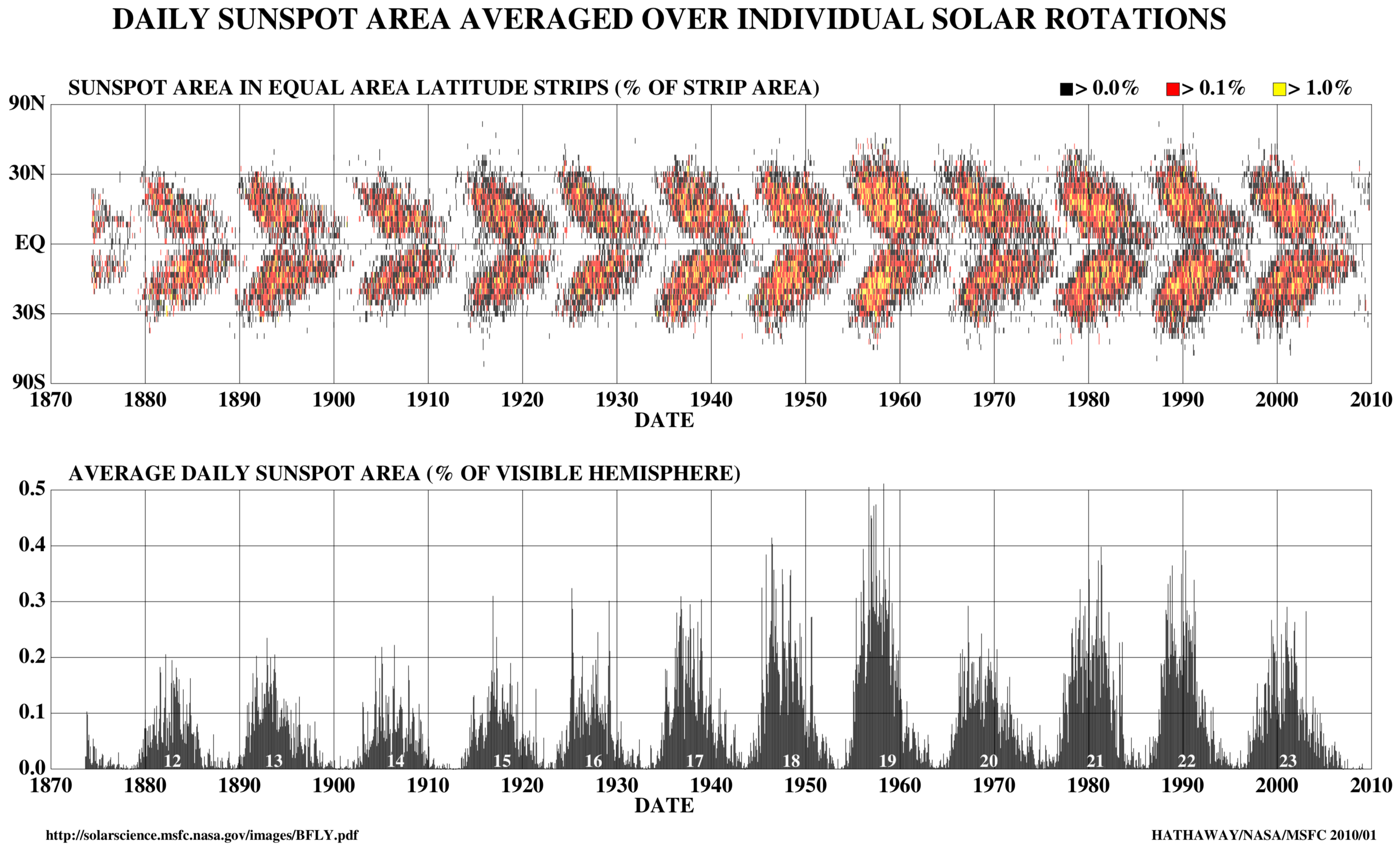
$$1G = 10^{-4}T$$

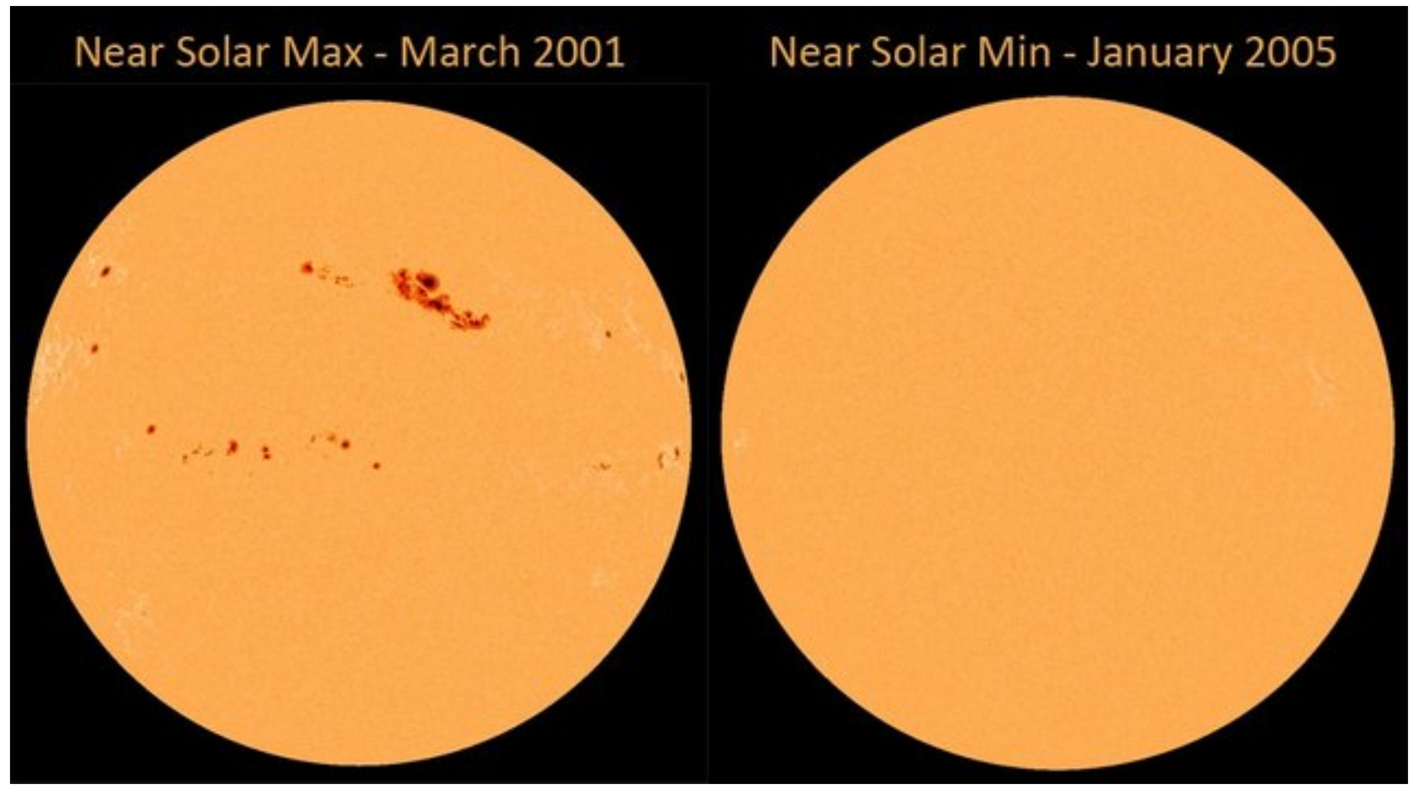
- earliest extant record of sunspots: *Book of Changes*
- dark spots on sun (Galileo)
- *have* lower temperature with respect to surrounding
- life time: days to weeks
- Regions of intense magnetic fields : 0.1~0.3T (the normal magnetic field of sun is ~10G; for earth, 0.5G)
- Often in pairs: leading and trailing sunspots
    - Hale's polarity law: opposite polarity from north to south hemisphere; the polarity changes each solar cycle

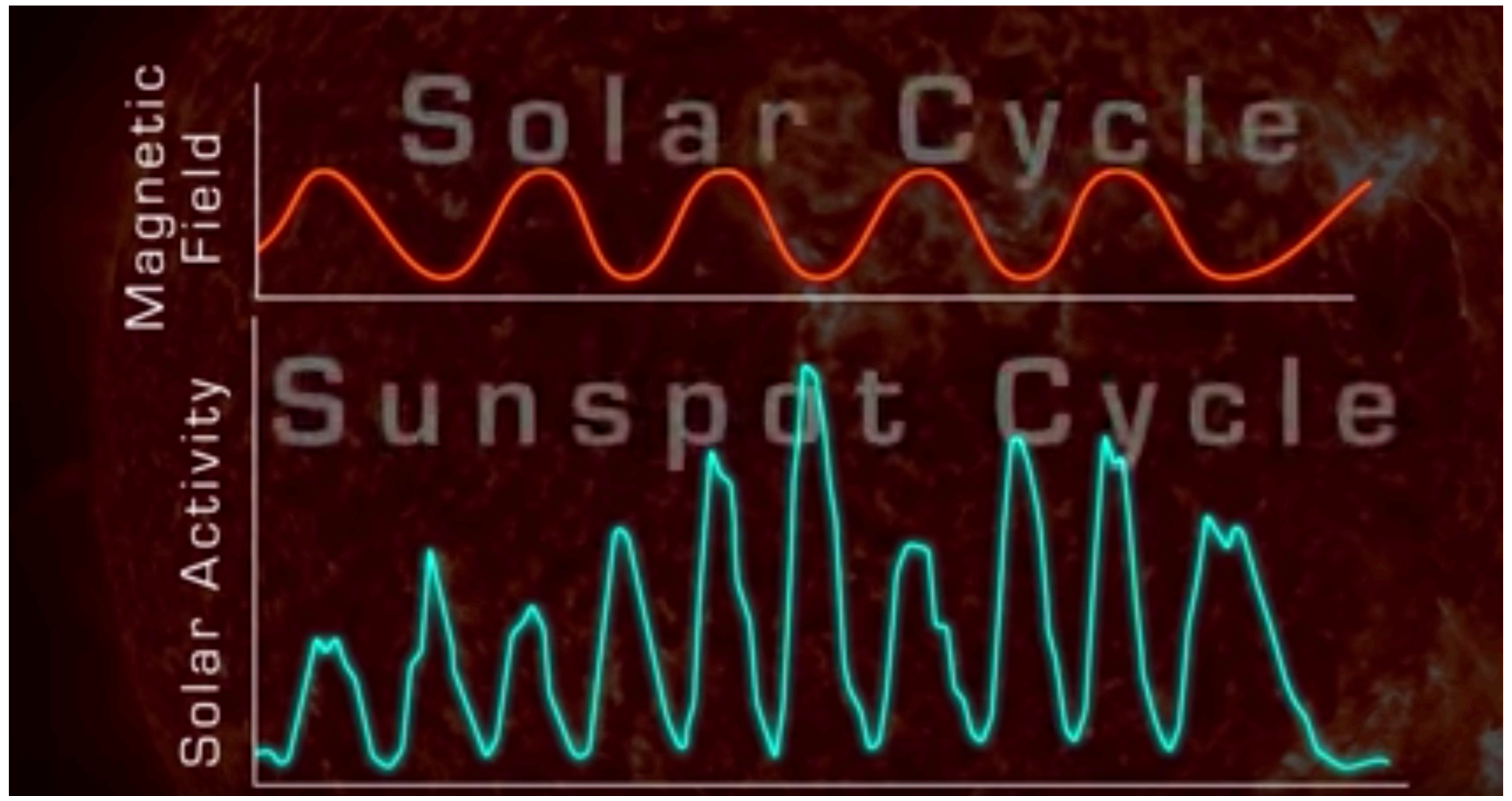




Observation: solar cycle







- Sunspot activity changes spatially and periodically
- Sunspot activity has a period of ~11 years with magnetic field reversed
- Solar cycle ~ 22 years

Sunspot activity caused by advection/diffusion?

Induction equation: $$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \qquad \eta = c^2/4\pi\sigma_e$$

Advection ($\nabla \times (\mathbf{u} \times \mathbf{B})$) **Diffusion** ($\eta \nabla \times \mathbf{B}$)

diffusion time scale : $\frac{B}{\tau_d} = \frac{\eta B}{L^2}$

advection time scale : $\frac{B}{\tau_a} = \frac{uB}{L}$

Reynolds number:

$$R_m = \frac{\tau_d}{\tau_a} = \frac{Lu}{\eta}$$

- Rm of sun>>1 => advection dominated; field line **frozen in the plasma flow**
- **But**, the diffusion time scale of sun ~ 10^{10}years $\gg$ solar cycle period
- Need other mechanism to explain solar activities

Solar dynamo theory

A solar dynamo model should…

- Sustain the magnetic field
- Cyclic polarity of 11year period
- Equator-ward migration of sunspots and pole-ward migration of diffuse surface field
- Polar field strength
- Observed antisymmetric parity
- …

Solar dynamo model

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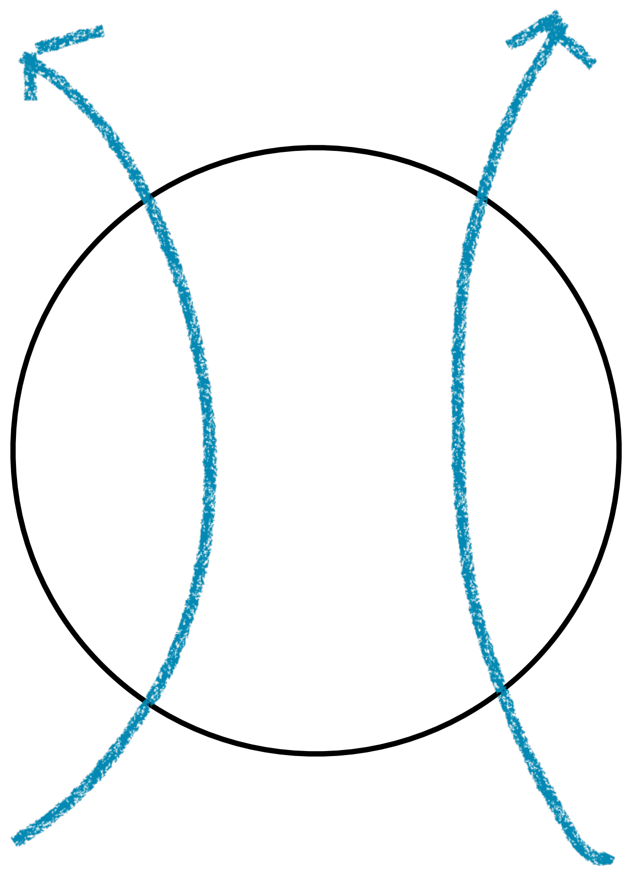
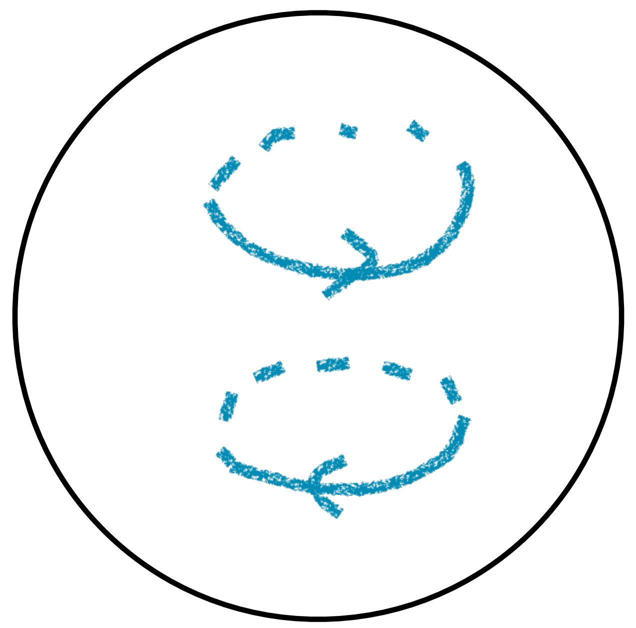
$\alpha\Omega$ dynamo



Poloidal field



Toroidal field
[related to sunspots]

- Cowling's anti-dynamo theorem(1934): an axisymmetric magnetic field cannot be maintained by dynamo action => NOT easy to set up a solar dynamo

- Parker's dynamo model(1955): P => T => P => T …

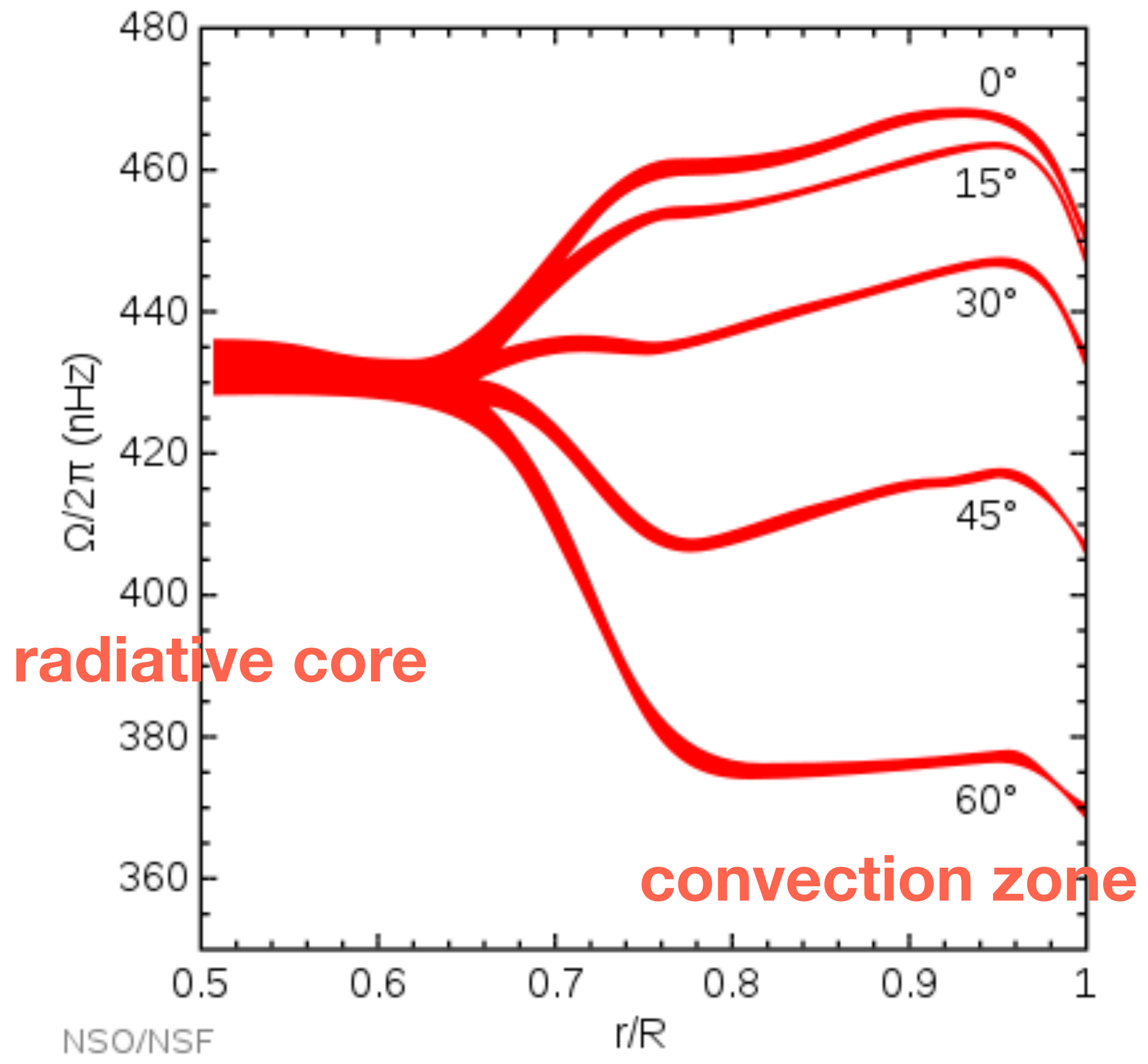
- Mean field theory: $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}', \qquad \mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$

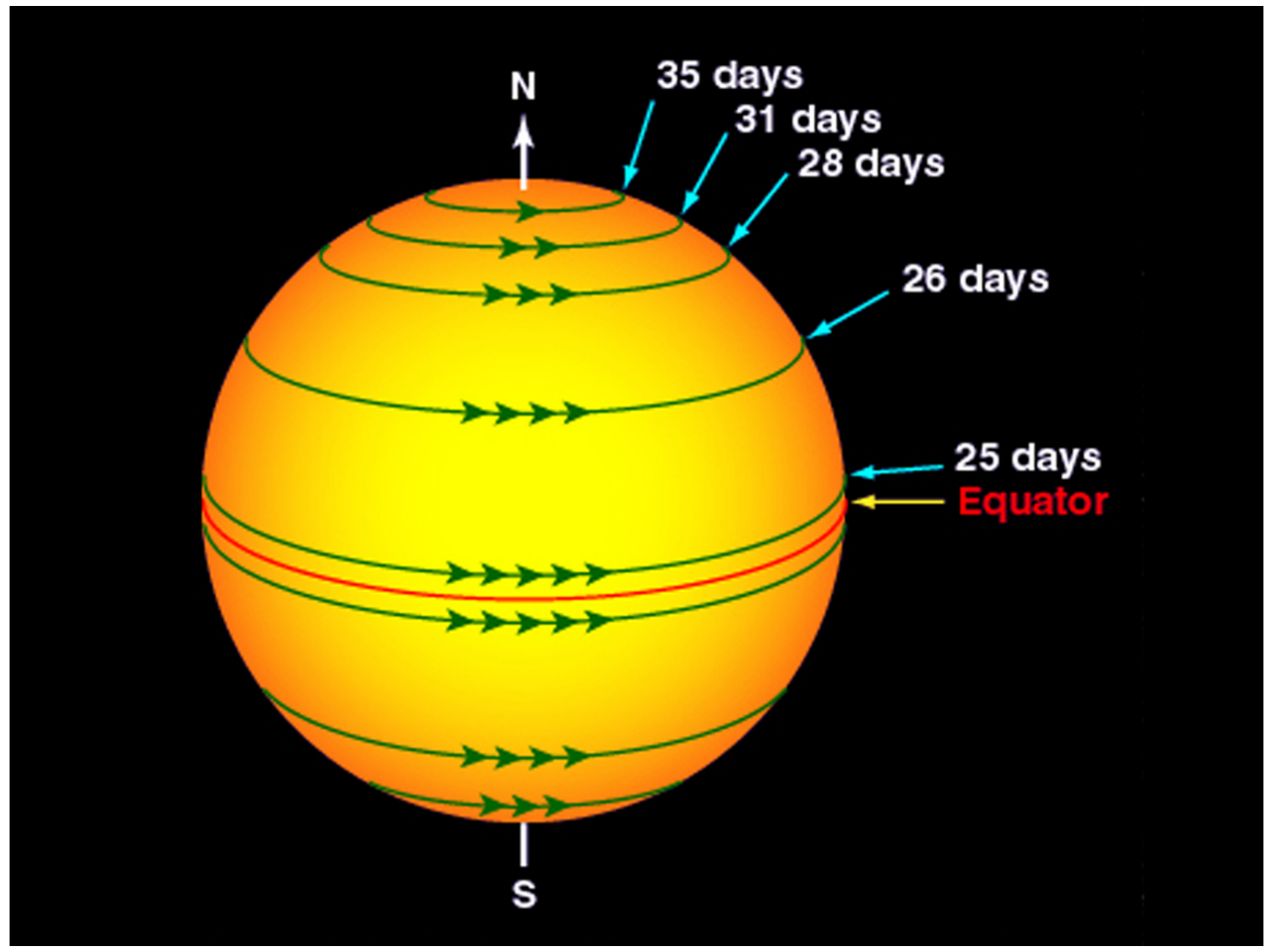
$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \underbrace{\nabla \times (\overline{\mathbf{u}} \times \overline{\mathbf{B}})}_{\Omega\text{ effect}} + \underbrace{\nabla \times \alpha \overline{\mathbf{B}}}_{\alpha\text{ effect}} - \nabla \times (\beta \nabla \times \overline{\mathbf{B}}) + \underbrace{\eta \nabla^2 \overline{\mathbf{B}}}_{\text{diffusion}}.$$

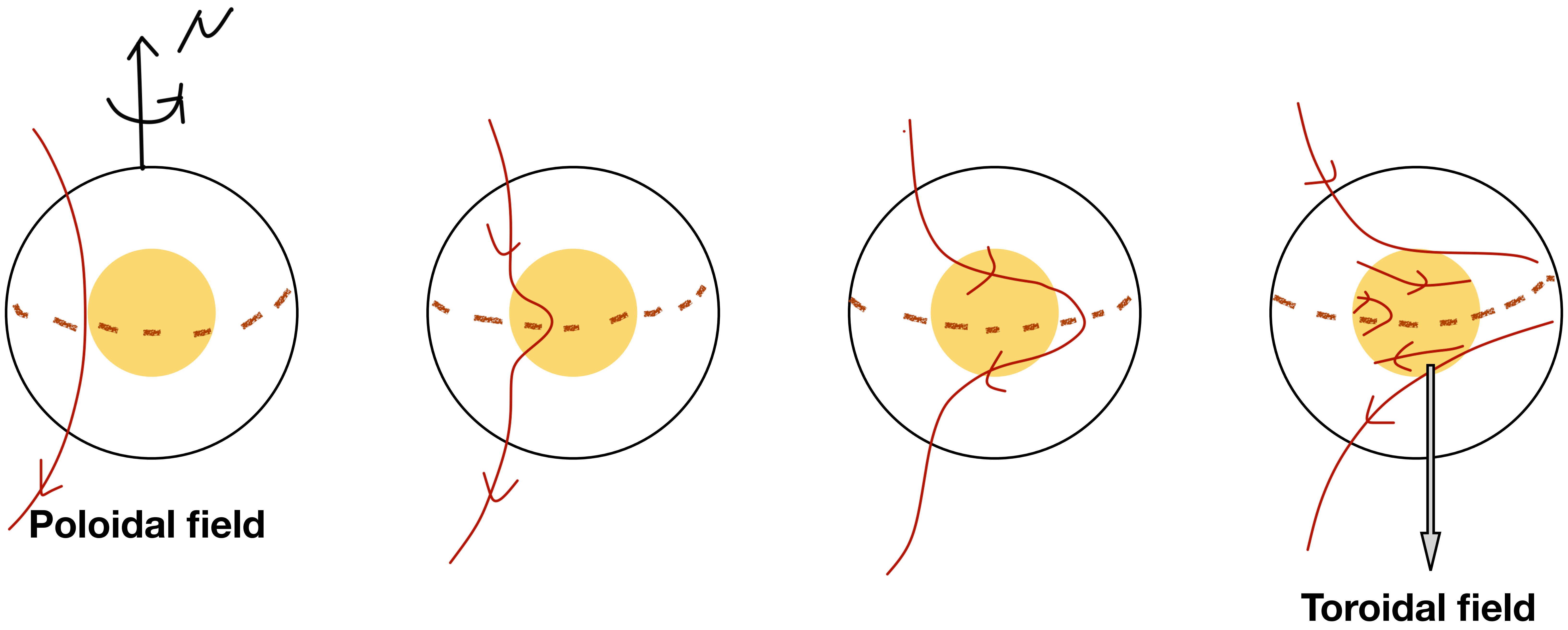
$$\alpha = -\frac{\tau}{3}\langle \vec{u}' \cdot (\underbrace{\nabla \times \vec{u}'}_{\vec{\omega}}) \rangle$$

Ω effect：Differential rotation

From helioseismology（日震学）





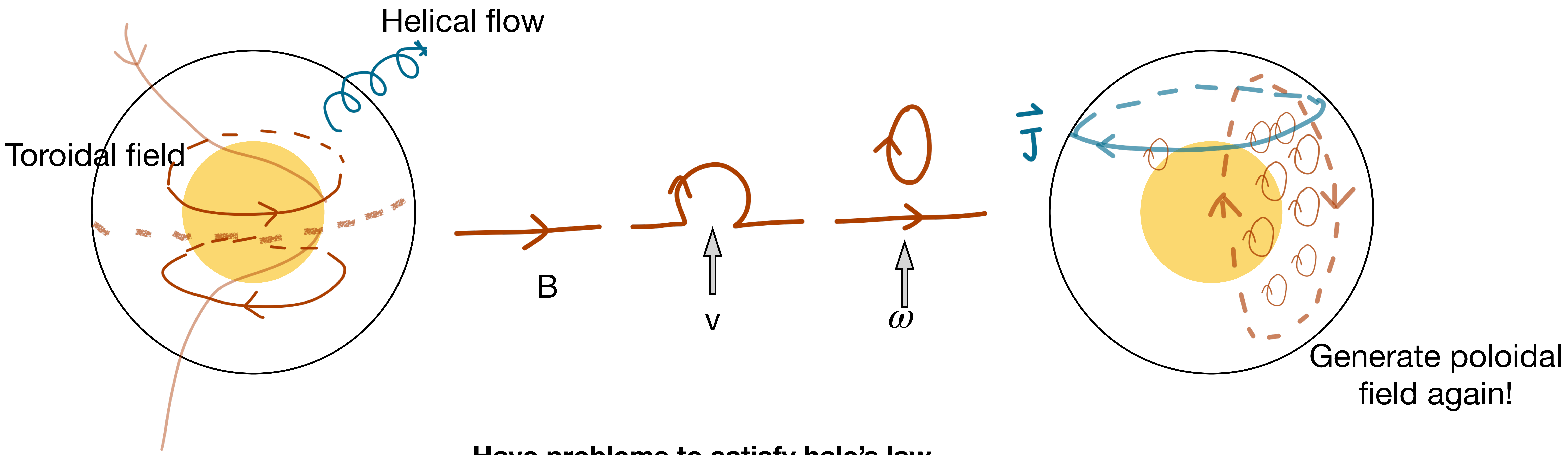


Due to differential rotation，poloidal field produces toroidal field

α effect : Toroidal to poloidal

Coriolis force + convection of flow =>helical flow =>twist toroidal field lines

Much smaller scale; can generate poloidal field from toroidal field



Have problems to satisfy hale's law

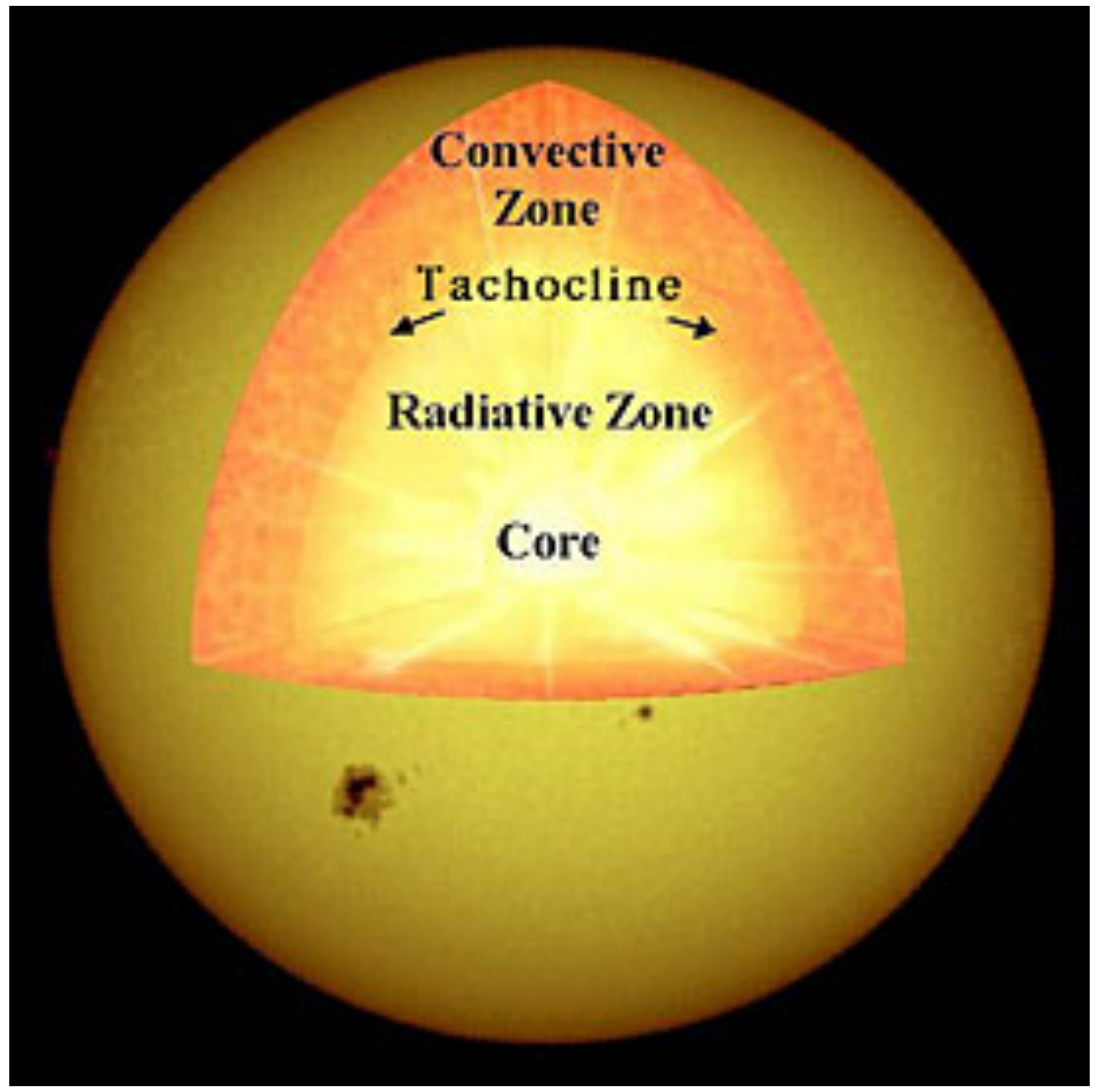
Solar dynamo model

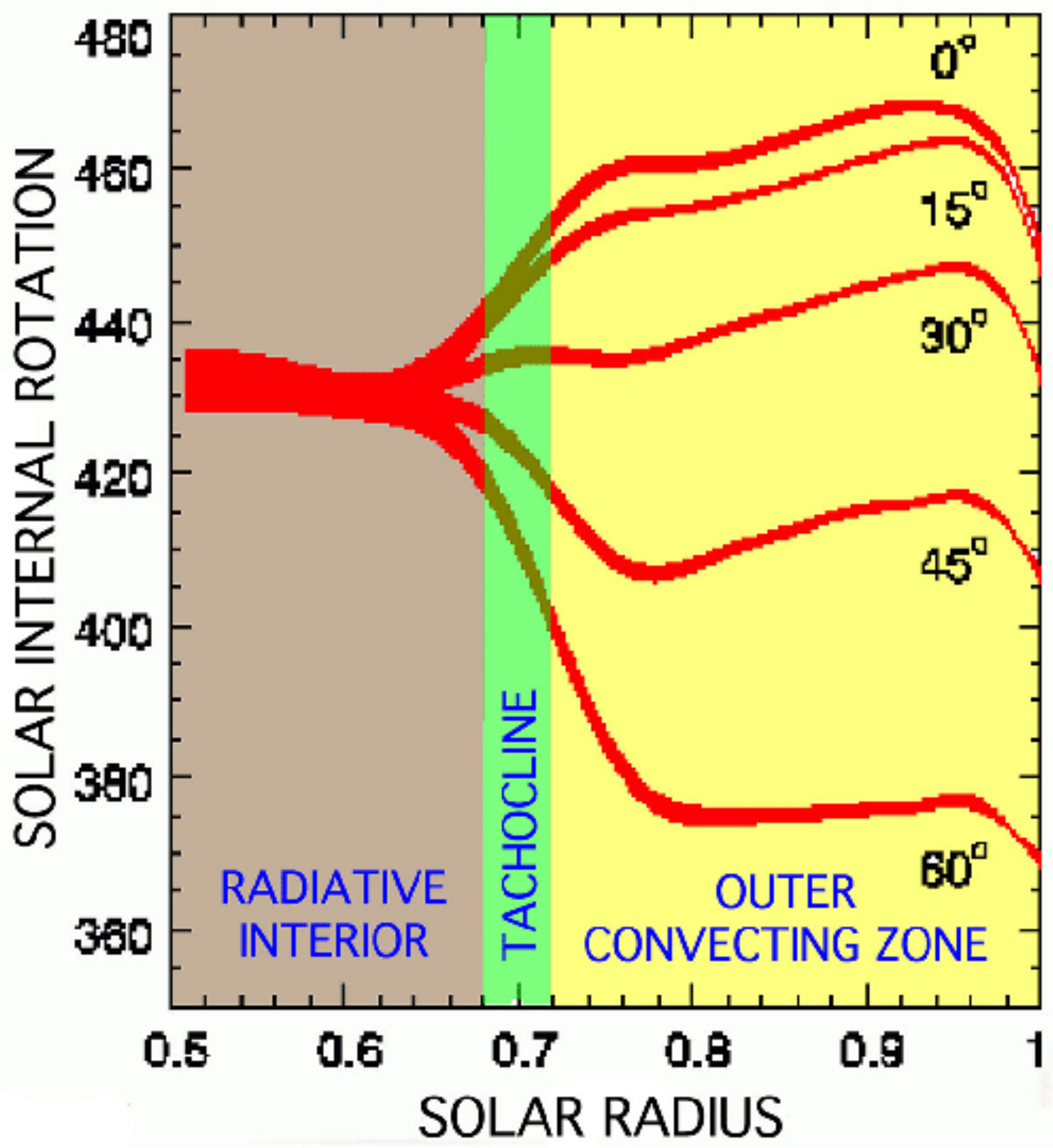
- $\alpha\Omega$ dynamo
- Interface dynamo (Babcock-Leighton mechanism)
- Flux transport dynamo

Babcock-Leighton dynamo: tachocline

Tachocline:

- the transition region of stars between the **radiative interior** and the differentially rotating outer **convective zone**
- at tachocline, the rotation abruptly changes to solid-body rotation
- the striation of tachocline can be detected through helioseismology（日震学）
- **a place that can store strong toroidal field!**





Babcock-Leighton dynamo: magnetic buoyancy

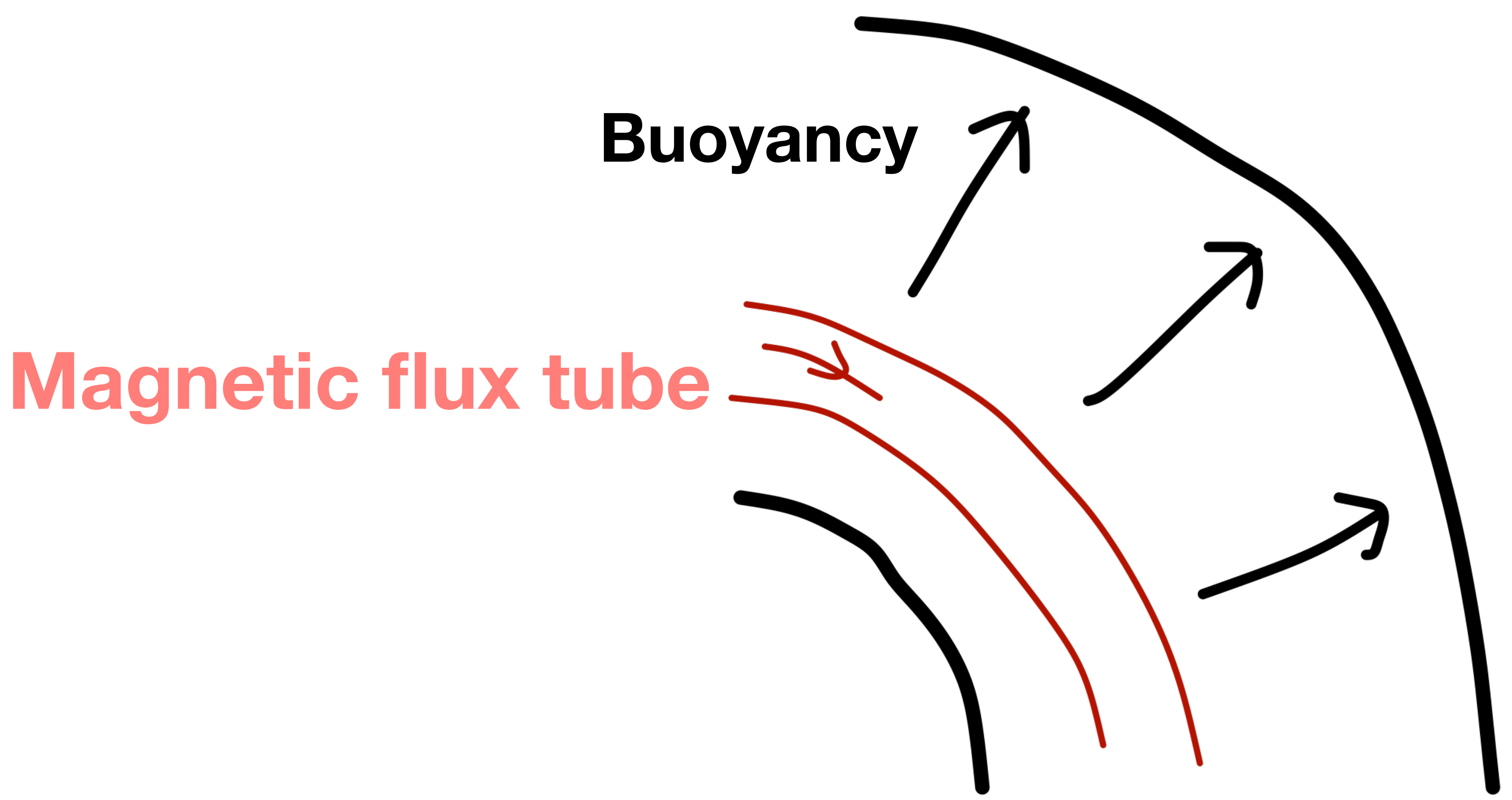
Consider a magnetic flux tube,
Hydrostatic equilibrium requires:

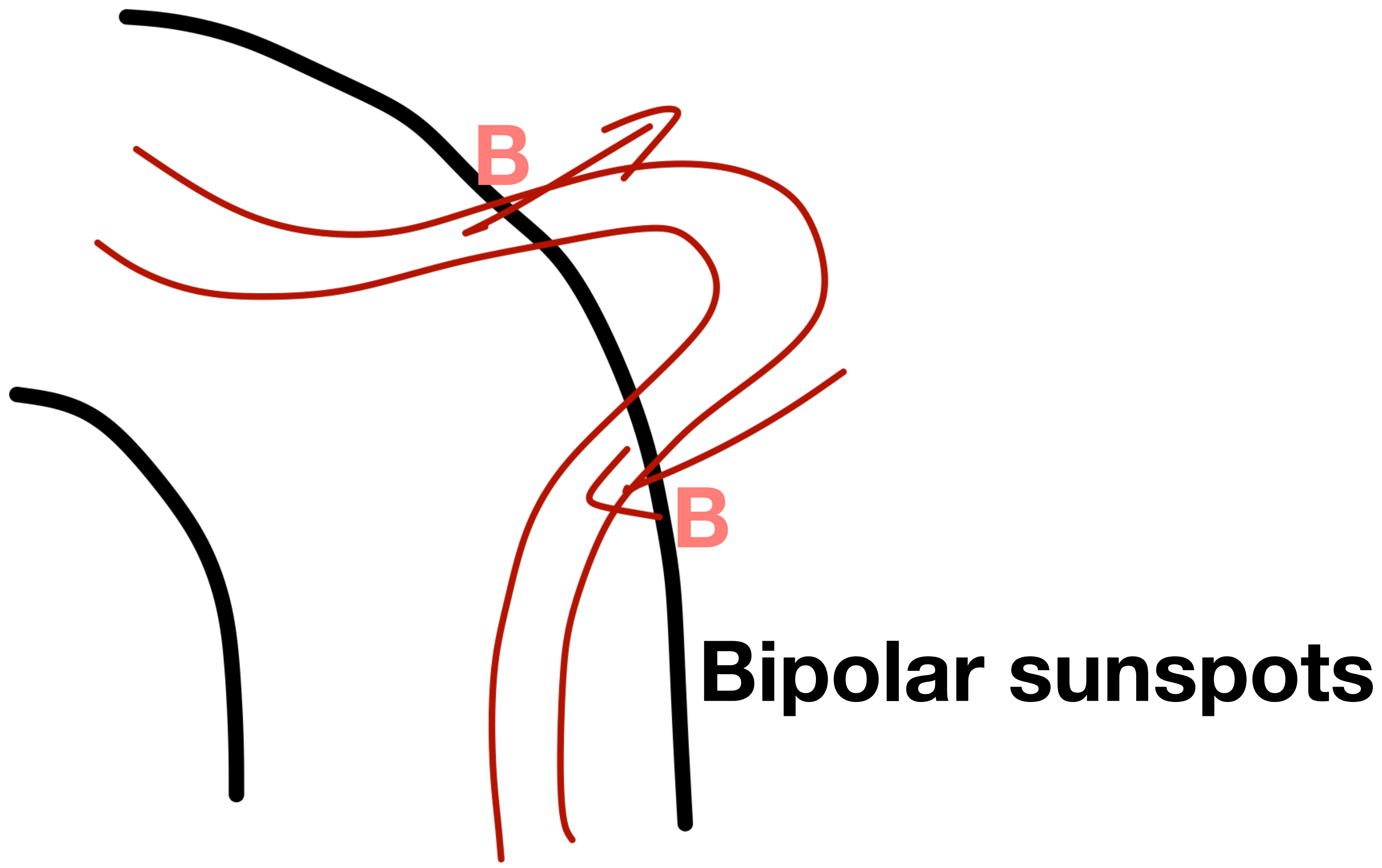
$$p_e = p_i + \frac{B^2}{2\mu}$$

- p_e: external gas pressure
- p_i: internal gas pressure
- $\frac{B^2}{2\mu}$: magnetic pressure

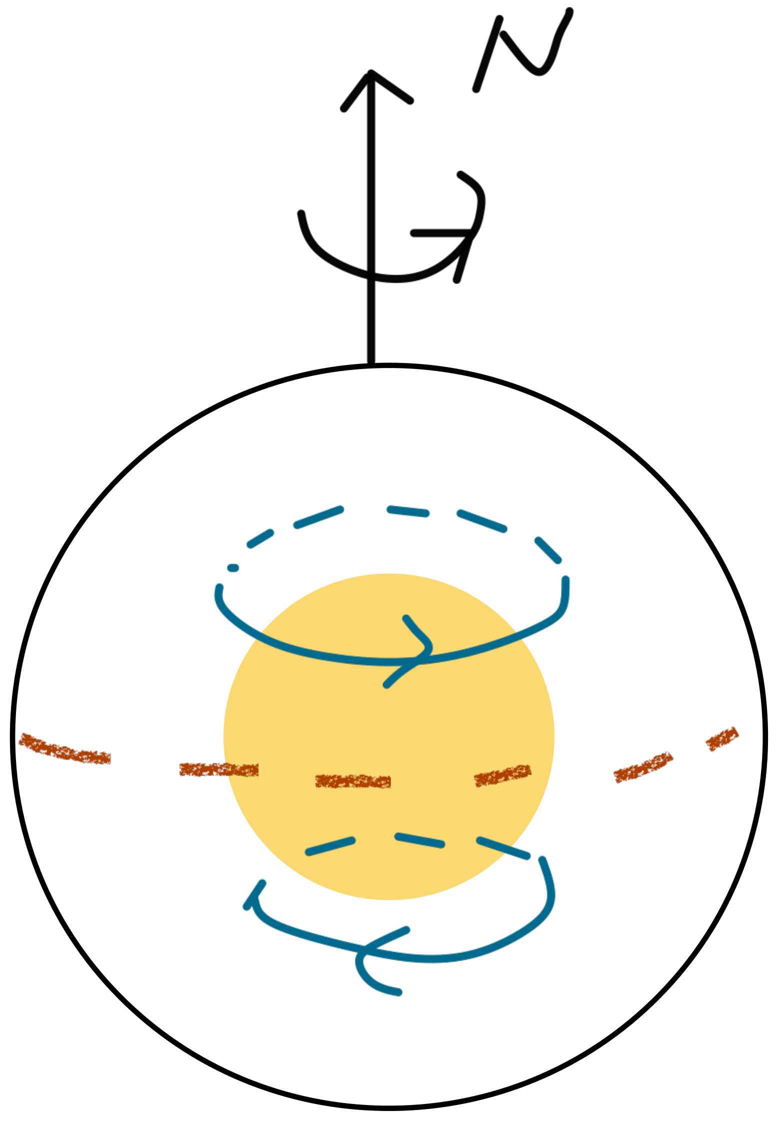
$p_i < p_e$ **+ same temperature** ⇒ $\rho_i < \rho_e$ → magnetic buoyancy!

If the magnetic field is **strong enough**, buoyancy would not be overwhelmed by other motions, such as convection and turbulence



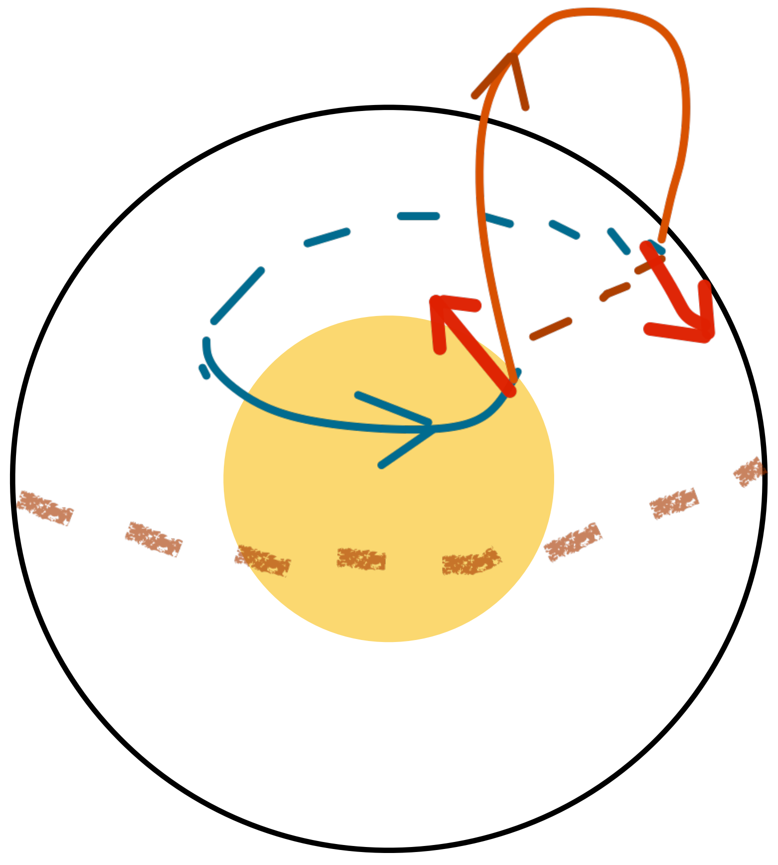


Babcock-Leighton dynamo: process



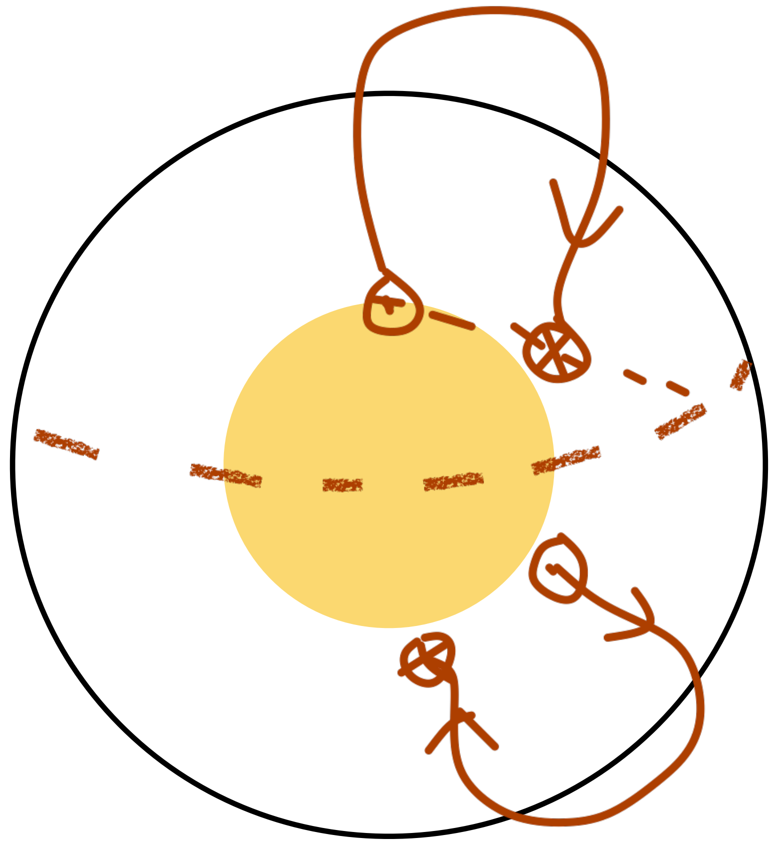
Initial state: toroidal field deep inside

Magnetic field buoyancy



Babcock-Leighton dynamo on surface

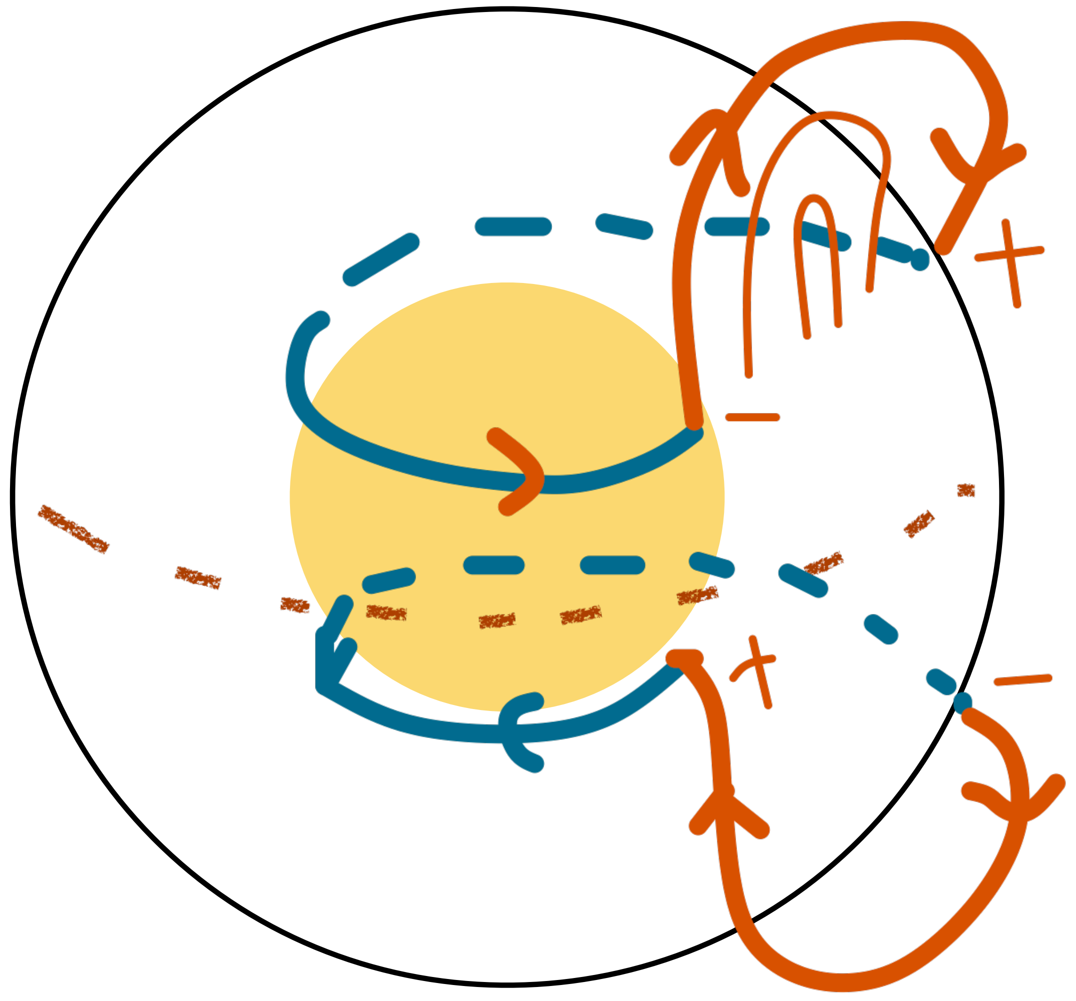
Coriolis force;
Differential rotation

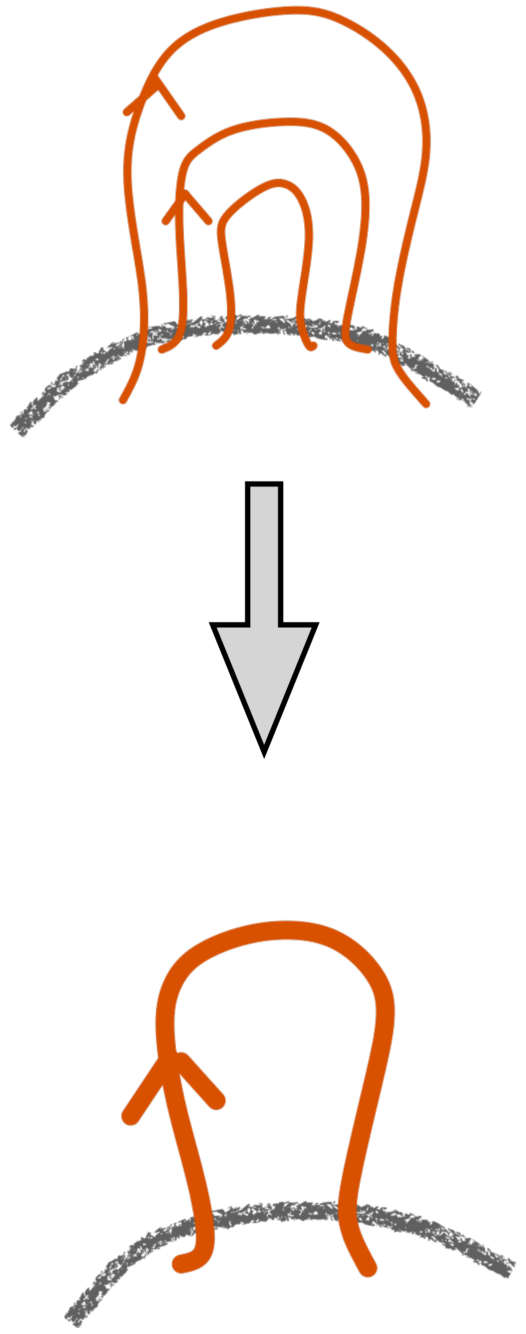


Hale's polarity law

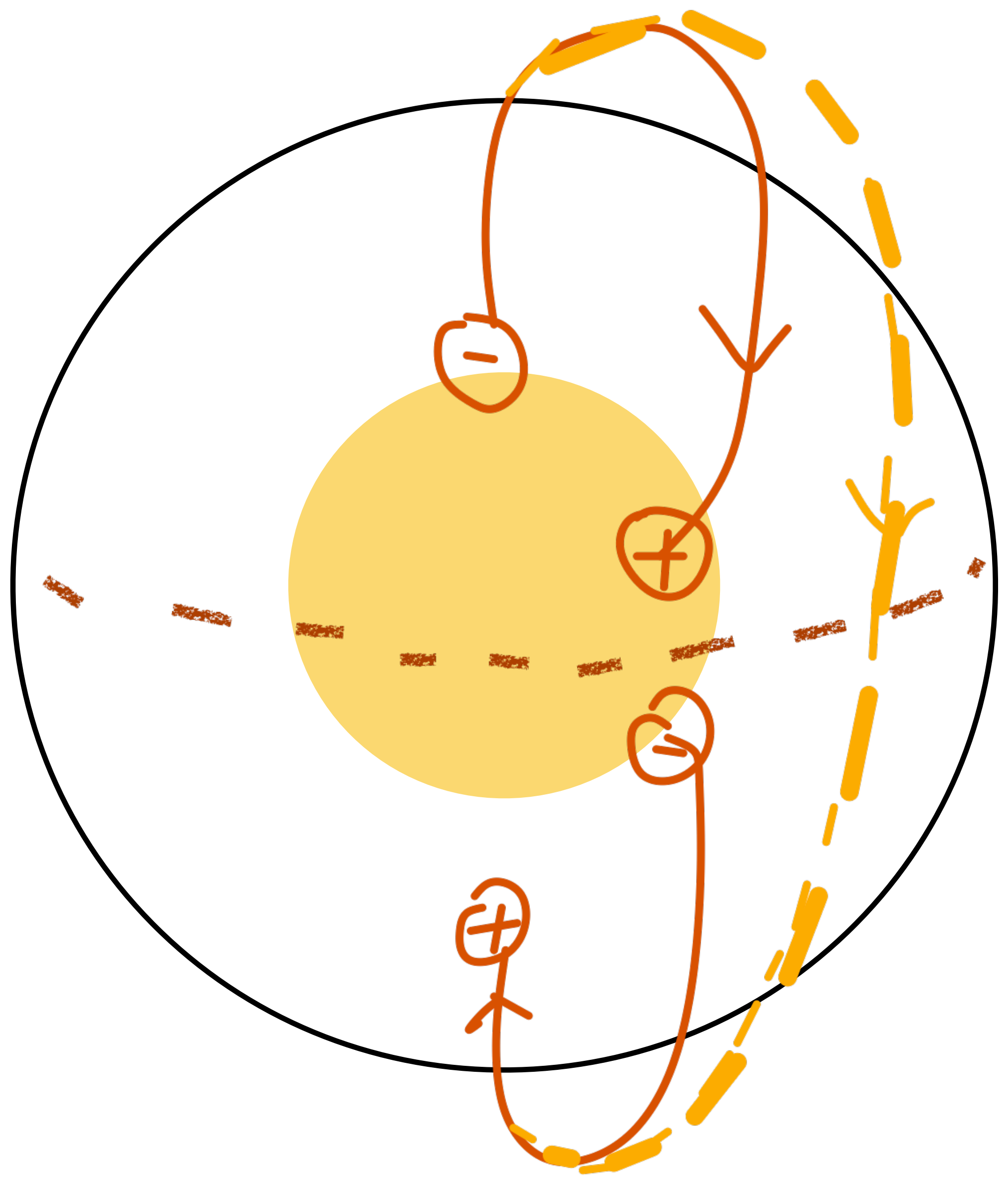
Tilted leading and trailing sunspots

Babcock-Leighton dynamo: toroidal to poloidal





Non-axisymmetric;
Mean polar field is too low



Magnetic reconnection

Solar dynamo model

- $\alpha\Omega$ dynamo
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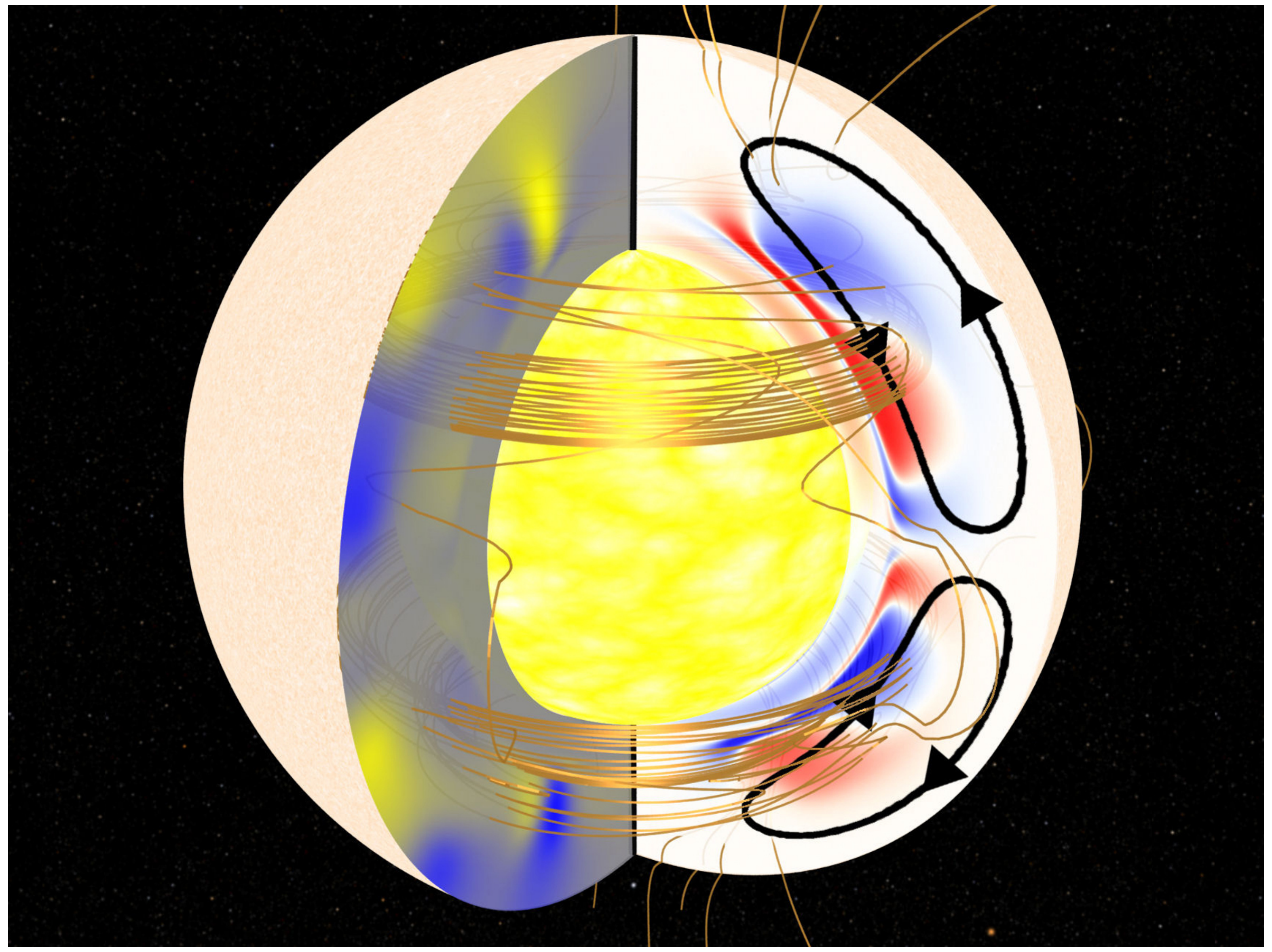
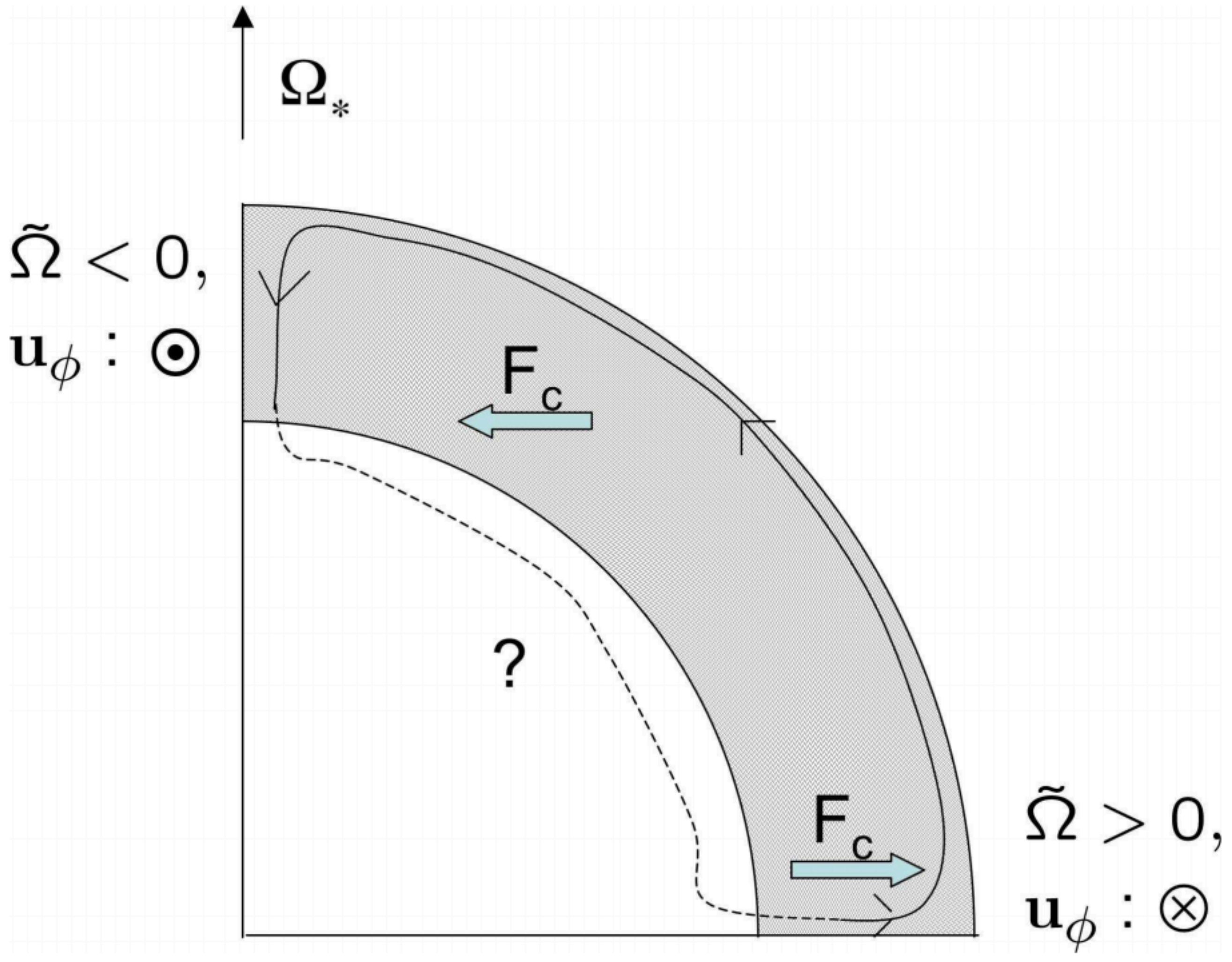
- Flux transport dynamo

Flux transport dynamo: meridional circulation

- Using helioseismology and magnetograms（磁力记录计）: meridional circulation in the outer half of the solar convection zone.
- Large-scale flow, the peak velocity at the surface is 10-20 m/s
- One possible model: gyroscopic pumping. Due to the differential rotation+gradient of Coriolis force.



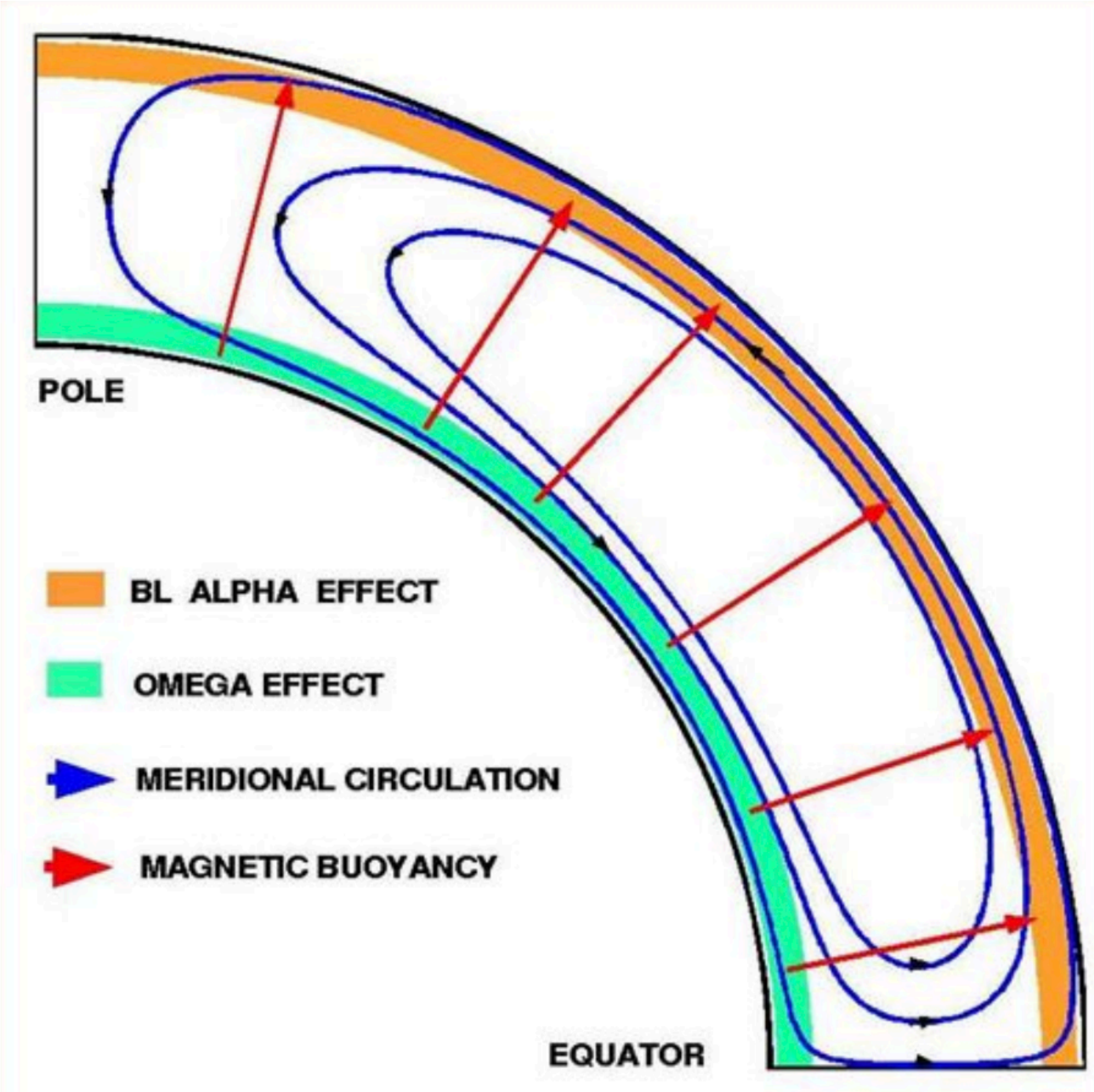


An illustration of gyroscopic pumping model.
Credit to P. Garaud & P. Bodenheimer 2010

Flux transport dynamo

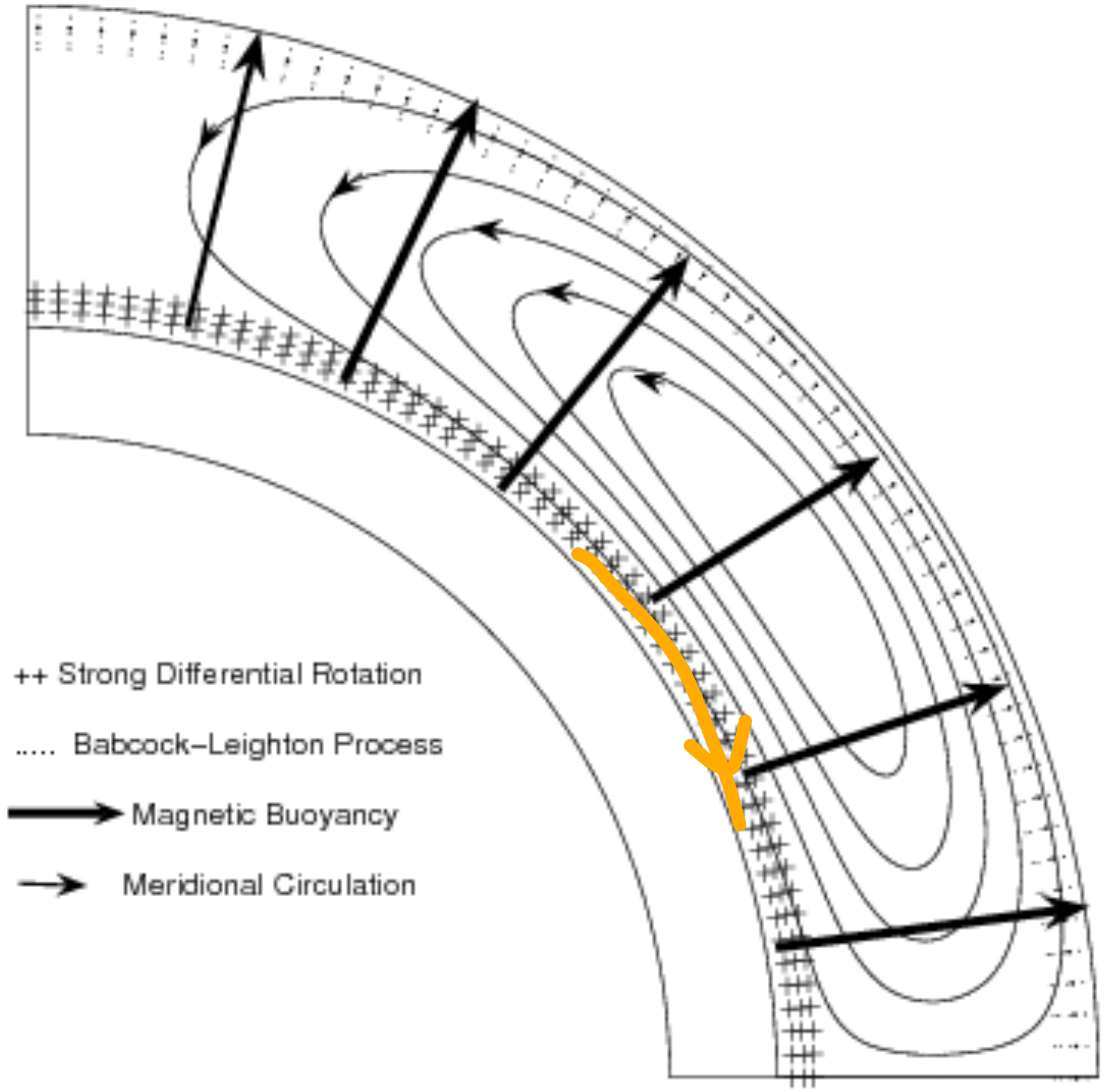
Flux transport dynamo:

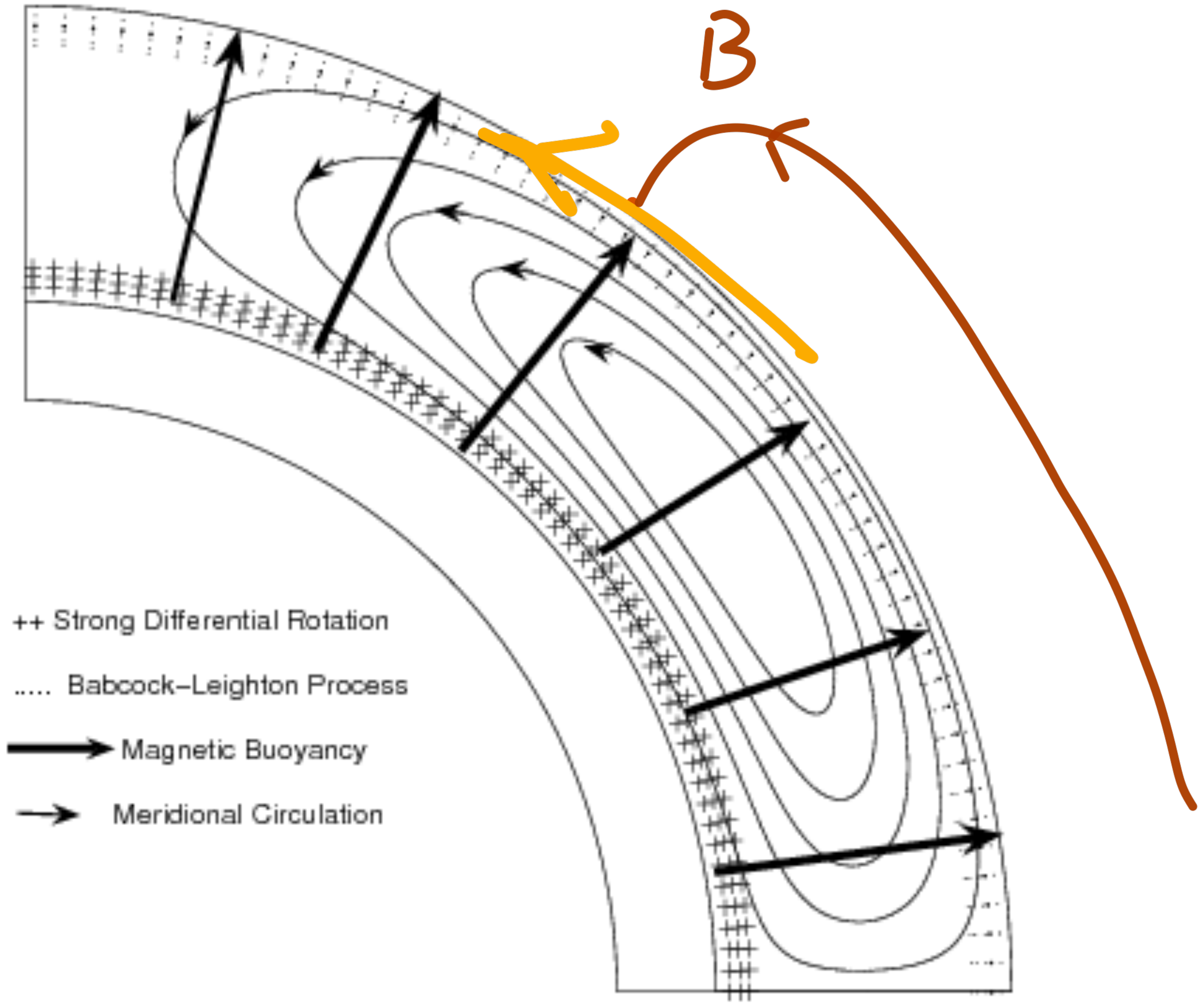
- Differential rotation generates toroidal field
- Babcock–Leighton mechanism turns toroidal field into poloidal field
- the **meridional circulation** produce the migration of magnetic field



Credit to: Arnab Rai Choudhuri

Flux transport dynamo: migration of magnetic field





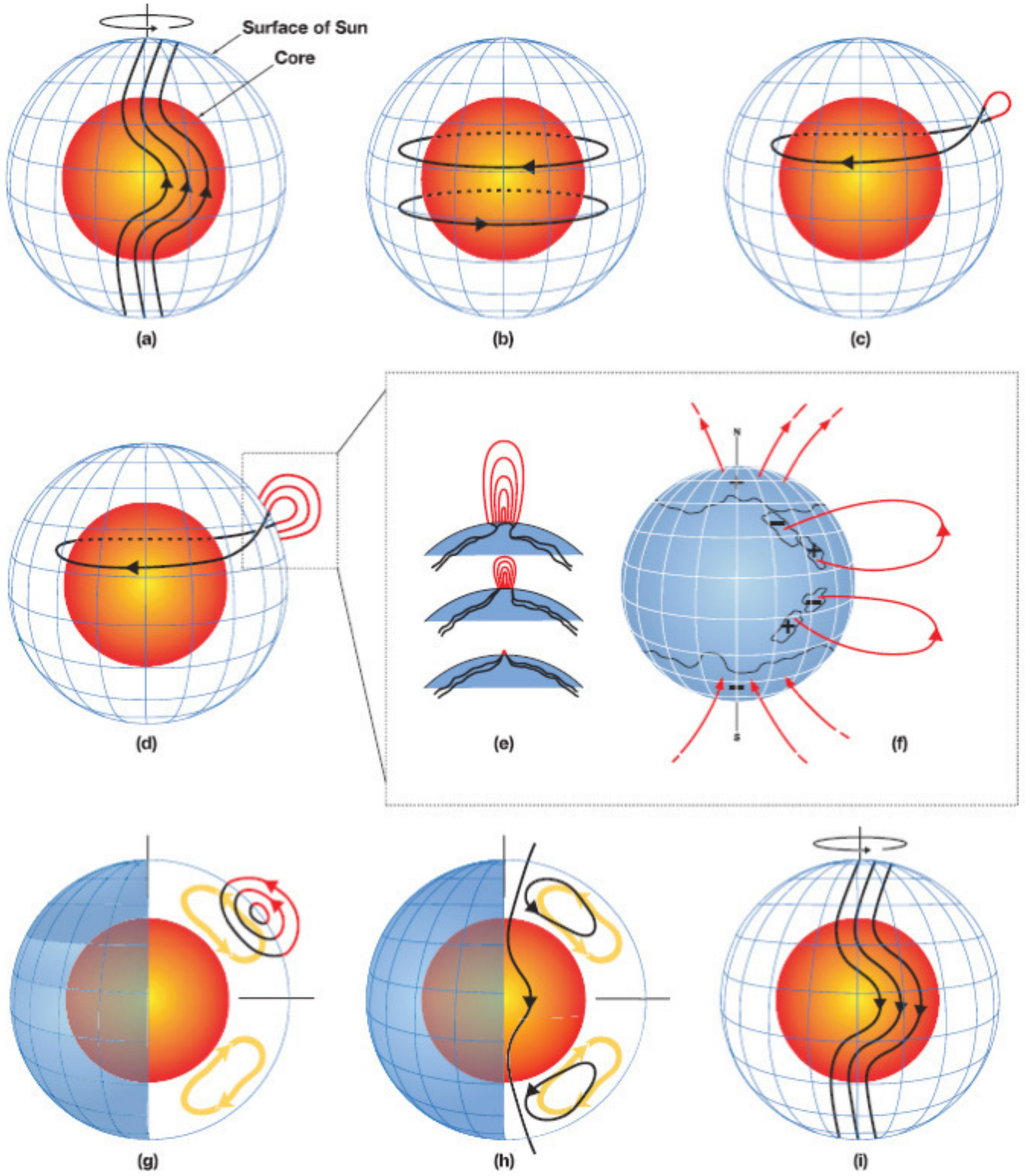
Pull the toroidal field equator-ward

Low latitude sunspots

Pull poloidal field pole-ward

poloidal field on the pole

Flux transport dynamo: the whole picture



→ **Differential rotation**

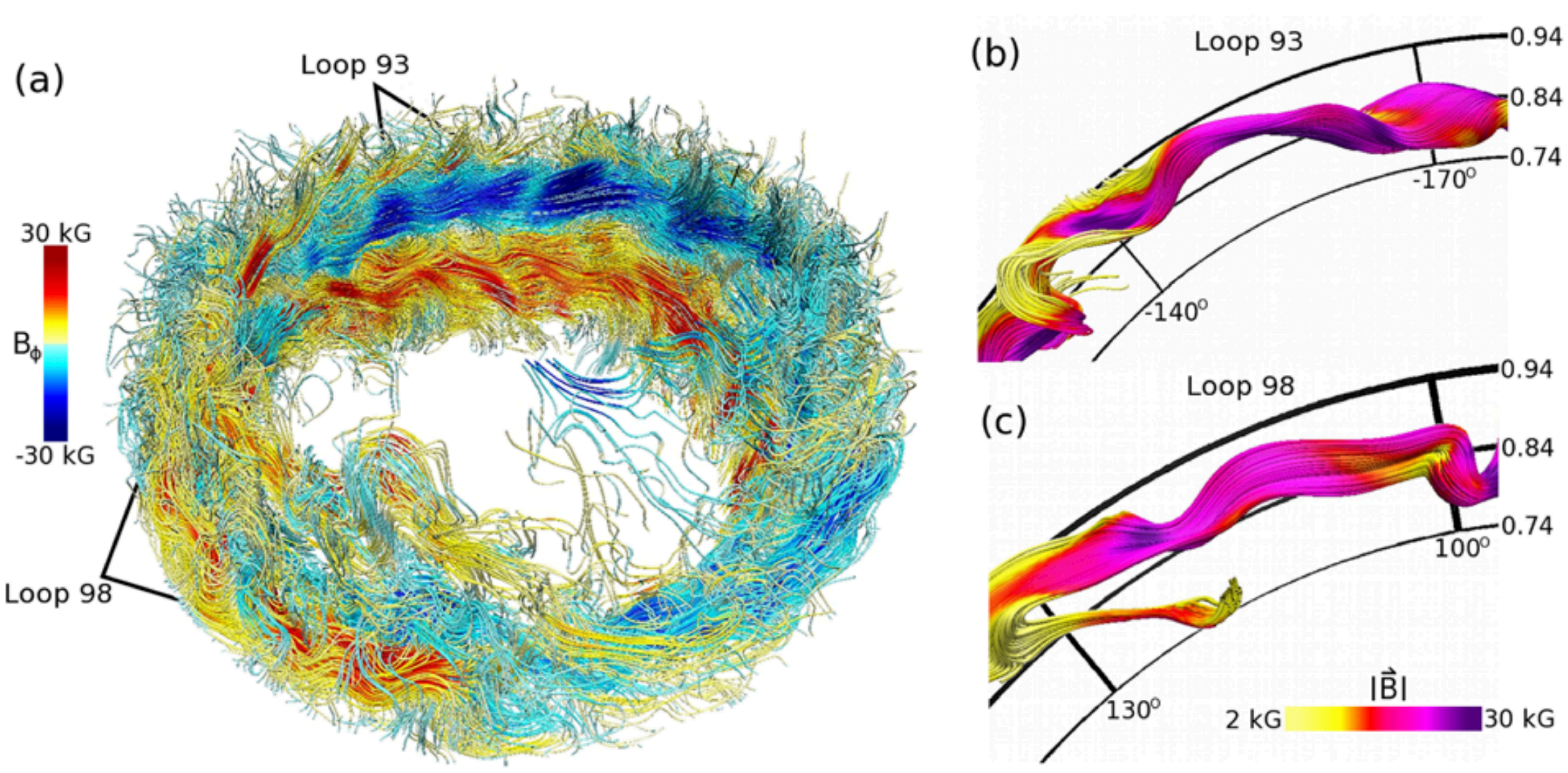
→ **Babcock-Leighton process+ Magnetic reconnection**

→ **Meridian circulation**

Simulations: The formation and rise of rope-like magnetic flux systems.

- Anelastic Spherical Harmonic (ASH) code (solve MHD equation in rotating spherical shells)

- Buoya



Credit to: Nelson et al.(2014)

Summary

- The sunspots shows periodic and spatially changing feature, which is related to the intense magnetic field.
- Differential rotation of sun can produce toroidal field from poloidal field.
- The flux transport dynamo can convert toroidal field to poloidal field due to meridional circulation.

Reference

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- Garaud, P. , & Bodenheimer, P. . (2010). Gyroscopic pumping of large-scale flows in stellar interiors, and application to lithium dip stars. *Astrophysical Journal, 719*(1), 313-334.
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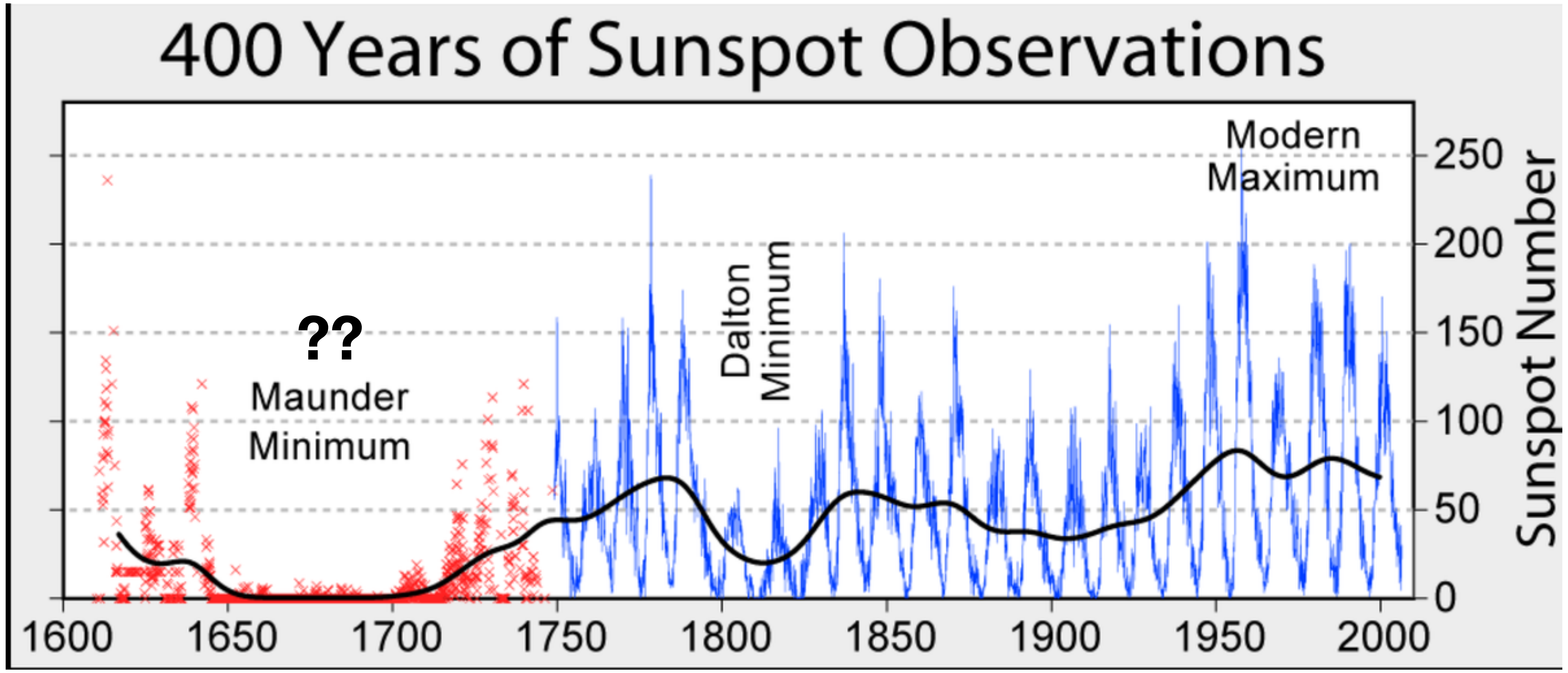
- Parker, E. N. . (1955). The formation of sunspots from the solar toroidal field. *Astrophysical Journal, 121*(121), 491.
- https://en.wikipedia.org/wiki/Sunspot
- https://en.wikipedia.org/wiki/Differential_rotation#Differential_rotation_of_the_Sun

Future issues

- What sets the dynamo period
- Is the tachocline important?
- Babcock-Leighton dynamo a mere by-product model?
- What triggered grand minima?
- …



Simulations: parameters

Table 1
Overview of Dynamo Cases

Case	N_r, N_θ, N_ϕ	Ra	Ta	Re	Re′	Rm	Rm′	Ro	Roc	ν	η	Pm	T_E
D3	97 × 256 × 512	3.28×10^5	1.22×10^7	173	104	86	52	0.374	0.315	13.2	26.4	0.5	61.6
D3a	97 × 256 × 512	5.84×10^5	2.41×10^7	244	154	122	77	0.447	0.295	9.40	18.8	0.5	67.1
D3b	145 × 512 × 1024	1.11×10^6	6.08×10^7	343	273	171	136	0.566	0.257	5.92	11.8	0.5	16.9
D3-pm1	145 × 256 × 512	2.98×10^5	1.22×10^7	149	102	149	102	0.372	0.300	13.2	13.2	1	18.8
D3-pm2	145 × 512 × 1024	3.08×10^5	1.22×10^7	145	101	291	202	0.370	0.306	13.2	6.60	2	13.6
S3	145 × 512 × 1024	7.68×10^8	4.46×10^{10}	8050	5750	4030	2880	0.581	0.262	0.218	0.435	0.5	4.01

Note. — Dynamo simulations at three times the solar rotation rate. All simulations have inner radius $r_{\rm bot} = 5.0 \times 10^{10}$cm and outer radius of $r_{\rm top} = 6.72 \times 10^{10}$cm, with $L = (r_{\rm top} - r_{\rm bot}) = 1.72 \times 10^{10}$cm the thickness of the spherical shell. Evaluated at mid-depth are the Rayleigh number $\mathrm{Ra} = (-\partial\rho/\partial S)(\mathrm{d}\bar{S}/\mathrm{d}r)gL^4/\rho\nu\kappa$, the Taylor number $\mathrm{Ta} = 4\Omega_0^2 L^4/\nu^2$, the rms Reynolds number $\mathrm{Re} = v_{\rm rms}L/\nu$ and fluctuating Reynolds number $\mathrm{Re}' = v'_{\rm rms}L/\nu$, the magnetic Reynolds number $\mathrm{Rm} = v_{\rm rms}L/\eta$ and fluctuating magnetic Reynolds number $\mathrm{Rm}' = v'_{\rm rms}L/\eta$, the Rossby number $\mathrm{Ro} = \omega/2\Omega_0$, and the convective Rossby number $\mathrm{Roc} = (\mathrm{Ra}/\mathrm{Ta\,Pr})^{1/2}$. Here the fluctuating velocity v' has the axisymmetric component removed: $v' = v - \langle v \rangle$, with angle brackets denoting an average in longitude. For all simulations, the Prandtl number $\mathrm{Pr} = \nu/\kappa$ is 0.25 and the magnetic Prandtl number $\mathrm{Pm} = \nu/\eta$ ranges between 0.5 and 4. The viscous and magnetic diffusivity, ν and η, are quoted at mid-depth (in units of 10^{11} cm^2 s^{-1}). The total evolution time T_E for each simulation is given in years. The values for case S3 with the dynamic Smagorinsky SGS model utilize the mean viscosity at mid-convection zone averaged on horizontal surfaces as well as in time. For case S3 using the dynamic Smagorinsky SGS model, the values quoted are based on the time-averaged rms viscosity, conductivity, and resistivity at mid-depth, noting that these diffusion coefficients have near hundred-fold spatial variations.